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A DUAL APPROACH TO MODELLING THE DAIRY INDUSTRY WITH PREDICTIONS ON THE IMPACT OF bST

by

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ABSTRACT

This study employs duality theory to model the dairy industry. Supply and demands for milk, cull cows, feed, labor and veterinary services were simultaneously estimated using Weighted Least Squares. Elasticities and partial adjustments were obtained for the Nation and the following regions, Appalachia, Cornbelt, Northeast, Pacific, Southern Plains and Upper-Midwest. Predictions for the change in quantity of goods demanded and supplied were made assuming a parallel shift in the supply of milk and demand for feed. In conclusion, predictions on the impact of bovine Somatotropin are made assuming all results are correct.

CHAPTER 1

Introduction

American agriculture has undergone a series of technological changes since the early 1900's(Conneman). Each innovation improves productive efficiency, lowers consumer prices and releases labor for work in other sectors. A recent field of research, biotechnology, applies genetic engineering to agricultural products. Inventions include a vaccine for the swine virus, psuedorabies, a frost inhibiting bacteria, a soil organism with insecticidal properties and bovine Somatotropin(bST). bST is anticipated to be the first of the new technologies to be used commercially. It can increase dairy cow milk production from 5 to 40%.

The required approval by the U.S. Food and Drug Administration(FDA) is predicted to occur in 1990. However, Professor Conneman of Cornell University notes "the FDA can delay for no good reason". One issue is the impact on dairy cow health. A second concern centers on whether bST alters dairy products or beef. Proponents argue the second concern is unfounded as bST is a naturally occurring protein which will digest without side effects.

The new invention has proved controversial. Issues raised by bST can be categorized into four groups(Offut and

Kuchler). First, technological advances tend to decrease the number of middle-size farms. A new technology shifts out the production function, lowering costs and consumer prices. If the price received by dairies falls below average total cost, then the firm must eventually drop out of business. Medium sized dairies are typically less efficient than larger ones. They tend to operate at a higher average variable cost. Prices may drop sufficiently low so that these dairies can not operate in the long run. Small dairies are also inefficient, but are not expected to diminish. These part-time operators typically earn income off the farm to offset negative returns.

Whether bST fits this scenario is unclear. The hormone will not require expensive fixed capital, so returns to size may not increase. However, less efficient farms may also be slow to adopt new technologies(Cochrane). Dairies that adopt early, could take their economic profits and buy out slow-to-adopt firms.

Before 1970 agricultural unemployment was low. Farmers typically left the agricultural sector to take advantage of higher paying jobs. After the oil embargo of 1973, foreclosures and involuntary farm unemployment has increased. Another temporary increase in farm unemployment could occur with the introduction of bST as dairy farmers are forced out of business. This study predicts regional disparities in the quantity of labor employed by dairies.

It even predicts that some areas will increase employment after the introduction of bST.

Third, technological change combined with government subsidies led to overproduction. Surplus stocks developed when price dropped below the support price(Fallert, McGuckin, Betts and Bruner). bST will increase production and contribute to surplus stocks. This means government is left with an unsavory political task of taking further reductions in price supports.

Finally, Professor Norton of Virginia Polytechnic Institute argued that the United States is on an international treadmill. To remain competitive, the nation must continue to research and develop new technologies. Today, this is not a problem because most countries subsidize milk production. For example, the United States employs import quotas to maintain subsidized prices. However, a reduction of trade barriers could give advantages to countries who adopt new technologies such as bST.

One objective of this study is to examine the regional winners and losers to bST. A second objective is to model the dairy industry using duality theory. Partial adjustments will be added in an attempt to marry dynamics and duality. Finally, regional estimates of elasticities and partial adjustments will be calculated.

CHAPTER 2

Literature Review

Bovine Somatotropin

At one time researchers used to visit slaughterhouses and scavenge pituitary glands for the bovine growth hormone. Now the genes can be reproduced using ordinary bacteria. The gene is isolated from the bacteria, purified and made available for commercial use. Daily injections of 44 mg will increase milk production between ten and forty percent over cows that are not injected with the hormone(Kalter).

A great deal of biological research has already been conducted. Dairy scientists studied why bST works, how much more milk is produced and differences between farm and laboratory results. Below is a brief summary of their results.

Bodily functions are divided into two categories. The first is homeostasis, it maintains steady state functions such as body temperature, feed intake, digestion and blood glucose levels. The second category, Homeorhesis covers major physiological changes such as growth, adolescence, pregnancy and lactation.

bST and other growth hormones control the homeorhetic function of growth. The earliest examination was in the 1920's. Crude extracts of the bovine pituitary gland were found to increase the growth rate for laboratory rats. Examples of recent studies have shown that growth hormone treatment increased carcass content, protein content and reduced body fat in calves and swine(Boyd; Bauman). Also, administration of bST to adolescent dairy heifers increased mammary parenchyma by 38 percent.

bST increases efficiency of converting nutrients into energy. This allows more proteins to be redirected to the homeorhetic function of lactation (Peel, pg. 1776). Brumby and Hancock reported several metabolic changes. bST alters heart rate, blood sugar and lipid levels. Also, feed intake increased providing energy for lactation and an increased metabolic rate.

The increased milk production occurs two to three days after the first injection. Continued injections heighten output throughout lactation. Furthermore, Methionyl Bovine Somatotropin(MBS), a recombinantly derived strain increased milk production over normal peak production for more than 100 days(Eppard & Bauman).

Scientists are unsure of the adverse effects upon cows. Feed intake adjusts to the increased milk production. Initially, feed intake falls short of increased milk production. However, after ten weeks the energy balance changed to the positive side. Eppard and Bauman's study reported that by week thirty all health parameters were at or better than the control's average. However, it is still unclear whether bST raises the occurrence of disease.

To date, research remains in the laboratory. Fallert, McGuckin, Betts and Bruner listed three situations where farm and laboratory output may differ.

Heat stress was found to lower bST response. Florida trials showed low summer production and good response in the cooler seasons. Arizona research has not found a large difference between the hot summer months and other times. Professor Huber of the University of Arizona, Animal Science Department felt response averaged about thirteen percent per lactation.

The University of Guelph grouped cows by milk production potential; low(11,000-14,300 lb./yr.), medium(14,301-17,600 lb./yr.) and high(17,601-22,000 lb./yr.). Surprisingly, low and medium producing cows respond best. High producing cows increased by three lb./day, while the low to medium group increased seven lb./day(Fallert et.al.).

Age is another issue, some argue that first lactation heifers respond better than older cows. However, the correlation between bST and age is inconclusive. One Canadian study found first lactation heifers do respond better than older cows. At the same time, U.S. trials found contrary evidence with smaller and equal responses(Fallert et.al.).

Finally, Conneman notes, bST still depends on the entire dairy management system. Optimal benefits require "high quality forage, feeding management and careful handling of dairy cows".

Economic Impact of bST

From an efficiency standpoint, bST is the same as any other technological innovation. The production function will shift upward with a corresponding shift downward and right-ward for the marginal cost function. Milk supply shifts out, while other demands and supplies change depending on whether they are complements or substitutes for the new technology.

This paper focuses on shifts in milk supply and demand for feed, labor and veterinary services. The speed of adjustment is estimated with a modified partial adjustment model.

Price of Milk

Hallberg and Parsens examined regional and national price changes. Using a linear programming model, the objective function maximized net social payoff. This is the sum of producer's and consumer surplus minus interregional transportation cost.

The blend price of milk was predicted for the nation and the following regions; Northeast, South Atlantic, South Central, Plains, Mountain, Southwest and Midwest. The model did not allow interregional trade. Thus the predictions on price differences are not equal to transportation cost.

The supply of milk was arbitrarily increased by fifteen percent. Supply was assumed to shift in a parallel fashion. increasing output by a constant percentage for all prices. The selection was arbitrary because of the uncertainty about the actual increase in milk supply.

Hallberg and Parsen's results use comparative statics. After the initial shock, the price of milk and related goods may change. For example, bST may lower milk price such that a neighboring region can transport the good and still receive a premium. If so, then interregional trade occurs until price differences equal transportation cost.

The nation's blend price was predicted to drop by 14.3 percent. Regional price changes ranged from -8.8 percent in the South Atlantic to -19.0 percent in the Southeast.

Magrath predicted a milk price decrease from twelve to thirty-five percent depending on the demand elasticity for milk and size of the production increase. Demand elasticity was varied from -0.1 to -0.4 with increments of -0.1. The thirty-five percent drop occurred at a thirty percent increase in output and -0.1 demand elasticity.

A particular expense curve(PEC) was used as proxy for supply. The curve orders farms from the most cost efficient to the least and then sums with respect to output. This results in a concave upward supply curve. The dual to the

PEC is what Magrath calls a partial output curve(POC). This is simply a production function that orders farms by milk output.

The first step in estimating the supply shift was to estimate a Cobb-Douglas partial output curve. The production response was calculated by increasing the Cobb-Douglas constant which implies marginal product curves are increased by the same percentage. Supply was later derived using profit maximization.

Fallert, McGuckin, Betts and Bruner predicted the blend price 1990 to 1996 under the assumption that bST receives approval in 1990. With four government price support scenarios, the model predicted a benchmark price assuming no availability of bST and milk price with the availability of bST. Under the first scenario, government price support for manufacturing grade milk is $10.10 /cwt. Milk prices for 1990 were assumed to be $11.13 /cwt. After 1990, without bSt the blend price rises steadily to $11.40 /cwt., while the presence of bST drops milk price to $10.80 /cwt.

The Food Security Act of 1985 limits price supports to $9.60/cwt. by 1990. With this support reduction, both benchmark and bST availability prices start at about $10.60/cwt. The non-adopted price increases to $11.20/cwt. by 1996. With bST adoption the price remains at $10.60/cwt.

The third scenario complies with the $9.60/cwt. ruling and follows with two fifty-cent annual reductions. Both blend prices begin at approximately $10.60/cwt. The non-bST price increases to $11.20/cwt., while with bST price drops to $9.80/cwt. in 1992. Afterwards, bST milk rises to $10.20/cwt. by 1996.

The final scenario maintains the $11.10/cwt. price support through 1996. Both prices start at $12.40/cwt. and remain through 1996.

The model employed by Fallert et. al. was created by Westcott. Four equations estimated milk cow inventories, milk production per cow, commercial milk use and farm milk price. Adoption rates were based on farmers' expected net-returns from bST. Profits vary with prices, so adoption rates must also vary amongst scenarios. Data was obtained from Cost of Production(Betts), a survey of U.S. farms.

Adoption Rate

bST offers rewards of lower consumer prices and could release labor for work in other sectors. However, rapid adoption shortens the time for adjustment. Kalter notes, "government stocks of surplus dairy products would jump at a high cost to the Federal treasury". Rural areas would suffer with a jump in unemployment, declines in land values and less demand for services.

Alternatively, Cochrane argues slow adoption allows larger more efficient dairies to buy out smaller, typically less efficient farms. Goldschmidt argues that the tendency towards larger farms lowers the quality of life in local communities.

Lesser, Magrath and Kalter predicted an adoption rate that surpasses 85% after three years. An S-shaped curve was applied to the data obtained from mail and personal interviews. To appear more realistic, the surveys were issued with a mock bST advertisements. Consistent and thoughtful answers were assured by repeating questions throughout the survey.

Adoption rates were estimated for both injection and implants of bST. A logistic function suggested by Pindyck and Rubenfeld yields an r^2 above 0.8. Results state at least eighty percent will experiment with bST after three years. Full adoption is at sixty-five percent for injections and seventy percent for implants. The author emphasized that survey results will be dampened if farmers were quoted a higher bST price.

Fallert, McGuckin, Betts and Bruner predicted adoption rates with four price scenarios. The range of diffusion was between 45 and 70 percent after six years. Scenario one, a minimum price support of $10.10/cwt. yields a 55 percent diffusion. The second scenario(price support at $9.60/cwt.) predicted an adoption rate at fifty percent by 1996. The forty-five percent and seventy percent adoption rates resulted from scenario three and four. The third scenario calls for a 9.60 support price and two annual reductions of fifty cents. The final scenario maintains the current $11.10/cwt. support price and yields a 70% adoption rate.

Adoption rates were dependent on net returns from bST. A floor of two dollars in revenue for every dollar of bST was required. Because production will increase and the price of milk will fall, adoption rates will vary. The production response was 8.4 lbs./day. The statistic was calculated by taking seventy-five percent of the laboratory responses. The last twenty five percent is compensation for farm conditions. The price of bST was assumed at 24 cents per cow per day. No variations in region, size of farm or management practices were considered. Finally, the adoption rates were tempered after hearing responses from about 100 researchers.

This study does not look to contradict past studies in modelling the dairy industry, but to add to the information. The major methodological contribution is the addition of partial adjustments to all netputs. No study to the knowledge of this author has used the ad hoc manner proposed by Nerlove. Lau appears to be the first to suggest such a procedure but empirical attempts to marry dynamics and duality theory has been limited to a dynamic optimization process(Epstien). The advantage to this study over dynamic optimization is the tremendous ease in application.

Empirical contributions include regional estimations of supply and demand elasticities as well as partial adjustments. Most studies in this field are only for the United States and/or yield unexpected results. For the most part, this study's results yield expected signs on elasticities. However, the difference in elasticities between regions is questionable. Also, the cross price elasticities used to predict the impact of bST are questionable. This can be explained by noting that bST and a reduction in price supports should lower milk price. However, milk price followed an upward trend for over ten years. Dairy farmers may not have produced at the lower prices predicted by bST in quite awhile. Technological change, increasing price supports and a change in resource endowments relevant to the dairy producer may bias the elasticities to reflect the current trend.

Except for the special case when partial adjustment coefficients are equal to one, this study is like Kalter et al and Fallert et al. The results will not differentiate amongst size of dairies. Equity questions about the distribution of dairy farms are not answered by the previous two authors. However, this study unlike others can examine regional disparities in adoption.

CHAPTER 3

Methodology

Duality Overview

Economists have used various models to simulate the U.S. dairy industry. Hallberg used a linear programming model. Kalter employed a particular expense curve. Supply curves have been estimated by Dahlgran and Huy et. al.

This study will employ duality theory to model dairy farm behavior under the assumption of profit maximization. A partial adjustment model is incorporated to predict the adjustment path resulting from an output shock. The model is then used to predict the price of milk after the introduction of bST.

The indirect profit function is used to derive supply for milk and cull cows along with demand for feed, veterinary services and labor. The profit function itself is not directly estimated. Instead, all output supply and input demand equations are estimated jointly.

In general, dual or indirect functions start with the "end product" of constrained or unconstrained maximization. It is consistent with economic theory only if the underlying production function contains the same properties as assumed in profit maximization. This chapter verifies that relationship.

The chapter begins with an introduction to netputs. Primal and dual approaches are later derived. Finally the paper will give a brief summary of different functional forms.

Netput Approach and Derivation of Net Supplies

A netput is simply defined as a good or commodity(Russell and Wilkinson). Its value is the amount supplied less the quantity demanded. If a netput is positive(negative) then the commodity is a net output(input). For example, dairies use milk as a net output since the quantity supplied exceeds the quantity demanded. For this study,

$$(3.1) \quad V = (v_{cull}, v_{feed}, v_{labor}, v_{milk}, v_{vet})$$

where:
- V - vector of netput commodities, called a netput bundle
- v_i - netput for commodity i $= y_i - x_i$
- y_i - gross amount of commodity i produced
- x_i - gross amount of commodity i consumed
- i - index for cull cows, feed, labor, milk and veterinary services

Two assumptions go with the netput bundle. First, V contains the origin. This allows the producer to shut down. Second, if any v_i is greater than zero, then there must be at least one v_k less than zero. That is, no net output can be produced without a resource. Figure 3.1 gives a example of a technology set for milk and feed.

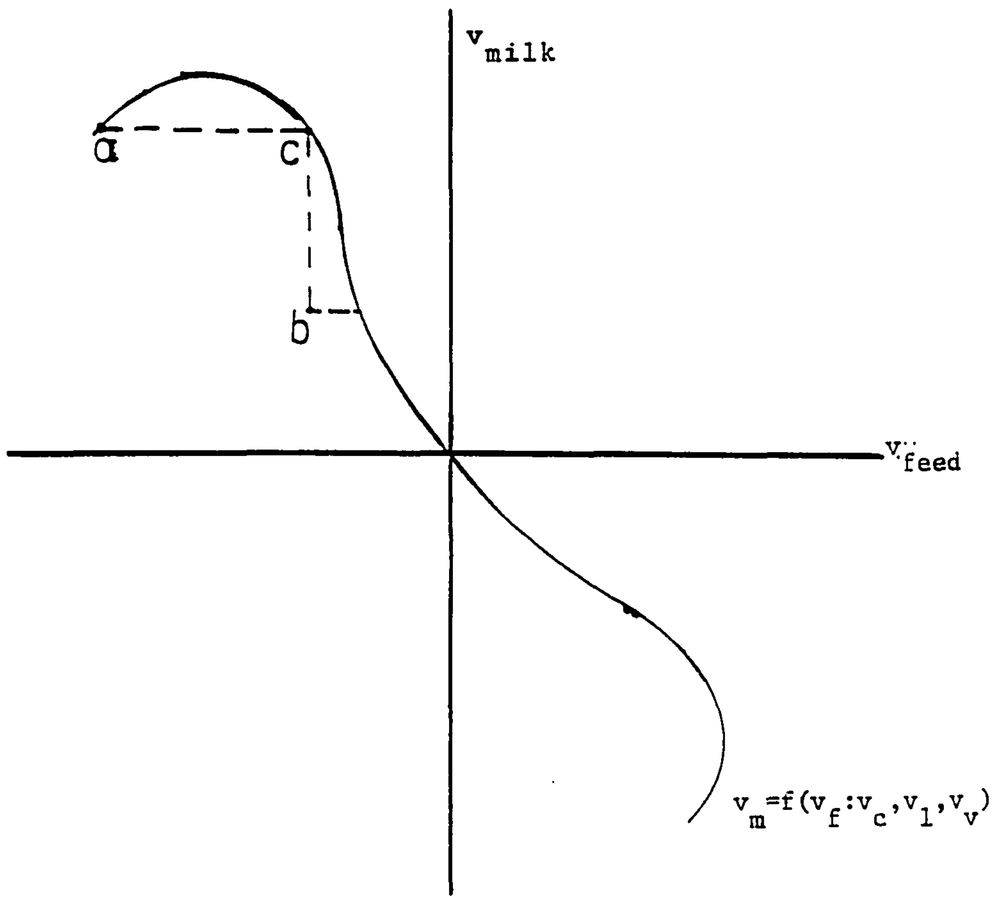

Figure 3.1. Example of a Technology Set for Milk

The technology set includes the solid line and the area below. It is the set of all netput bundles given quantities for cull cows, labor and veterinary services. Efficient points fall along the downward sloped portions of the boundary. More specifically a bundle V^* is technologically efficient when there does not exist an alternative bundle V such that $v_i \geq v^*_i$, and $v_k > v^*_k$ for some k where i and k stand for cull cows, feed, labor, milk and veterinary services. Inefficient points lie along an upward slope of the border and below the boundary of the technology set. For example, point a is inefficient since the same quantity of milk can be produced with less feed. Point b is inefficient since output can be increased with the same or lower quantity of feed. However, c is efficient, output can not be increased without increasing the quantity of feed.

The netput perspective recognizes that a price change can convert net outputs to net inputs. For example, if the price of labor becomes extremely high all other prices held fixed, then a farmer would sell his skills. Of course, net outputs can become net inputs. If the price of milk rose ceteris paribus, then dairies would hold off selling cull cows keeping them as an input for milk production.

Primal Approach

Profits are equal to total revenue minus total cost. In equation form,

(3.2) $\Pi = P \cdot V = \sum_{i=1}^{n}(p_i y_i - p_i x_i)$

i - cull cows, feed, labor, milk and veterinary services

Two assumptions are made to assure profit maximization.

1. Boundedness: There exists a netput bundle V^* such that $v^*_i \geq v_i$ for all i. This assures an upper-limit on all outputs so profits can be calculated.

2. Regularity: The technology set contains it's boundaries. So a profit maximum can be found along the edge of the technology set.

The netput bundle is expressed as a production function by taking the supremum or maximum of v_i given a vector of remaining netputs. For example,

(3.3) $v^*_{cull} = \sup(v_{cull}: v_{feed}, v_{labor}, v_{milk}, v_{vet})$

Uniqueness of v_i is proven by showing the contrary can not hold true. Suppose for the moment, v_{ia} and v_{ib} maximize output given the same vector of remaining netputs. Also suppose that v_{ia} is not equal to v_{ib} i.e. $v_{ia} = f(V^{-i})$, $v_{ib}=f(V^{-i})$ and $v_{ia} \neq v_{ib}$. The V^{-i} stands for all goods except i.

If $v_{ia} > v_{ib}$ then v_{ia} is the maximum value for the technology set. Likewise if $v_{ib} > v_{ia}$ then v_{ib} is the supremum. But this contradicts the assumption that v_{ia} and

v_{ib} both maximize output. So the production function offers only one value (v_i) for a given vector of all other netputs.

The producer's problem is,

$$(3.5a)\quad \text{Max } \Pi = P \cdot V \text{ s.t. } F(V) = 0$$
where:
$$F(V) \equiv v_j - f(V^{-j})$$
$$\equiv \text{implicit production function}$$

In order to solve this maximization problem, it is reformulated in Lagrangean form as:

$$(3.5b)\quad L\Pi = P \cdot V + \lambda(F(V) - 0)$$

First order conditions(FOC) are:

$$(3.5c)\quad dL\Pi(V,\lambda)/dV_i = Pi + \lambda dF(V)/dV_i = 0$$
$$(3.5d)\quad dL\Pi(V,\lambda)/d\mu = F(V) = 0$$

A unique solution is assured if the underlying technology set is continuous and strictly convex. Second order conditions which require the Hessian matrix to be positive definite can then be used to assure profit maximization. The example below is the Hessian for milk supply, F stands for the implicit production function and subscripts stand for derivatives with respect to the ith good.

$$(3.5e)\quad |H| = \begin{vmatrix} 0 & F_c & F_f & F_l & F_v \\ F_c & \lambda F_{c,c} & \lambda F_{c,f} & \lambda F_{c,l} & \lambda F_{c,v} \\ F_f & \lambda F_{f,c} & \lambda F_{f,f} & \lambda F_{f,l} & \lambda F_{f,v} \\ F_l & \lambda F_{l,c} & \lambda F_{l,f} & \lambda F_{l,l} & \lambda F_{l,v} \\ F_v & \lambda F_{v,c} & \lambda F_{v,f} & \lambda F_{v,l} & \lambda F_{v,v} \end{vmatrix}$$

where, c-cull cows, f-feed, l-labor and v-vet

As demonstrated in Figure 3.2 , profit maximization occurs at the tangency between an equal profit hyperplane and the technology set. The equal profit hyperplane holds profit and prices fixed while varying the level of netputs. The equal profit hyperplane appears as the isoprofit line in Figure 3.2 where only two netputs are considered. The line is derived as,

$$\Pi_o = p_{milk}v_{milk} - p_{feed}v_{feed}$$

or,

$$v_{milk} = \Pi_o/p_{milk} - (p_{feed}/p_{milk})v_{feed}$$

Note that profit maximization can occur only along technologically efficient portions of the set.

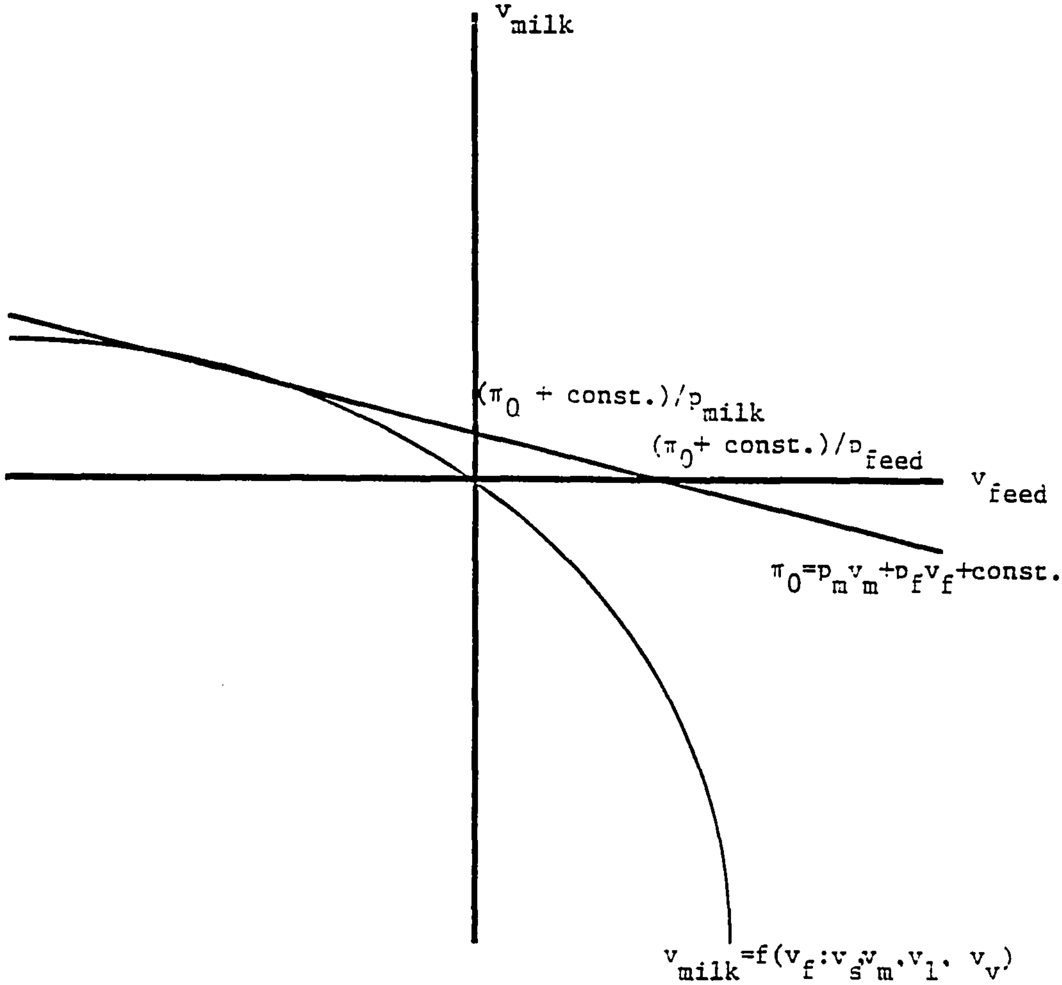

Figure 3.2. Example of Profit Maximization

Hotelling's Lemma

The envelope theorem allows the derivation of demand and supply from a dual function. Hotelling's lemma is an application of the envelope theorem to the profit function. Examples of other applications are Roy's theorem which is used to derive marshallian demand from the indirect utility function and Shephard's lemma which is used for the cost function. Each of these applications is merely a special case of the envelope theorem.

Net supplies are derived by taking the total differential of the first order conditions (3.5c) and (3.5d) and solving the system of equations for optimal quantities of v_i. If the optimal quantities are less than zero then the commodity is demanded for the production process. If positive, then the commodity is supplied.

The indirect profit function is obtained by substituting the net supplies into the profit equation.

(3.6a) $\Pi(P) = \Sigma_i p_i \cdot \phi_i(P)$

where,

- P - vector of prices
- $\Pi(P)$ - profit equation
- $\phi_i(P)$ - net supplies for the ith commodity
- i - cull cows, feed,labor, milk and veterinary services

Thus the indirect function gives optimal profits for a given vector of prices.

Hotelling's lemma states, the first derivatives of the profit function is equal to net supplies(See Appendix A for proof)

$$(3.6b)\quad \frac{d\Pi(P)}{dp_i} = \Sigma_j \frac{d\phi_j(P)}{p_j dp_i} + \phi i(P)$$

Beattie argues "the left-hand side is the change in the maximum value of $\Pi(P)$ allowing all $\phi_i(P)$ to adjust. Whereas, the right-hand side is the change in $\Pi(P)$ holding all $\phi_i(P)$ fixed."

Comparative Statics

Comparative statics relationships are derived by taking the total differential of the vector of net-supply functions. For this study the matrix is,

$$(3.7)\quad D^2\Pi(P) = \begin{bmatrix} \frac{dv_c}{dp_c} & \frac{dv_c}{dp_f} & \frac{dv_c}{dp_l} & \frac{dv_c}{dp_m} & \frac{dv_c}{dp_v} \\ \frac{dv_f}{dp_c} & \frac{dv_f}{dp_f} & \frac{dv_f}{dp_l} & \frac{dv_f}{dp_m} & \frac{dv_f}{dp_v} \\ \frac{dv_l}{dp_c} & \frac{dv_l}{dp_f} & \frac{dv_l}{dp_l} & \frac{dv_l}{dp_m} & \frac{dv_l}{dp_v} \\ \frac{dv_m}{dp_c} & \frac{dv_m}{dp_f} & \frac{dv_m}{dp_l} & \frac{dv_m}{dp_m} & \frac{dv_m}{dp_v} \\ \frac{dv_v}{dp_c} & \frac{dv_v}{dp_f} & \frac{dv_v}{dp_l} & \frac{dv_v}{dp_m} & \frac{dv_v}{dp_v} \end{bmatrix}$$

Own price effects are along the diagonal (i=j), while cross-price effects are along the off-diagonal. Also, the matrix is symmetric because Young's theorem requires: $d\Pi(P)/dp_i dp_j = d\Pi(P)/dp_j dp_i$.

Conditions on Profit Function

The first and second order conditions for maximization of the profit function result in the following restrictions on the indirect profit function(See Appendix B for proofs).

1. Weak Monotonicity - profits are non-decreasing (increasing) in net-output (input) prices.

2. Linear Homogeneity - the dual function is homogeneous of degree one in all prices i.e. $\Pi(tP)=t\Pi(P)$.

3. Convexity - The profit function is convex upwards(downwards) for output(input) prices.

4. Continuity - The profit function is continuous in prices.

Duality Theory and Implications for the Technology Set

The model is,

$$\Pi = \Pi(p_{cull}, p_{feed}, p_{labor}, p_{milk}, p_{vet}) \tag{3.8a}$$

First derivatives yield net supplies.

$$\begin{aligned} v_{cull} &= \phi(p_{cull}, p_{feed}, p_{labor}, p_{milk}, p_{vet}) \\ v_{feed} &= \phi(p_{cull}, p_{feed}, p_{labor}, p_{milk}, p_{vet}) \\ v_{labor} &= \phi(p_{cull}, p_{feed}, p_{labor}, p_{milk}, p_{vet}) \\ v_{milk} &= \phi(p_{cull}, p_{feed}, p_{labor}, p_{milk}, p_{vet}) \\ v_{vet} &= \phi(p_{cull}, p_{feed}, p_{labor}, p_{milk}, p_{vet}) \end{aligned} \tag{3.8b}$$

Researchers typically estimate these net supply systems with seemingly unrelated regression models. However, only satisfaction of the regularity conditions guarantees a proper technology set. This section verifies that the assumed technology set will have the same properties as proposed in the primal approach.

Lau roughly categorized three approaches for determining the characteristics of an underlying technology set (Diewert, 178). The first category is based on conjugacy correspondence by Fenchel and Rockafellar. A second approach uses symmetric duality between gauge functions. Shephard, Gorman, McFadden, Hanoch and Jacobsen all provided major contributions. Finally, Diewert, Uzawa and McFadden examine the set of production possibilities and the support function.

The discussion follows Varian who credits the arguments to McFadden and Diewert's work on support functions. The argument involves an introduction to the Weak Axiom of Profit Maximization (WAPM), derivation of the underlying technology set and graphical analysis of the function's shortcomings.

The weak axiom of profit maximization (WAPM) uses actual price-quantity combinations selected by a firm. The firm's behavior is symbolized with equation 3.9. That is, given a price vector(P_i) the chosen bundle must maximize profits. Otherwise, a firm would have selected a different combination. Thus, any other bundle(V_j) represents lower profits or can not be produced.

(3.9) $P_iV_i \geq P_iV_j$ for all i,j

where: P_i - observed price vector

V_i - chosen netput bundle

i,j - quantity vectors of cull cows, feed, labor, milk and veterinary services.

Points chosen by WAPM must coincide with the indirect profit function. Both assume profit maximization and free competition. In addition, optimal quantities are derived from a given vector of prices. So both approaches must yield identical quantities.

WAPM derives the appropriate slope for the net supply functions. For example, let (p_{milk}, v_{milk}) and (p'_{milk}, v'_{milk}) be two observations. Now, $p_{milk}(v_{milk}-v'_{milk})\geq 0$ since $p_{milk}v_{milk}\geq p_{milk}v'_{milk}$ and $p'_{milk}(v_{milk}-v'_{milk})\geq 0$ Adding the two together, $(p_{milk}-p'_{milk})(v_{milk}-v'_{milk})\geq 0$ which implies taking limits $dp/dv\geq 0$

WAPM predicts the underlying technology set as regular, monotonic and convex. This is because the axiom assumes the outer bound of the netput set as the technology set. That is,

$$(3.10)\quad OB(V) = (V_i: P_i \cdot V \leq P_i \cdot V_i) \text{ for all } i$$

where: V_i - chosen quantity vector of netputs
V - set of all possible netput bundles
P_i - vector of prices
$OB(V)$ - outer bound of the set V

Weak monotonicity is proven by choosing two bundles V_1 and V_2. If $V_1 \leq V_2$ then, at least one dimension(product) $V_1 < V_2$. By weak monotonicity, $V_1 \leq V_2$ if $\Pi(V_1) \leq \Pi(V_2)$. We can separate both bundles into a vector of goods which are equal, V_0, and a vector of goods where $V_1 < V_2$. Thus:

$$(3.11)\quad \Pi(V_1) = P_i \cdot V_0 + P_j \cdot V_{1j} \leq \Pi(V_2) = P_i \cdot V_0 + P_j \cdot V_{2j}$$
$$P_j \cdot V_{1j} \leq P_j \cdot V_{2j}$$
$$V_1 \leq V_2$$

The technology set is concave in netputs. Let V_1, V_2 and V^* stand for three netput bundles on the boundary of the technology set. Also, let P^* be the profit maximizing price vector for V^*. WAPM implies $P^*V^* \geq P^*V_1$ and $P^*V^* \geq P^*V_2$. Thus, $P^*V^* \geq tP^*V_1 + (1-t)P^*V_2$ which implies $V^* \geq tV_1 + (1-t)V_2$.

By definition the set OB(V) is regular if and only if it contains all it's boundary points. To show regularity, two bits of information are required. First is completeness. Choose any two profit maximizing points V_i and V_j. Both must be on the technology set's surface, otherwise net outputs could increase without further use of inputs and profits would thereby increase. Select one commodity v_1 out of each bundle. Using the equation for the production function(3.3) v_i can be written as, $v_i{}^1 = \sup(v^1 : v_i{}^2, v_i{}^3, \ldots, v_i{}^n)$ and $v_j{}^1 = \sup(v^1 : v_j{}^2, v_j{}^3, \ldots, v_j{}^n)$. Completeness states that $v_i{}^1 \geq v_j{}^1$, $v_i{}^1 \leq v_j{}^1$ or $v_i{}^1 = v_j{}^1$. In other words, given a vector of n-1 netputs, a value for the n^{th} netput exists and can be compared with other v^n. The second piece of information is that OB(V) must be continuous. Varian notes that if OB(V) is concave then it is continuous. The two characteristics, continuity and completeness imply OB(V) is closed.

As shown in Figure 3.3, OB(V) is given with three price vectors. As the number of price combinations approaches infinity, the curve takes on the three conditions of regularity, monotonicity and concavity [1].

1/ Lau provides a proof requiring only quasi-convexity in the technology set.

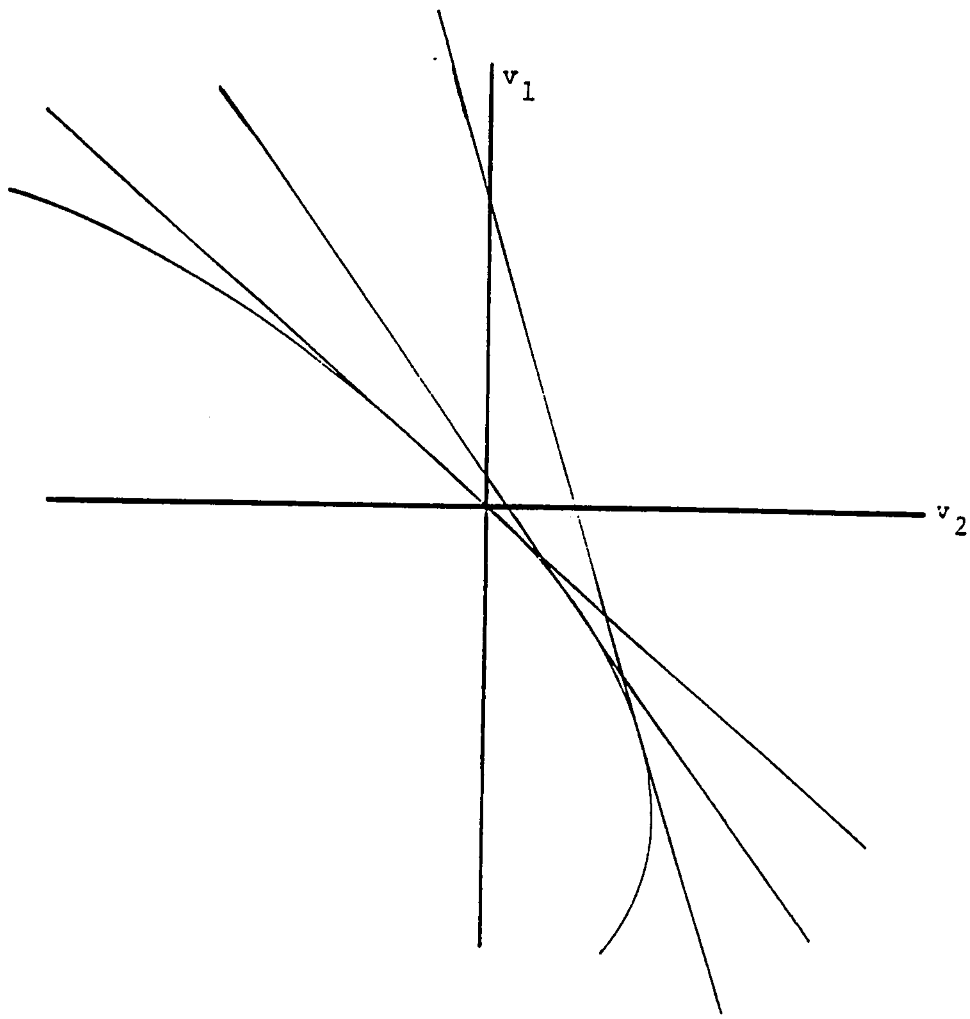

Figure 3.3. Example of How the Outer Bound of V Approximates the True Technology Set

So an estimated profit function specifies the underlying technology as closed, monotonic and concave. But the actual technology set can assume a variety of shapes(see figure 3.4 below). The actual boundary is the thick black line. The outer bound includes the netput set along with the cross-hatched areas. This means that the profit function may include areas that are not technologically feasible.

The graph on the left represents culling or buying replacement cows so that milk production will increase in a stepwise fashion. On the right is an example of a netput function which includes a portion of increasing returns to scale. Neither example can be correctly estimated by the profit function.

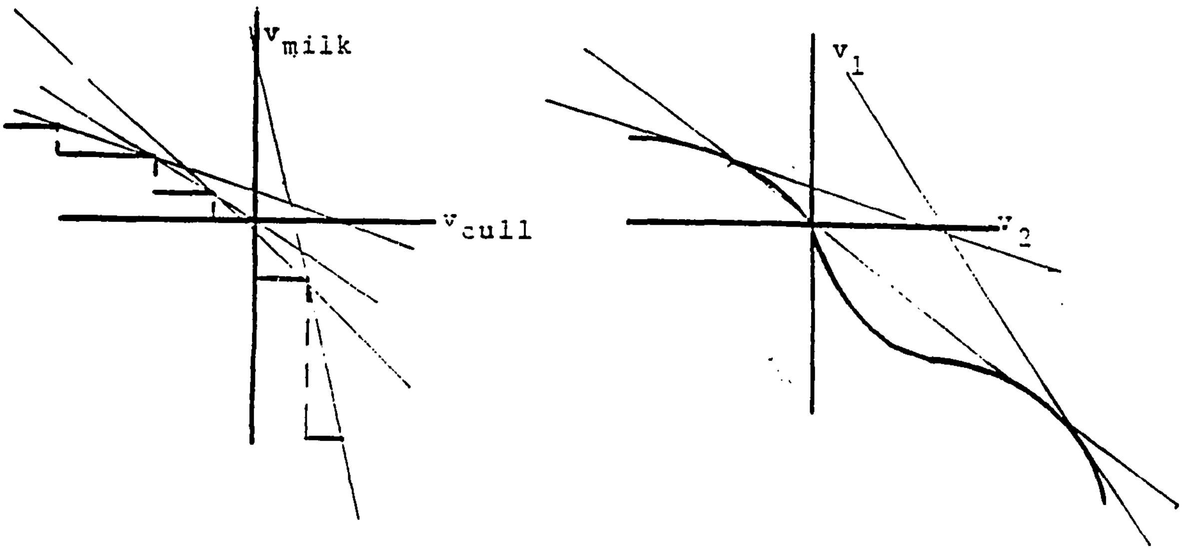

Figure 3.4. The Outer Bound of V Will Trace Out a Weakly Concave Technology Set Even if the True Technology is Different

The principle advantage to dual functions is the ease in calculation. First derivatives yield net supplies and the second derivatives give comparative static results. Amongst duality options, the indirect profit function was chosen because, it easily applies to multiple net outputs.

A major concern is meeting all the regularity conditions specified by indirect functions. Properties of homogeneity, convexity, symmetry and monotonicity should be tested. Symmetry and homogeneity were imposed while tests for monotonicity and convexity were conducted.

Flexible Form and the Normalized Quadratic

A variety of functional forms can estimate the profit function. The choices can be broken into two categories, global and local flexibility. The global flexible form attempts to match the true function over a range of data. Local flexibility may match a true function at a particular point, however Wohlgenant, note that "this point is in general unknown and may not even be in the range of the data(Caves and Christensen and White)". Moreover, the approximation error from a Taylor's series can be large for small departures from the (unspecified point of expansion(White)."

One attempt at global flexibility uses the Sobolev norm. This type of functional flexibility minimizes a modified expected error over m derivatives(Gallant).

$$(3.12)\quad \|e\|_{m,r} = \sum_{i=0}^{m} \int_a^b [d^i e(x)/dx^i]^\rho f(x)dx\}^{1/\rho}$$

where:
$0<a<b<2\Pi$
$1\leq\rho\leq\infty$
$e(x) = g(x) - g(x:\theta)$, the difference between the actual and estimated observation
$f(x)$ is the density function

If the function poorly estimates the k^{th} derivative $0\leq k\leq i$, then $\|e\|$ grows in size. An accurate estimation implies $\|e\|$ approaches zero. According to Gallant "a form that is Sobolev flexible can consistently estimate elasticities over all prices, it will not reject spuriously and will have negligible prediction bias." The major Sobolev flexible function is a Fourier series. However, questions remain on the accuracy of this expansion for empirical studies(Weaver).

Local flexibility is sometimes referred to as Diewert flexibility requires a function $g(x:\theta)$ to provide a second order local approximation to $g(x)$. This is at the point x_o, where θ_o is a corresponding choice of parameters(Gallant). In equation form,

$$(3.13a)\quad g(x_o:\theta_o) = g(x_o)$$
$$(3.13b)\quad \left.\frac{dg(x:\theta_o)}{dx}\right|_{x=x_o} = \left.\frac{dg(x)}{dx}\right|_{x=x_o}$$
and
$$(3.13c)\quad \left.\frac{d^2g(x:\theta_o)}{dxdx'}\right|_{x=x_o} = \left.\frac{d^2g(x)}{dxdx'}\right|_{x=x_o}$$

Such forms have enough parameters to allow elasticities of substitution to assume any value[2]. However, Diewert flexibility can not guarantee an accurate estimation of elasticities for just any price. Consistency is assured only at the point of expansion. Extrapolating away from the data must be done with caution.

Aside from flexibility are regularity conditions, linear homogeneity, monotonicity and convexity. Most Diewert flexible forms locally satisfy monotonicity and convexity. Global satisfaction requires Sobolev flexibility.

2/ Another definition of Diewert flexibility is:

$$\lim_{h \to 0} : g(x_0+h) - g(x_0+h:\theta) | / |h|^2 = 0$$

A modified normalized quadratic was chosen for estimation. The function is Diewert flexible and is a second order Taylor's expansion, [3]

$$(3.14)\quad \Pi(P{:}Z) = 1/2[\ 1\ P'\ t\ ]\cdot\Gamma\cdot\begin{bmatrix} 1 \\ P \\ t \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} \gamma_{o,o} & \gamma_{o.c} & \gamma_{o,f} & \gamma_{o,l} & \gamma_{o,v} & \gamma_{o,t} \\ \gamma_{c,o} & \gamma_{c,c} & \gamma_{c,f} & \gamma_{c,l} & \gamma_{c,v} & \gamma_{c,t} \\ \gamma_{f,o} & \gamma_{f,c} & \gamma_{f,f} & \gamma_{f,l} & \gamma_{f,v} & \gamma_{f,t} \\ \gamma_{l,o} & \gamma_{l,c} & \gamma_{l,f} & \gamma_{l,l} & \gamma_{l,v} & \gamma_{l,t} \\ \gamma_{v,o} & \gamma_{v,c} & \gamma_{v,f} & \gamma_{v,l} & \gamma_{v,v} & \gamma_{v,t} \end{bmatrix}$$

where:
- Γ - matrix of coefficients
- P - column vector of n-1 netput prices normalized by the n^{th} price, i.e. (p_{cull}/p_{milk}, p_{feed}/p_{milk}, p_{labor}/p_{milk}, p_{vet}/p_{milk})
- t - trend variable

The function is normalized by assuming homogeneity of degree one. Russell and Wilkinson state "f is homogeneous

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3/ Taylor polynomials approximate the value of f(x) in the neighborhood of X = A. The polynomial which approximates f(X) is,

$$f(X) = f(A)+f'(A)X+\frac{f''(A)}{2!}\cdot X^2+\ldots+\frac{f^{(n)}(A)}{n!}\cdot X^n + \text{remainder}$$

where:

$$f(X) = F(a_1+th_1, a_2+th_2, \ldots, a_n+th_n)$$
$$f'(A)X = \Sigma_i F_1 x_1$$
$$f''(A)X^2 = \Sigma_i \Sigma_j F_{ij} x_i x_j$$

A second order expansion would take the first three terms and append an error term as remainder.

A standard modified quadratic is,

$$F(x) = \gamma_o + \sum_{i=1}^{n}\gamma_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{m}\gamma_{ij}x_i x_j$$

The only difference is that F(X)'s intercept(γ_o) and first order terms($\gamma_i x_i$) are twice as large as the corresponding terms in (3.14). Coefficients measure the change in the dependant variable per unit change in the independent variable. Since γ_o is constant, the coefficients on the 1 and elements of P' of (3.14) should be twice as large.

of degree 1 in x if the multiplication of each of the variables, x_i, i=1,...,n by a given positive scalar, multiplies the image by this same scalar raised to the first power"(pg. 53). The indirect function is $\Pi(p_1,p_2,...,p_i,...,p_n)$. Each price is a scalar s.t. $\Pi/p_i = \Pi(p_1/p_i,p_2/p_i,...,1,...p_n/p_i)$. The equation above arbitrarily chose the n^{th} price.

Beattie and Taylor state that "first derivatives of a function homogeneous of degree k are homogeneous of degree k-1"(pg.50). From above,

$$(3.15) \qquad \begin{aligned} (1/p_i)\Pi(P) &= \Pi(P/p_i) \\ (1/p_i)d\Pi(P)/dp_j &= d\Pi(P/p_i)/dp_j \\ &= (1/p_i)x_j \\ d\Pi(P)/dp_j &= x_j \end{aligned}$$

The normalized quadratic is Diewert flexible, but can globally satisfy convexity. Such quadratic forms are also "self dual", they assure convexity for the underlying technology set(Shumway).

Previous Studies That Applied Duality Theory to Dairy Farms

Huy, Elterich and Gempesaw estimated dairy production with a translog profit function. The multi-equation model estimated net supplies for milk, livestock, concentrate, roughage, labor and miscellaneous inputs. As in most other duality studies, the estimation method was Zellner's seemingly unrelated regression. A goal of the study was to test for the existence of a "meaningful aggregate United States production function". In theory there does exist an aggregate production function. However, barriers to trade such as milk marketing orders and transportation costs gives each region a different price. Thus, production falls inside the production possibility frontier.

Unfortunately, the test cannot suggest less than optimal production. Huy et. al. used intercept dummy variables to separate each region. Using an F-test, the study rejected the hypothesis of no differences in cost or revenue shares at constant prices. This only implies that for a given price, the quantity supplied differs from region to region. An appropriate test asks whether price differences has significantly lowered aggregate production.

Other goals were to estimate own and cross price elasticities. Test were conducted for economies of size and a shadow price for fixed inputs.

Dahlgran formed a multi-equation model of the dairy industry. The full model held nine equations. After several substitutions, the reduced form consisted of milk supply, feed demand, dairy cow adjustment and milk price capitalization rate. Each of four equations required different estimation techniques yielding estimates for elasticities and partial adjustment coefficients.

Howard and Shumway applied a dynamic dual model to the dairy industry. The study employed a generalized Leontiff and normalized quadratic profit function to estimate elasticities for milk supply, feed demand and quasi-fixed demands for labor and cows. Both were suggested by Epstien for meeting the required conditions on an intertemporal profit function. The dynamic model results in estimating a partial adjustment coefficient for quasi-fixed inputs. Method of estimation was nonlinear three stage least squares. The estimated elasticities and partial adjustment coefficients are compared with estimates in this study.

Chapter 4
Empirical Model

The model used to represent dairy farmers' profit maximization brings together several concepts discussed in the previous chapter. First profits are defined as,

$$(4.1a)\quad \Pi^*_t = V^*_t{}' P_t$$

V^*_t - column vector of optimal netput quantities
P_t - column vector or prices normalized by the price of milk in period t
Π^*_t - optimal level of profits normalized by milk price in period t

Then representing Π^*_t with a normalized quadratic indirect profit function gives,

$$(4.1b)\quad \Pi^*_t = (1/2)[1 \; P_t \; t]\Gamma \begin{bmatrix} 1 \\ P_t \\ t \end{bmatrix}$$

In this formulation, t represents a trend variable. It is used to capture the linear changes in all variables. The trend variable fits into the profit function by treating it as a fixed factor. Γ is the matrix of coefficients as specified in equation 3.14.

Hotelling's lemma states,

$$(4.1c)\quad d\Pi^*_t/dp_{i,t}=v^*_{i,t}=\Gamma_i\begin{bmatrix}1\\ P_t\\ t\end{bmatrix}$$

where:
- t - trend variable
- $v^*_{i,t}$ - netput quantities for good i in period t
- Γ_i - row vector of coefficients for good i
- i - cull cows, feed, labor and veterinary services
- t - 1,...,T

The first equality is given from (4.1b) and the second results from computing the derivative of (4.16).

Partial Adjustment Model

Suppose dairy farms do not instantaneously adjust to profit maximizing conditions. Instead, firms change net supplies according to the partial adjustment process whereby the change in net supplies is a fraction of the difference between desired net production and actual net production in the previous period, i.e.

$$(4.2a)\quad v_{i,t} - v_{i,t-1} = k_i(v^*_{i,t}-v_{i,t-1})$$

where:
- $v_{i,t}$ - quantity of netput i in period t
- $v^*_{i,t}$ - optimal netput use for commodity i in period t
- $0\le k_i\le 2$

or upon rearranging terms,

$$(4.2b)\quad v^*_{i,t} = \frac{v_{i,t}}{k_i} - \frac{(1-k_i)v_{i,t-1}}{k_i}$$

Substituting (4.2b) into (4.1c) and rearranging gives,

$$(4.3) \quad v_{i,t} = (1-k_i)v_{i,t-1} + k_i\Gamma_i\begin{bmatrix} 1 \\ p_t \\ t \end{bmatrix}$$

Expansion on i to include all netputs gives,

$$v_{c,t}=(1-k_c)v_{c,t-1}+\gamma_{o,c}+\gamma_{c,c}p_c+\gamma_{c,f}p_f+\gamma_{c,l}p_l+\gamma_{c,v}p_v+\gamma_{c,t}t$$
$$v_{f,t}=(1-k_f)v_{f,t-1}+\gamma_{o,f}+\gamma_{f,c}p_c+\gamma_{f,f}p_f+\gamma_{f,l}p_l+\gamma_{f,v}p_v+\gamma_{f,t}t$$
$$v_{l,t}=(1-k_c)v_{l,t-1}+\gamma_{o,l}+\gamma_{l,c}p_c+\gamma_{l,f}p_f+\gamma_{l,l}p_l+\gamma_{l,v}p_v+\gamma_{l,t}t$$
$$v_{v,t}=(1-k_v)v_{v,t-1}+\gamma_{o,v}+\gamma_{v,c}p_c+\gamma_{v,f}p_f+\gamma_{v,l}p_l+\gamma_{v,v}p_v+\gamma_{v,t}t$$

Traditional partial adjustment models lag only one dependent variable. In contrast, this model lags all netput quantities. Like the traditional models, the adjustment path depends on the magnitude of the partial adjustment coefficient. In figure 4.1, let x_o represent the optimal netput quantity before period t_o. At t_o there is a shock pushing the new optimal level to x_1. The firms adjustment after t_o depends on k. For example, if k=0, as in Figure 4.1b then no adjustment occurs. The quantity produced by the industry remains constant on both sides of t_o. Figure 4.1c shows that if k<1, some adjustment will occur after the first period. The adjustment path asymptotically approaches the new optimal level.Figure 4.1d shows that k=1 implies full adjustment at t_o. If 1<k<2, the adjustment path over-compensates but eventually converges upon x_1(figure 4.1e). And finally, for k=2 the adjustment cycles(figure 4.1f) while k>2 results in instability.

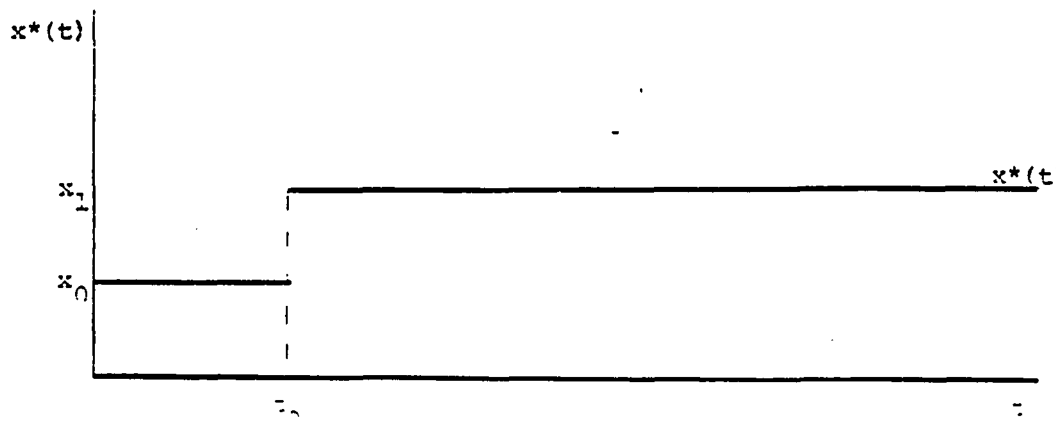

Figure 4.1a. Optimal Quantity of Netputs Given a Shock in Period t_0

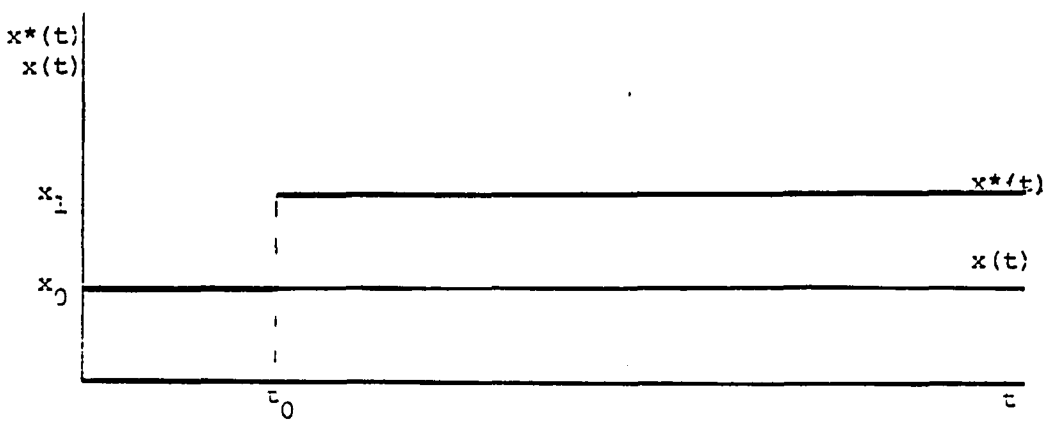

Figure 4.1b. Adjustment Path With k=0

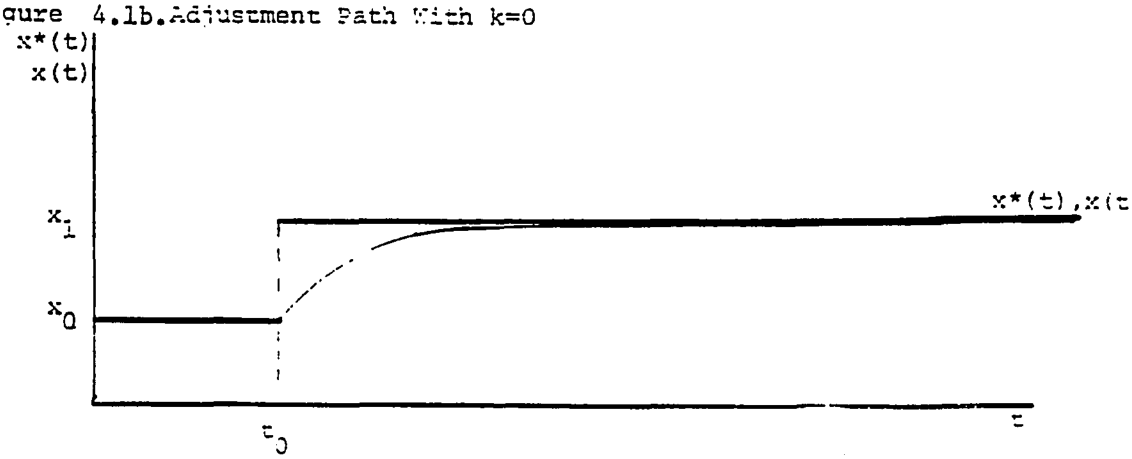

Figure 4.1c. Adjustment Path with k<1

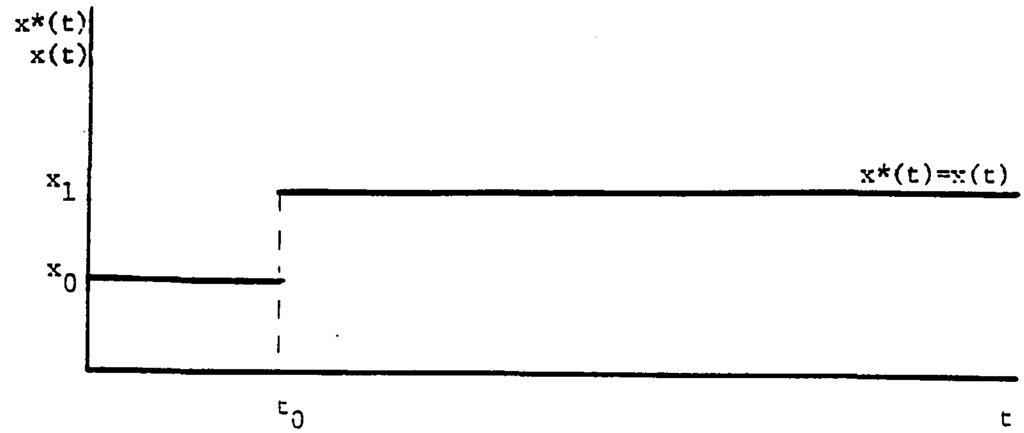

Figure 4.1c. Adjustment Path With k=1

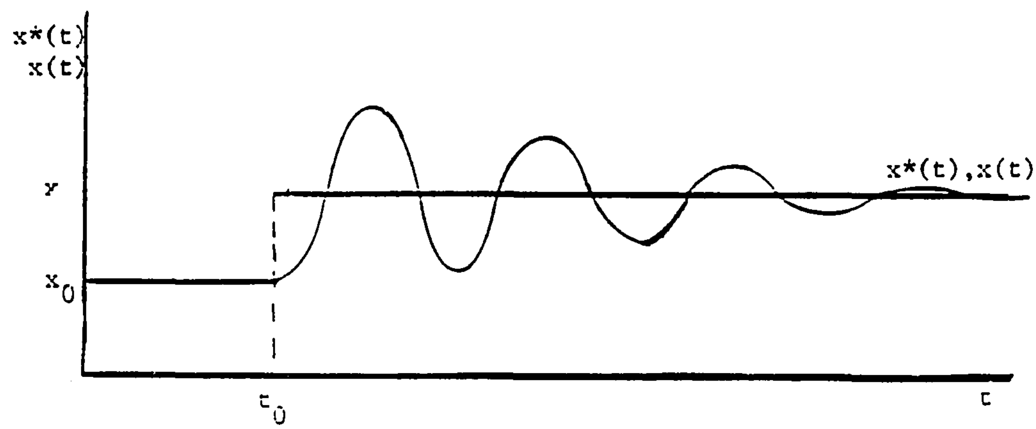

Figure 4.1d. Adjustment Path With 1<k<2

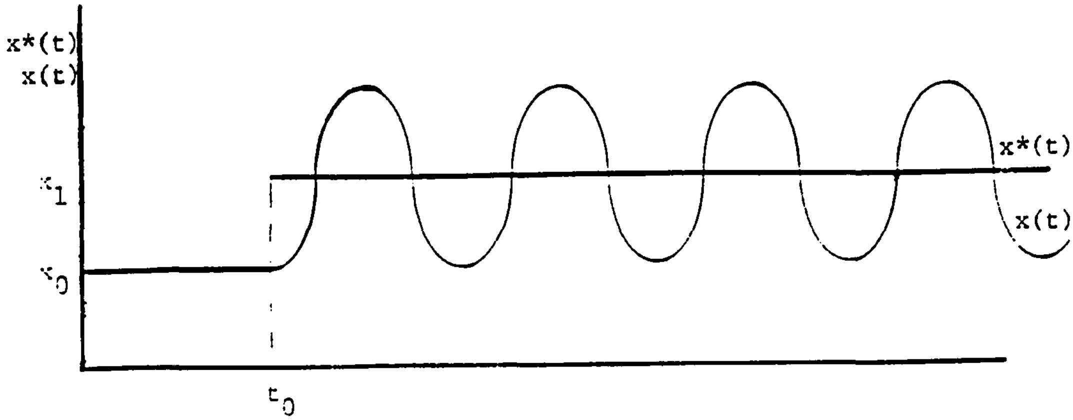

Figure 4.1e. Adjustment Path With k=2

Imposing and Testing Conditions

Four restrictions on the profit function are homogeneity, symmetry, convexity and monotonicity. Homogeneity is automatically satisfied with a normalized quadratic. Symmetry was imposed at estimation. The remaining two properties are tested for validity.

The first derivatives of optimal net supplies are,

$$(4.4a)\quad \frac{dv^*_{j,t}}{dp_{i,t}} = \frac{dv^*_{j,t}}{d(p_j/dp_{milk})} \cdot \frac{d(p_j/p_{milk})}{dp_j} = \frac{\gamma^*_{i,j}}{p_{milk,t}}$$

$$(4.4b)\quad \frac{dv^*_{j,t}}{dp_{milk,t}} = \frac{dv^*_{j,t}}{d(p_j/dp_{milk})} \cdot \frac{d(p_j/dp_{milk})}{dp_j}$$

$$= \sum_{i=1}^{4} \gamma^*_{i,j} \cdot \frac{-p_{i,t}}{p^2_{milk,t}}$$

One can show that homogeneity is satisfied. The net supplies are homogeneous of degree zero if,

$$(4.5a)\quad \sum_{i=1}^{4} \frac{dv^*_{j,t}}{dp_{i,t}} \cdot p_{i,t} + \frac{dv^*_{j,t}}{p_{n,t}} \cdot p_{n,t} = 0$$

If they are not homogeneous of degree zero then the left hand side of 4.5a is not equal to zero. Substitute 4.4a and 4.4b into 4.5a and the desired result occurs. Homogeneity also holds for milk supply.

$$(4.5b)\quad \sum_{i=1}^{5} \frac{dv^*_{milk,t}}{dp_{i,t}} \cdot p_{i,t} = 0$$

Symmetry requires all cross price effects to be equal as Young's theorem states $d^2\Pi/dp_i dp_j = d^2\Pi/dp_j dp_i$. Thus,

$$\text{(4.6a)} \quad \frac{dv^*_{j,t}}{dp_{i,t}} = \frac{dv^*_{i,t}}{dp_{j,t}} = \gamma^*_{i,j}/p_{milk,t} \text{ for all } i,j$$

For the milk supply equation, symmetry requires,

$$\text{(4.6b)} \quad \frac{dv^*_{milk,t}}{dp_{j,t}} = \frac{dv^*_{j,t}}{dp_{milk,t}} = \sum_{i=1}^{n-1} \gamma^*_{i,j} \cdot \frac{-p_{i,t}}{p^2_{milk,t}}$$

that implies,

$$v^*_{milk,t} = \int \sum_{i=1}^{4} \gamma^*_{i,j} \cdot \frac{-p_{i,t}}{p^2_{milk,t}} \cdot dp_{j,t} + \text{const.}$$

$$= \sum_{i \neq j} -\gamma^*_{i,j} \cdot \frac{p_{i,t} p_{j,t}}{p^2_{milk,t}} \cdot - \gamma^*_{j,j} \cdot \frac{p^2_{j,t}}{p^2_{milk,t}} + \text{const.}$$

The symmetry calculations allow milk supply to be added to the system of equations, so that a partial adjustment coefficient can be estimated for this netput. The coefficients now represent the change in quantity of a netput given a change in nominal prices.

Symmetry can be imposed as follows. Let Γ^* represent a vector of coefficients which is not restricted by symmetry. Also, let Γ represent the matrix of restricted coefficients. Γ is a linear transformation of Γ^*. A linear equation can be written as,

$$\text{(4.7a)} \quad Y = \Gamma X = \Gamma^* Q X$$

where, $\Gamma^* Q = \Gamma$

Symmetry is thus imposed by multiplying X by Q prior to estimating the matrix of coefficients.

Monotonicity states that profits are non-decreasing in output prices and non-increasing in input prices. The test checks the first derivative of the profit function. The first derivative of net supplies must be positive for net outputs and negative for net inputs. The normalized quadratic can be checked for local satisfaction. However, global satisfaction requires a Sobolev flexible form.

Convexity requires the Hessian matrix to be positive semi-definite. Since the second derivative of a quadratic is constant, convexity is globally satisfied if the Hessian is positive semi-definite.

Seemingly Unrelated Regression (SUR)

Seemingly unrelated regression is often employed to estimate supply systems. A special case of generalized least squares, SUR recognizes that an error term on one net supply is contemporaneously correlated with the error term of another. That is, the of error terms between equations are not equal to zero. Binswanger justifies the regression with the following argument; "If restrictions across equations($\gamma_{ij}=\gamma_{ji}$) are imposed, OLS estimators are no longer efficient." Like other generalized least squares procedures, SUR transforms the dependent and independent variables so that covariances are equal to zero and the variance is constant throughout. Then ordinary least squares can be applied to give efficient estimators. The general SUR model is,

(4.8a) $Y = X\Gamma + u$

or

(4.8b)

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_N \end{bmatrix} = \begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdots & X_N \end{bmatrix} \cdot \Gamma + \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_N \end{bmatrix}$$

where $i=1,...,N$

Each Y_i stands for a column vector of the dependent variables for equation $i(y_{i1}, y_{i2}, ..., y_{iT})$. The total length of Y is NT, which multiplies the number of observations(T) against the number of equations(N). X_i stands for a matrix of k independent variables for equation i as columns and T observations or rows. Each zero on the off diagonal stands for a matrix with columns and rows corresponding to to X_i. Thus, the entire matrix of independent variable is made up of smaller X_i matrices with columns and rows that do not overlap. The dimensions on the entire X matrix is NT X Nk. Gamma is a column vector of coefficients of length Nk. Note that when Γ is multiplied against the matrix of independent variables, only the coefficients multiplied against X_i are assumed to influence y_i.

For this study, i stands for cull cows, feed, labor, milk and veterinary services. Y_i is a column vector of netput quantities with ten observations. Each X_i is a matrix of ten observations(T=10) and seven independent variables(k=7) that correspond to equation 4.3. More specifically the columns or independent variables of X_i are $v_{i,t-1}$, 1, p_{cull}/p_{milk}, p_{feed}/p_{milk}, p_{labor}/p_{milk}, p_{vet}/p_{milk} and t. Contrary to equation 4.3, Γ is a column vector that stacks each Γ_i. That is, $\Gamma=[\Gamma_{cull},\Gamma_{feed},\Gamma_{labor},\Gamma_{milk},\Gamma_{vet}]'$ where each Γ_i can be expanded into the coefficients written in 4.3.

u_{it} stands for the i^{th} element in the vector u_t. The error, u_{it}, is assumed distributed as a multivariate normal random variable with contemporaneous covariance equal to σ_{ij}(Amemiya). Assumptions about the expectations of $u_{i,t}$'s are,

$$(4.8a)\quad E\{u_{it}\} = 0 \qquad t=1,\ldots,T$$

$$(4.8b)\quad E\{u_{it},u_{jt'}\} = \begin{bmatrix} \sigma_{ij} & \text{if } t=t' \\ & \text{or} \\ 0 & \text{otherwise} \end{bmatrix}$$

The covariance matrix Ω, is written as,(4.9)

$$\Omega = \Sigma\otimes I_T = \begin{bmatrix} \sigma_{11}I_T & \sigma_{12}I_T & \cdots & \sigma_{1N}I_T \\ \sigma_{21}I_T & \sigma_{22}I_T & \cdots & \sigma_{2N}I_T \\ \vdots & \vdots & & \vdots \\ \sigma_{N1}I_T & \sigma_{N2}I_T & \cdots & \sigma_{NN}I_T \end{bmatrix} = \begin{bmatrix} \begin{smallmatrix}\ddots & & 0\\ & \sigma_{11} & \\ 0 & & \ddots\end{smallmatrix} & \begin{smallmatrix}\ddots & & 0\\ & \sigma_{12} & \\ 0 & & \ddots\end{smallmatrix} & \cdots & \begin{smallmatrix}\ddots & & 0\\ & \sigma_{1N} & \\ 0 & & \ddots\end{smallmatrix} \\ \begin{smallmatrix}\ddots & & 0\\ & \sigma_{21} & \\ 0 & & \ddots\end{smallmatrix} & \begin{smallmatrix}\ddots & & 0\\ & \sigma_{22} & \\ 0 & & \ddots\end{smallmatrix} & \cdots & \begin{smallmatrix}\ddots & & 0\\ & \sigma_{2N} & \\ 0 & & \ddots\end{smallmatrix} \\ \vdots & \vdots & & \vdots \\ \begin{smallmatrix}\ddots & & 0\\ & \sigma_{N1} & \\ 0 & & \ddots\end{smallmatrix} & \begin{smallmatrix}\ddots & & 0\\ & \sigma_{N2} & \\ 0 & & \ddots\end{smallmatrix} & \cdots & \begin{smallmatrix}\ddots & & 0\\ & \sigma_{NN} & \\ 0 & & \ddots\end{smallmatrix} \end{bmatrix}$$

The identity matrix(I_T) is of dimension T. Σ represents the covariance matrix with N columns and rows. Each σ_{ij} is the covariance between equations i and j. The symbol $\otimes$ represents a Kroneker product. The product multiplies each term in Σ against the matrix I_T. The matrix on the far right is the result. The entire matrix is made up of sub-matrices that recognize dependence between error terms of different net supplies for a given period t. The size of Ω is NT X NT.

The errors are assumed to have no autocorrelation or heteroskedasticity within equations. No autocorrelation implies $E(u_{i,t}, u_{j,t+1})=0$ $i,j=(1,\ldots,N), t=(1,\ldots,T)$[4]. Homoskedasticity means $\sigma_{i,j}$ is constant regardless of time. Thus, every submatrix of Ω will contain zeros in the off-diagonal and a constant(σ_{ij}) on the diagonal. That is why Ω can be written in compact form ($\Sigma \otimes I_T$).

Inefficient Estimators

Applying OLS to the whole system would yield unbiased, consistent but inefficient estimators [5]. The OLS estimator for Γ is,(Johnston, pg291)

$$\hat{\Gamma} = \Gamma + (X'X)^{-1}X'u \qquad (4.10a)$$

Since u is independent of X, estimators are unbiased,

$$E(\hat{\Gamma}) = \Gamma \qquad (4.10b)$$

4/ See Guilkey for a large sample test of autocorrelation.
5/ Pindyck and Rubenfeld note that applying OLS to individual equations would also yield inefficient estimators.

Assume $E\{uu'\}=\sigma^2 V=\Omega$ where σ^2 is a constant and V is a symmetric positive definite matrix. The OLS estimators can be shown to be inefficient as,

$$\begin{aligned}(4.11a)\quad var(\hat{\Gamma}) &= E\{(\hat{\Gamma} - \Gamma)(\hat{\Gamma} - \Gamma)'\} \\ &= E\{[(X'X)^{-1}X'u][(X'X)^{-1}X'u]\}' \\ &= E\{(X'X)^{-1}X'uu'X(X'X)^{-1}\} \\ &= (X'X)^{-1}X'E\{uu'\}X(X'X)^{-1} \\ &= (X'X)^{-1}X'\sigma^2 VX(X'X)^{-1} \\ &= \sigma^2(X'X)^{-1}X'VX(X'X)^{-1}\end{aligned}$$

Under appropriate ordinary least squares conditions $E\{uu'\}=\sigma^2 I_{NT}$. Where I is the identity matrix of length NT.

$$\begin{aligned}(4.11b)\quad var(\hat{\Gamma})_{OLS} &= (X'X)^{-1}X'\sigma^2 I_{NT}X(X'X)^{-1} \\ &= \sigma^2(X'X)^{-1}X'I_{NT}X(X'X)^{-1} \\ &= \sigma^2(X'X)^{-1}\end{aligned}$$

However, V is not equal to the identity matrix, so using OLS may not yield the smallest possible variances. The SUR procedure corrects for the inefficient variance.

<u>SUR and Efficiency</u>

Theil notes that if V is a symmetric and positive definite then "there exists a non-singular NTxNT matrix V, such that RVR' = I_{NT}". Multiplying the linear regression model, equation 4.6, by R gives,

$$(4.12)\quad Y^* = X^*\Gamma + u^*$$

where

$$\begin{aligned}Y^* &= RY \\ X^* &= RX \\ u^* &= Ru\end{aligned}$$

Since R is fixed in each time period the expected value of Ru is still zero RE{u}=0. The variance of Ru is,

$$(4.13)\quad var(Ru) = E\{Ru(Ru)'\} = E\{Ruu'R\} = RE\{uu'\}R = R\sigma^2 VR' = \sigma^2 I_{NT}$$

From equation 4.9 the estimators are efficient,

$$(4.14)\quad var(\hat{\Gamma}) = E\{(\hat{\Gamma} - \Gamma)(\hat{\Gamma} - \Gamma)'\} = E\{(X^{*\prime}X^*)^{-1}X^{*\prime}Ruu'R'X^*(X^{*\prime}X^*)^{-1}\} = \sigma^2(X^{*\prime}X^*)^{-1}$$

where, $X^* = RX$

And the linear regression is,

$$(4.15)\quad \Gamma = [(RX)'RX]^{-1}(RX)'RY = (X'R'RX)^{-1}X'R'RY = (X'V^{-1}X)^{-1}X'V^{-1}Y = (\sigma^2/\sigma^2)(X'V^{-1}X)-1(X'V^{-1}Y)$$

Since σ^2 is a scalar, the distributive law can be applied such that,

$$(4.15)\quad \Gamma^*_i = (X'(\sigma^2V)^{-1}X)^{-1}(X'(\sigma^2V)^{-1}Y) = (X'\Omega X)^{-1}(X'\Omega Y)$$

Unfortunately, Ω and Γ are unknown. We are left with more unknown variables than equations. Zellner has proposed the following method to estimate Ω and carry out the SUR regression.

1. Apply OLS separately to each equation to obtain, $\hat{\Gamma}=(X_i'X_i)X_i'Y_i$.
2. Calculate estimates for u.

$$\hat{u}_i = Y_i - X_i\hat{\Gamma} = [I - X_i(X_i'X_i)^{-1}X_i']Y_i \qquad i=1,\ldots,N$$

3. Construct the covariance matrix $\hat{\Omega} = \hat{\Sigma} I_T$.

$$s_{ij} = \frac{\hat{u}_i{}'\hat{u}_j}{(n-k_i)^{\frac{1}{2}}(n-k_j)^{\frac{1}{2}}}$$

$$\hat{\Sigma} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & & \vdots \\ s_{n1} & s_{n2} & & s_{nn} \end{bmatrix}$$

4. Find estimates for Γ

$$\hat{\Gamma} = (X'\hat{\Omega}^{-1}X)X'\hat{\Omega}^{-1}Y$$

Like generalized least squares, SUR can be iterated over steps 2 to 4 until the maximum likelihood value converges to the desired level.

Data Characteristics and SUR Modifications

Data were obtained from a variety of sources. Netput quantities were calculated from Betts and Bruner's cash budgets. Wages, for veterinary services and labor, were taken from Employment and Wages and used as prices for each netput. The blend price of milk was obtained from Agricultural Prices, while feed price is available in Agricultural Statistics.

Netputs were aggregated into five groups, cull cows, milk, feed, labor and veterinarian services. Feed costs are silage, concentrates, hay, pasture and haylage. Medical expenses include payments for veterinarians, artificial insemination and DHIA fees. Finally cull cows include calves, replacements and cull cows.

Betts and Bruner listed each category (silage, concentrates, etc.) as revenue or cost. Quantities for each group was calculated by taking the total cost for each good in period t and dividing by its respective price. For example, the quantity of feed is the total amount spent on silage concentrates, hay, pasture and haylage then divided by the price of feed. This procedure assumes each group is homothetically separable from the others. That is, the quantity ratios of individual items within a group do not depend on quantity ratios outside of the group. Shumway states, "homothetic separability is sufficient for consistent aggregation of items, thus a two stage choice is possible." Homothetic separability is testable, however it requires data on netput quantities which is not available.

One reason for aggregating netputs was to increase degrees of freedom without sacrificing an accurate representation of all costs and revenues. By including most of the budget, each category can influence profits. A few of the more expensive excluded goods are transportation, taxes and machinery and one generally fixed factors. A second reason is that prices for each good are not available. Each netput quantity is given on a per cow basis. This can be done assuming constant returns to scale or linear homogeneity in the production function.

Standardized Industrial Codes (SIC, U.S. Bureau of Labor Statistics) SIC 074 and SIC 07 were used as price for veterinary services. Dairy wages came from SIC 02 and SIC 0241. Both data sets are from Employment and Wages Annual Survey. The four or three digit code was available from 1979 to 1985, while the two digit code dates from 1975 to 1982. This study's data set ran from 1975 to 1985. So the two time series had to be joined together. Regressions were run from two digit onto three digit data. Fitted values for 1975 to 1985 were used to replace missing values of the four digit series.

The blend price for milk and the feed price were given by state. Average prices were calculated for each region by summing over states and dividing by the quantity employed.

Existing data allowed eleven observations per equation. However, the partial adjustment lags one value so the number of observations decreases to ten. As shown in (4.3), the model requires seven parameters per equation. This leaves only three degrees of freedom for error. The statistical and economic reliability of a model estimated under these conditions is questionable.

To overcome the limited degrees of freedom two modifications were made. First, the SUR was modified. Usually the first step is to estimate each equation by OLS. Instead, all equations were combined and then estimated with OLS. Second, symmetry was imposed to further reduce the number of estimated parameters. The number of parameters decreased from 35 to 25, while the number of observations increased to 50. The final degrees of freedom for error is 25.

A third technique involved aggregating regions. Data was available for six regions, Appalachia, Cornbelt, Northwest, Pacific, Southwest, Upper-Midwest and for the United States. In an attempt to increase the degrees of freedom one regression was run using dummy variables to account for regional differences. However, the data matrix proved too large for the software's memory. Thus, of necessity, each region was estimated separately.

To assure the convergence of the algorithm, all variances across equations were assumed equal to zero. The diagonal $(\sigma_{11}, \sigma_{22}, \ldots, \sigma_{nn})$ was kept, but any variances along the off-diagonal were set at zero. This converts the estimation into a weighted least squares, acknowledging different variances per equation. Regressions were carried out with symmetry and homogeneity imposed.

Elasticities

Elasticities can be estimated for both the short and long run(Johnston). In the short run,

$$(4.16) \quad E_{ij} = \frac{dv_i}{dp_j} \cdot \frac{p_j}{v_i} = \frac{k_i \cdot \gamma_{ij} \cdot p_j}{v_{i,t}}$$

where,

i,j - cull cows, feed, labor, milk and veterinary services

In the long run, actual profits equal the optimal level,

$$(4.17) \quad E^*_{ij} = \frac{dx^*_i}{dp_j} \cdot \frac{p_j}{v_i} = \frac{\gamma_{ij} p_j}{v^*_i}$$

All own and cross price elasticities will be put into one matrix (E).(4.18)

$$E = \begin{bmatrix} e_{c,c} & e_{c,f} & e_{c,l} & e_{c,m} & e_{c,v} \\ e_{f,c} & e_{f,f} & e_{f,l} & e_{f,m} & e_{f,v} \\ e_{l,c} & e_{l,f} & e_{l,l} & e_{l,m} & e_{l,v} \\ e_{m,c} & e_{m,f} & e_{m,l} & e_{m,m} & e_{m,v} \\ e_{v,c} & e_{v,f} & e_{v,l} & e_{v,m} & e_{v,v} \end{bmatrix}$$

CHAPTER 5

RESULTS

The iterative seemingly unrelated regression algorithm, subject to the modifications described was used to derive parameter estimates. This iterative procedure also gives maximum likelihood estimates. All estimations converged in fewer than twenty iterations and are given in Appendix D.

Monotonicity was satisfied for all equations in all regions. That is, the first derivative of the profit function is positive(negative) with respect to output(input) prices. In other words the estimated direct price effect for each equation carries the correct sign.

Convexity of the profit function failed for all regions and the United States. The test is that all principal minors of the Hessian matrix must be greater than zero. However, the test is not a statistical one. Probabilities can not be assigned to the determinants. Only the first principal minor (dP/dpidpi=cii) can be statistically tested. However, when cii was less than zero, the p-value was insignificant.

Adjustment of the Dairy Industry to Economic Change

Partial adjustment coefficients are reported for cull cows, feed, vet, wages and milk using the partial adjustment model in tables 5a through 5e. The majority of values were greater than one, however only five are significantly different from one. The null hypothesis corresponding to the significance levels is that immediate adjustment occurs i.e. k=1. The values for k are assumed to follow a t-distribution. The p-value corresponds to a two tail test giving the probability of k being further away from unity. In this regression there are 25 degrees of freedom. A t-statistic at the 0.05 significance level is 2.060. If the partial adjustment coefficient is significantly different from one then the model implies a lagged adjustment(see figures 4.1a to 4.1f).

Table 5.1a. Partial Adjustment For Cull Cows

REGION	k_{cull}	t-stat	p-value
United States	1.483391	-6.09266	0.0000
Appalachia	1.064048	-0.15694	0.8759
Cornbelt	1.385323	-2.57019	0.0284
Northeast	2.085222	-3.98740	0.0002
Pacific	1.406356	-1.22971	0.2246
Southern Plains	1.832227	-5.40048	0.0000
Upper-Midwest	1.413866	-1.38148	0.1733

Table 5.1b. Partial Adjustment for Milk Supply

REGION	k_{milk}	t-stat	p-value
United States	1.135095	-0.78317	0.4372
Appalachia	1.045922	-0.40979	0.6837
Cornbelt	1.017803	-0.05236	0.9584
Northeast	1.443308	-2.00116	0.0508
Pacific	0.783822	0.45977	0.6477
Southern Plains	0.941819	0.21240	0.8327
Upper-Midwest	1.343026	-1.10010	0.2765

Table 5.1c. Partial Adjustment for Feed Demand

REGION	k_{feed}	t-stat	p-value
United States	1.181344	-0.75289	0.4550
Appalachia	0.833615	0.73155	0.4679
Cornbelt	0.886587	0.35315	0.7255
Northeast	1.832122	-2.34316	0.0231
Pacific	1.340168	-2.06860	0.0438
Southern Plains	1.323372	-1.14937	0.2559
Upper Midwest	1.094865	-0.37185	0.7116

Table 5.1d. Partial Adjustment for Veterinary Services

REGION	k_{vet}	t-stat	p-value
United States	1.075272	-0.54159	0.5905
Appalachia	1.131907	-0.80399	0.4252
Cornbelt	1.529720	-1.76880	0.0830
Northeast	1.045711	-0.38854	0.6993
Pacific	0.956207	0.14510	0.8852
Southern Plains	1.173430	-1.41473	0.1633
Upper Midwest	1.545773	-3.20562	0.0023

Table 5.1e. Partial Adjustment for Labor Demand

REGION	k_{labor}	t-stat	p-value
United States	1.009629	-0.08063	0.9361
Appalachia	1.399456	-2.58865	0.0126
Cornbelt	1.274030	-1.74389	0.0873
Northeast	1.157687	-0.70904	0.4816
Pacific	1.289008	-1.95515	0.0562
Southern Plains	1.109178	-0.39068	0.6977
Upper Midwest	1.278268	-1.10776	0.2733

For cull cows, all the estimated partial adjustment coefficients are greater than unity. However, only the Northeast, Southern Plains and United States yield values significantly different from one. The coefficients that exceed unity imply over adjustment in the first period. These results roughly coincide with Dahlgran's results. His partial adjustment was insignificantly different from one for the United States. Howard and Shumway's estimates for the United States were mixed. The Generalized Leontif partial adjustment was estimated to be 0.14. The Normalized Quadratic estimated a partial adjustment equal to -0.4. The negative value implies the quantity demanded moves away from the optimal equilibrium value.

Most of the partial adjustments for milk supply were not significantly different from one. This implies an immediate adjustment after the first period. The only coefficient significantly different from one was for the Northeast region. The partial adjustment is equal to 1.44 and is significant at the 0.05 level. This implies an adjustment path that exceeds the optimal quantity of milk supplied after the first period. The path eventually converges to the optimal level.

The remaining coefficients are for feed, labor and veterinary services. For the most part, these partial adjustments are not significantly different from one.

The model assumes adoption rates are reflected by the partial adjustment coefficient. Most of the values are equal to one and suggest immediate adoption. Thus, concerns about a jump in surplus stocks and short term rural unemployment may arise. However, these results conflict with previous studies. Lesser, Magrath and Kalter predicted an adoption rate to exceed three years. While, Fallert, McGuckin, Betts and Bruner suggested at least six years.

A number of statistical factors may explain why the partial adjustments coefficients are so high. First, the error terms may be serially correlated, which would tend to bias the coefficients on lagged dependent variables(Pindyck and Rubenfeld). Second, SIC codes for veterinary services and dairy wages include a number of related occupations. For example, the SIC for veterinary services includes care for horses, bees, fish, goats and house pets. Variations in these wages could occur without a logical connection to the quantity used by dairies. Thus, results may not reflect this industry.

Also, as is often done, a trend variable was used to capture any linear change over time. However, this may include technological change. Thus, there may be a high level of multicollinearity between the lagged netput for the partial adjustment coefficient and the trend variable.

A third issue is functional form. There are questions on the robustness of various flexible forms. As stated earlier, global flexibility estimates the dual throughout the domain of the data while local flexibility provides an accurate estimation at one point. A choice other than the normalized quadratic may have yielded different estimates.

Finally, the partial adjustment includes dairy farmers' reaction to changes in price and technology. Farmers may react quickly to a change in price for variable netputs. Whereas, adoption of new technology may take some time. For example, computers were introduced in the 1960's to dairy farmers in order to save time with bookkeeping. Today, only 30 percent of the dairies have adopted the new technology(Conneman).

If all statistical problems are resolved and if some partial adjustments are still greater than one, then explanations do exist. For example, the data set for this study ran from 1975 to 1985. After the oil embargo of 1973, much of the 1970's was characterized by inflation. Suppose milk and feed price rose but producers only noticed the increase in milk price. Then dairy farmers may sell too much milk without realizing the higher feed price has shifted back supply. Thus, overadjustment occurs until farmers realize the increased cost for inputs. Imposing Homogeneity may correct for some of the inflation, however an uneven fluctuation in prices may not be corrected.

<u>Elasticities</u>

Elasticities are defined as,

$$E_{ij} = \%\Delta Q_i / \%\Delta P_j$$

where: Q_i-quantity of good i
P_j-price of good j
$i,j=1,\ldots,n$

They are computed by setting partial adjustments equal to one and using equation 4.16. Table 5.2 lists own price elasticities along the diagonal and cross-price elasticities on the off-diagonal. The table corresponds to the equation above by letting quantity i stands for rows and price j for columns. The computations used average netput quantities and prices over the period 1975 to 1985.

The partial adjustment coefficients were set equal to unity because so many values were not significantly different from one. However, if k is greater than one, then elasticities are more elastic in the short run. Conversely, k less than one implies a more inelastic response occurs in the short run. Finally, a k equal to one implies long run elasticities are equal to short run elasticities. In other words, immediate adjustment occurs.

The a priori expectations are that own price elasticities must be positive for net outputs and negative for inputs. Expectations on cross-price elasticities are as follows. If i is a net output, j a net input then the elasticity is negative. This states an increase in the price of a factor should decrease output. Conversely if i is a net input, j an output then the elasticity is positive. An increase in output price will increase the quantity of factors employed.

There are no a priori expectations about cross price elasticities for netputs with the same sign. For net inputs if E_{ij} is greater than zero, i and j are substitutes. If E_{ij} is less than zero then the netputs are complements. For net outputs the signs are reversed. Complementary outputs have positive elasticities while competitive outputs have negative elasticities.

Table 5.2a. Cross-Price Elasticities for the United States

	w.r.t. the price of CULL	FEED	VET	LABOR	MILK
CULL	0.0132	0.0277	0.0398	0.0256	-0.1062
FEED	-0.0068	-0.0314	-0.0031	-0.0780	0.1193
VET	-0.1285	-0.0401	-0.6549	0.2982	0.5254
LABOR	-0.0357	-0.4432	0.1290	-0.2909	0.6408
MILK	-0.0099	-0.0451	-0.0151	-0.0427	0.1128

Table 5.2b. Cross Price Elasticities for Appalachia

	w.r.t. the price of CULL	FEED	VET	LABOR	MILK
CULL	-0.0073	-0.0366	0.0187	-0.0169	0.0421
FEED	0.0063	-0.1955	-0.0013	0.0271	0.1634
VET	-0.0553	-0.0228	-0.6283	0.4852	0.2211
LABOR	0.0152	0.1419	0.1475	-0.4230	0.1183
MILK	0.0032	-0.0726	-0.0057	-0.0100	0.0851

Table 5.2c. Cross-Price Elasticities for the Cornbelt

	w.r.t. the price of CULL	FEED	VET	LABOR	MILK
CULL	0.0122	0.0036	0.0274	0.0170	-0.0602
FEED	-0.0009	-0.0632	-0.0023	0.0279	0.0385
VET	-0.0796	-0.0286	-0.1583	0.1442	0.1223
LABOR	-0.0290	0.2013	0.0846	-0.5159	0.2590
MILK	-0.0054	-0.0147	-0.0038	-0.0137	0.0376

Table 5.2d. Cross Price Elasticities for the Northeast

	w.r.t. the price of CULL	FEED	VET	LABOR	MILK
CULL	0.0030	0.0030	0.0223	-0.0445	0.0011
FEED	-0.0007	0.1022	0.0184	-0.0085	-0.1113
VET	-0.0625	0.2092	-0.1941	0.1156	-0.0682
LABOR	0.0498	-0.0385	0.0462	0.0897	-0.1472
MILK	0.0014	0.0393	0.0021	0.0114	-0.0543

Table 5.2e. Cross Price Elasticities for the Pacific

	w.r.t. the price of				
	CULL	FEED	VET	LABOR	MILK
CULL	0.0049	0.0207	0.0273	-0.0460	-0.0069
FEED	-0.0034	-0.3560	0.0014	-0.0820	0.4399
VET	-0.0907	0.0293	-0.0707	0.2198	-0.0876
LABOR	0.0479	-0.5258	0.0689	0.2323	0.1767
MILK	-0.0005	-0.2119	0.0021	-0.0133	0.2237

Table 5.2f. Cross-Price Elasticities for the Southern Plains

	w.r.t. the price of				
	CULL	FEED	VET	LABOR	MILK
CULL	-0.0006	0.0118	-0.0190	-0.0068	0.0145
FEED	-0.0017	0.0855	-0.0022	-0.0488	-0.0328
VET	0.0795	-0.0621	-0.6824	0.6969	-0.0318
LABOR	0.0060	-0.2950	0.1482	-0.4130	0.5537
MILK	0.0010	0.0157	0.0005	-0.0439	0.0267

Table 5.2g. Cross Price Elasticities for the Upper-Midwest

	w.r.t. the price of				
	CULL	FEED	VET	LABOR	MILK
CULL	0.0050	0.0227	-0.0162	0.0200	-0.0316
FEED	-0.0078	-0.0687	0.0106	-0.0019	0.0678
VET	0.0602	0.1152	-0.1053	0.3277	-0.3978
LABOR	-0.0442	-0.0123	0.1946	-0.0316	-0.1065
MILK	-0.0036	-0.0224	0.0121	0.0055	0.0088

For the most part own price elasticities carry the expected signs. All milk price elasticities are less than 0.25 and only the Northeast is less than zero. The inelastic results coincide with other studies. Dahlgran estimated a milk supply elasticity of less than 0.8. Huy's elasticities were less than 0.75 for most regions while the upper-midwest was slightly negative. Howard and Shumway estimated a 0.05 milk supply elasticity for the United States using a normalized quadratic. However, their estimates with a generalized Leontiff yielded a -0.12 elasticity.

The inelastic results for all own price elasticities reflect a lack of substitutes for each good. For example feed is necessary for milk production. Also, demand for milk is inelastic which implies the equilibrium quantity of milk will not fluctuate. Therefore, production plans for dairy farmers should remain fairly constant. Demand and supply curves for the dairy farmer should remain inelastic.

As expected, most of the elasticities for cull cows are positive. Only the Southern Plains had a negative elasticity, while the largest elasticity of 0.013 was estimated for the United States. These results coincide with Huy and Elterich's study. Their estimates for own price elasticities were less than 0.5. The results are also consistent with Dahlgran's. His elasticity for cull cows ranged from 0.0 in the short run to 1.6 after 20 years.

The elasticities for feed are highly inelastic. The largest one being -0.20 for Appalachia. A couple of the regions, the Northeast and Southern Plains have positive elasticities. But these are highly inelastic at 0.10 and 0.08. The own price elasticity of demand for feed complies with Howard and Shumway's study. Howard and Shumway's elasticities are slightly more elastic at -0.58 for the Generalized Leontiff and -0.28 for the Normalized Quadratic. Elasticities out of Huy's study are broken into feed and roughage. Their elasticities for roughage contained the wrong sign for the Northeast, Upper-Midwest and Cornbelt. Their demand for concentrate was negative and inelastic for all regions except the Upper-Midwest. This studies signs conflict with Huy's study. Unexpected signs do not match up between regions. The inelastic estimates are as expected since there are few substitutes for feed. A increase in feed price will not greatly affect the production plans of farmers.

Elasticities for labor are also inelastic. The Northeast and Pacific contain an unexpected sign, but the remaining regions are negative. Appalachia and the Southern Plains are the most elastic at -0.4. This implies a greater ability to substitute labor for other goods such as capital. Howard and Shumway came up with a positive sign for the elasticity on labor. Huy et al calculated negative elasticities for all regions except the Upper-Midwest and

Cornbelt. However, their study estimated Appalachia as the most inelastic demand for labor.

Elasticities between input and outputs were mixed in terms of satisfying the a priori expectations. However, it should be noted, that no statistical test was conducted. So an unexpected sign for any of the elasticities may not be significantly different from zero6 .

Also, it is fairly common to estimate inaccurate elasticities. Young notes that "elasticities are often very sensitive to data composition and variable construction procedures". Several studies noted conflicting results. Young's elasticities of demand for agricultural land contradicted results by Lopez and Binswanger.

6/ See Anderson and Thursby for a test on elasticities for the translog function.

CHAPTER 6

FORECAST OF THE ECONOMIC IMPLICATIONS OF BOVINE SOMATOTROPIN FOR THE UNITED STATES DAIRY INDUSTRY

This section predicts initial changes in netput quantities due to the introduction of bST. The theoretical reasoning and procedure are as follows. First, bST shifts out the production function which is represented by a shift in milk supply and feed demand. Figure 6.1 represents the dairy industry at market level. Infinitely elastic supplies are assumed for all inputs and infinitely elastic demands are assumed for the outputs except milk. For milk, equilibrium price decreases as supply moves down the demand curve. The new equilibrium price then shifts all related net supplies. Table 6.3 lists the percent change in milk price given an increase in milk output and the elasticity of demand. Figure 6.1 shows the change as the difference between q_i^o and q_i^1. The price of feed does not change because it is assumed the dairy industry is a pure competitor for this good. Quantities of net supplies later adjust to the new milk price. The percent change in quantity of net supplies are given in Table 6.4. The partial adjustment coefficient was assumed to be unity which implies immediate adoption and adjustment to bST.

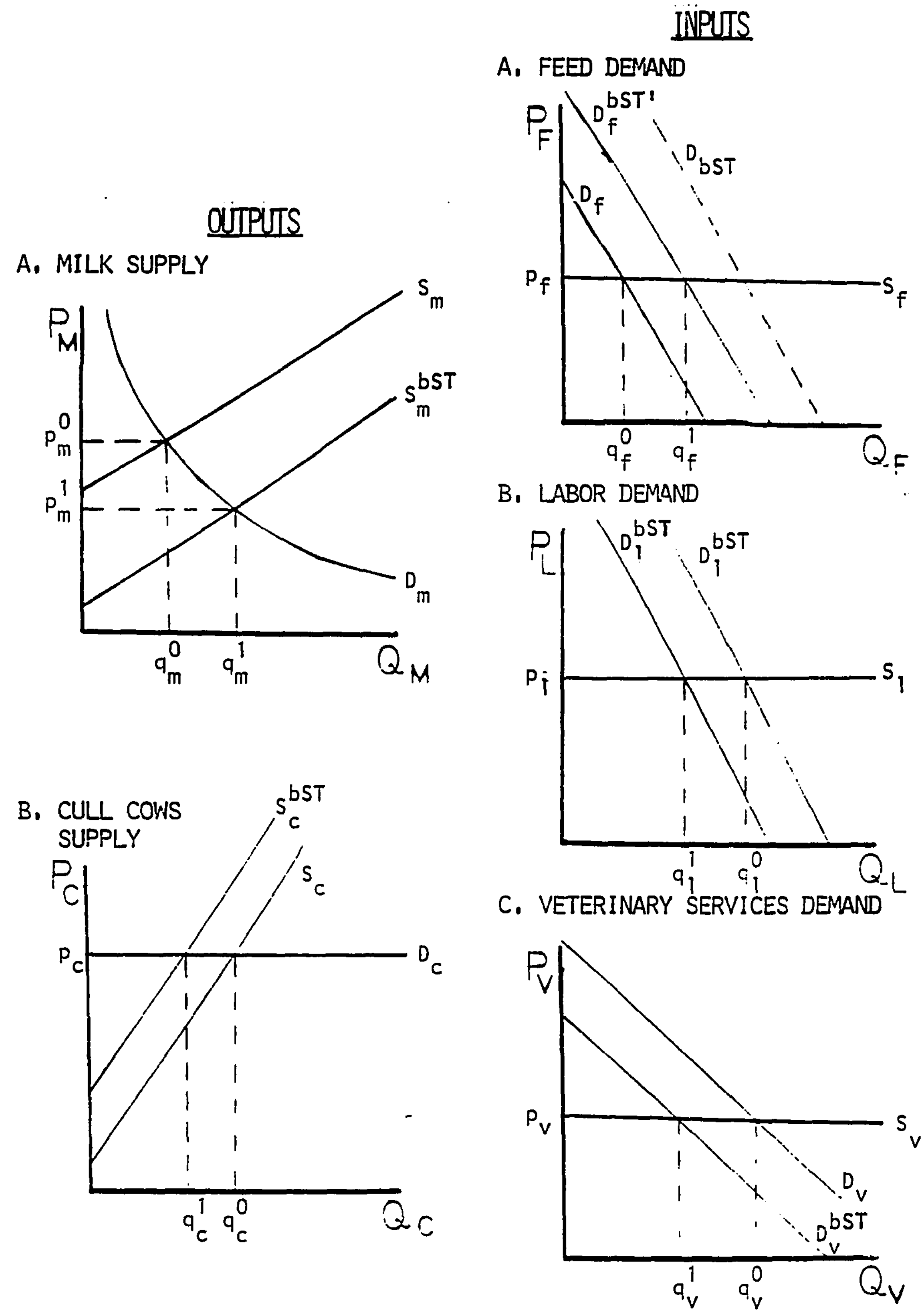

FIGURE 6.1. GRAPHICAL DEPICTION OF THE IMPACT OF BST

Supply and Demand Shift

The introduction of bST will shift the production function and net supplies. Demand for feed, veterinary services and supply of milk will shift outwards. The magnitude or direction of change in the remaining netputs is unknown, so no shift was assumed.

Feed demand was assumed to increase by ten percent. Brumby and Hancock estimated a twelve percent increase in feed intake. While, Professor Huber of the University of Arizona felt five or six percent was more appropriate. This study chose ten percent as a conservative approximation to Brumby and Hancock.

Past research indicates increases in milk production from ten to thirty percent. Table 6.1 lists several examples.

Table 6.1. Previous Studies of bST and the Increase in Milk Production

Author	Period	Dosage	Production Response
Bines and Hart	25days	30mg/day	14.8%
Brumby and Hancock	12wks	50mg/day	45.3%
Machlin	10wks	NA	33.3%
Peel et. al.	22wks	50mg/day	17.7%
Eppard and Bauman	30wks	27mg/day	
Pituitary bovine somatotropin			16.5%
Methionyl bovine somatotropin			36.2%

Because the period for none of these studies covered an entire lactation, the average increase in output will be different from that shown in Table 6.1. For example, Eppard and Bauman's study period was over the second and third trimester of lactation. The average increase in output over the entire lactation is calculated to be 25.6 percent(Yonkers).

Furthermore, laboratory responses use high quality feed and better herd management techniques. Actual farm conditions may lower the increase in output. The study examined changes in milk supply at five percent increments which should cover a range of possible outcomes.

The veterinary services input was originally included to reflect how the use of intensive herd health management services by the dairy industry, responds to changing economic conditions. That is, it was felt that a shift in veterinary services would serve as a proxy to the newly acquired demand for bST. However, the magnitude of the shift was uncertain. There are several ways to approximate this shift, all of which are arbitrary in their use of assumptions[7].

7/ For example, Fallert et.al. argued that a floor of two dollars in revenue must be earned for every dollar of bST. For this study, that change in revenue is the difference in profits before and after the shift in milk supply and feed demand. The change in the quantity of veterinary services would be calculated as all other netputs. The increased revenue would be divided by wages for veterinary services.

A second example would employ Kalter's cash budgets on the cost of manufacturing bST. Kalter varied output

Because of the arbitrary nature of the choice, net supply for veterinary services was not changed. Instead, note that cross elasticities with respect to the quantity of veterinary services become more inelastic. Let v^{*}_{ii} stand for veterinary services in equations 4.16 and 4.17. Now shock the production function by introducing bST, the quantity of veterinary services would increase and e_{ij} would decrease.

Parallel shift in net supplies were arbitrarily assumed[8]. This is the equivalent to changing the first order terms in the normalized quadratic profit function. Net supplies are given by,

$$(6.1a)\quad x_{i,t} = (1-k_i)x_{i,t-1} + \gamma_{o,i} + \sum_{i=0}^{n}\gamma_i p_i$$

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obtaining point estimates for the average cost of bST. Assuming pure competition, one can take price equal to the minimum average cost, then multiply against the quantity administered per cow per year. This yields the change in netput cost for veterinary services. The change in quantity is the change in cost divided by wages for veterinary services. However, Kalter's estimates show considerable returns to scale. A plants minimum average cost manufactures enough bST to administer to 6.5 million cows. The average number of cows in the United States is approximately 11 million. Depending on conditions for international trade of bST, manufacturers may be able to exercise some market power. Thus, price may not be at the minimum average cost.

8/ An alternative approach to representing a technological change such as bST is to directly shift the production function. Equation 4.3 lists the model of net supplies. After imposing symmetry a fifth equation for milk supply was included. The five net supplies are a function of five prices, lagged quantities and a trend variable. The production function can be found by eliminating all price variables.

An exogenous increase in this function by a proportion δ is,

$$(6.1b)\quad \delta = \frac{x'_{i,t} - x_{i,t}}{x_{i,t}} = \frac{c'_{o,i} - c_{o,i}}{\gamma_{o,i}}$$

However, $\gamma_{o,i}$ is the first order approximation in a Taylor series.

$$(5.1c)\quad \gamma'_{o,i} = (1+\delta)\gamma_{o,i}$$

Decrease In Milk Price

After the transformation function shifts outward the quantity of milk supplied at the original price exceeds the quantity demanded. The blend price falls until the quantity supplied is equal to the quantity demanded. The change in milk price is given by varying the demand elasticity and output of milk. The rows of Table 6.3 vary the elasticity of demand from -0.2 to -0.6. These values were chosen because the demand for most food items are inelastic and greater than -0.6(Knutson, Penn and Boehm). The columns of table 6.3 represent the percent increase in output. Finally, inside the table is the change in milk price. An example of one of the tables results is a 15% increase in output with demand elasticity at -0.4 decreases price by 37.5%.

Table 6.3. Percent Decrease In Price

demand elast.	Percent Increase in Output 0.05	0.10	0.15	0.20	0.25	0.30
-0.2	-25.0	-50.0	-75.0	-100.0	-125.0	-150.0
-0.4	-12.5	-25.0	-37.5	-50.0	-62.5	-75.0
-0.6	-08.3	-16.6	-25.0	-33.3	-41.7	-50.0

Some of the results in Table 6.3 are meaningless. For example, price will not decrease by 150 percent. Instead, firms are forced to drop out of business as price falls below fixed costs. Supply will cut back and price will remain positive.

Changes In The Quantities of Related Goods

Once milk price decreases. Net supplies shift and the quantity demanded or supplied changes. The initial change can be calculated using cross-price elasticities($\%\Delta Q_i/\%\Delta P_m$). The change in quantities is represented by $q_i^1 - q_i^o$ on the horizontal axes of Figure 6.1. Tables 6.4 to 6.7 and Figure 6.2 to 6.4 show the percent change in the quantity of netputs given an increase in the quantity of milk. Across the top of each table are percentage increases in milk supply. While each row represents a different region. Three tables are given for each netput. They represent different elasticities of demand for milk. Finally, figures 6.2 - 6.4 give an example of the relationship between an increase in milk supply and the change in netput use. The graphs assume a -0.4 demand elasticity for milk and a ten percent increase in the quantity of feed.

Table 6.4. PERCENT CHANGE IN QUANTITY OF CULL COWS

Table 6.4a. Percent Change in Cull Cows With the Elasticity of Demand for Milk Equal to -0.2

Region	Percent Increase In Milk Output					
	5	10	15	20	25	30
USA	-2.66	-5.31	-7.96	-10.62	-13.28	-15.93
Appalachia	1.05	2.11	3.16	4.21	5.26	6.31
Cornbelt	-1.51	-3.01	-4.51	-6.02	-7.53	-9.03
Northeast	0.41	0.81	1.21	1.62	2.03	2.43
Pacific	0.17	0.35	0.51	0.69	0.86	1.05
Southern Plains	-0.36	-0.72	-1.08	-1.45	-1.81	-2.17
Upper Midwest	0.79	1.58	2.37	3.16	3.95	4.74

Table 6.4b. Percent Change in Cull Cows With the Elasticity of Demand for Milk Equal to -0.4

Region	Percent Increase In Milk Output					
	5	10	15	20	25	30
USA	-1.33	-2.66	-3.98	-5.31	-6.64	-7.96
Appalachia	0.53	1.05	1.58	2.11	2.63	3.16
Cornbelt	-0.75	-1.51	-2.26	-3.01	-3.76	-4.51
Northeast	0.20	0.41	0.61	0.81	1.01	1.21
Pacific	0.08	0.17	0.25	0.34	0.43	0.51
Southern Plains	-0.18	-0.36	-0.54	-0.72	-0.90	-1.08
Upper Midwest	0.39	0.79	1.18	1.58	1.97	2.37

Table 6.4c. Percent Change in Cull Cows With the Elasticity of Demand for Milk Equal to -0.6

Region	Percent Increase In Milk Output					
	5	10	15	20	25	30
USA	-0.89	-1.77	-2.66	-3.54	-4.43	-5.31
Appalachia	0.35	0.70	1.05	1.40	1.75	2.11
Cornbelt	-0.50	-1.00	-1.51	-2.01	-2.51	-3.01
Northeast	0.14	0.27	0.41	0.54	0.68	0.81
Pacific	0.05	0.11	0.17	0.23	0.28	0.34
Southern Plains	-0.12	-0.24	-0.36	-0.48	-0.60	-0.725
Upper Midwest	0.26	0.52	0.79	1.05	1.31	1.58

Table 6.5. PERCENT CHANGE IN THE QUANTITY OF FEED

Table 6.5a. Percent Change in the Quantity of Feed With Elasticity of Demand for Milk Equal to -0.2

Region	Percent Increase In Milk Output					
	5	10	15	20	25	30
USA	2.71	5.42	8.13	10.84	13.55	16.26
Appalachia	3.71	7.43	11.14	14.86	18.57	22.28
Cornbelt	0.88	1.75	2.63	3.5	4.38	5.25
Northeast	-2.53	-5.06	-7.59	-10.12	-12.65	-15.18
Pacific	-8.49	-16.99	-25.49	-33.99	-42.487	-50.98
Southern Plains	0.74	1.49	2.235	2.98	3.72	4.47
Upper Midwest	-0.96	-1.92	-2.88	-3.84	-4.80	-5.76

Table 6.5b. Percent Change in Quantity of Feed With Elasticity of Demand for Milk Equal to -0.4

Region	Percent Increase In Milk Output					
	5	10	15	20	25	30
USA	1.36	2.71	4.07	5.42	6.78	8.13
Appalachia	1.86	3.71	5.57	7.43	9.28	11.14
Cornbelt	0.44	0.88	1.31	1.75	2.19	2.63
Northeast	-1.26	-2.53	-3.79	-5.06	-6.32	-7.59
Pacific	-4.24	-8.49	-12.74	-16.99	-21.24	-25.49
Southern Plains	0.37	0.74	1.11	1.49	1.86	2.23
Upper Midwest	-0.48	-0.96	-1.44	-1.92	-2.40	-2.88

Table 6.5c. Percent Change in Quantity of Feed With Elasticity of Demand for Milk Equal to -0.6

Region	Percent Increase In Milk Output					
	5	10	15	20	25	30
USA	0.90	1.81	2.71	3.61	4.52	5.42
Appalachia	1.24	2.48	3.71	4.95	6.19	7.43
Cornbelt	0.29	0.58	0.88	1.17	1.46	1.75
Northeast	-0.84	-1.69	-2.53	-3.37	-4.22	-5.06
Pacific	-2.83	-5.66	-8.49	-11.33	-14.16	-16.99
Southern Plains	0.24	0.49	0.74	0.99	1.24	1.49
Upper Midwest	-0.32	-0.64	-0.96	-1.28	-1.60	-1.92

Table 6.6. PERCENT CHANGE IN THE QUANTITY OF LABOR

Table 6.6a. Percent Change in the Quantity of Labor With Elasticity of Demand for Milk Equal to -0.2

Region	Percent Increase In Milk Output					
	5	10	15	20	25	30
USA	16.02	32.04	48.06	64.08	80.10	96.12
Appalachia	2.96	5.92	8.87	11.83	14.79	17.74
Cornbelt	6.48	12.95	19.42	25.90	32.38	38.85
Northeast	-3.68	-7.36	-11.04	-14.72	-18.40	-22.08
Pacific	-4.41	-8.83	-13.25	-17.67	-22.08	-26.50
Southern Plains	-13.84	-27.68	-41.52	-55.37	-69.21	-83.05
Upper Midwest	2.66	5.32	7.98	10.65	13.31	15.97

Table 6.6b. Percent Change in the Quantity of Labor With Elasticity of Demand for Milk Equal to -0.4

Region	Percent Increase In Milk Output					
	5	10	15	20	25	30
USA	8.01	16.02	24.03	32.04	40.05	48.06
Appalachia	1.48	2.96	4.44	5.92	7.39	8.87
Cornbelt	3.24	6.48	9.71	12.95	16.19	19.42
Northeast	-1.84	-3.68	-5.52	-7.36	-9.2	-11.04
Pacific	-2.20	-4.41	-6.62	-8.83	-11.04	-13.25
Southern Plains	-6.92	-13.84	-20.76	-27.68	-34.60	-41.52
Upper Midwest	1.33	2.66	3.99	5.32	6.65	7.98

Table 6.6c. Percent Change in the Quantity of Labor With Elasticity of Demand for Milk Equal to -0.6

Region	Percent Increase In Milk Output					
	5	10	15	20	25	30
USA	5.34	10.68	16.02	21.36	26.70	32.04
Appalachia	0.99	1.97	2.96	3.94	4.93	5.92
Cornbelt	2.16	4.32	6.48	8.63	10.79	12.95
Northeast	-1.23	-2.45	-3.68	-4.91	-6.13	-7.36
Pacific	-1.47	-2.94	-4.41	-5.89	-7.36	-8.83
Southern Plains	-4.61	-9.22	-13.84	-18.45	-23.07	-27.68
Upper Midwest	0.88	1.77	2.66	3.55	4.43	5.32

Table 6.7. PERCENT CHANGE IN THE QUANTITY OF VETERINARY SERVICES

Table 6.7a. Percent Change in the Quantity of Veterinary Services With Elasticity of Demand for Milk Equal to -0.2

Region	Percent Increase In Milk Output					
	5	10	15	20	25	30
USA	13.14	26.27	39.4	52.54	65.68	78.81
Appalachia	5.53	11.06	16.58	22.11	27.64	33.16
Cornbelt	3.06	6.12	6.17	12.23	15.29	18.34
Northeast	-1.71	-3.41	-5.11	-6.82	-8.52	-10.23
Pacific	2.19	4.38	6.57	8.76	10.95	13.14
Southern Plains	0.79	1.59	2.38	3.18	3.97	4.77
Upper Midwest	9.94	19.89	29.83	39.78	49.72	59.67

Table 6.7b. Percent Change in Quantity of Feed With Elasticity of Demand for Milk Equal to -0.4

Region	Percent Increase In Milk Output					
	5	10	15	20	25	30
USA	6.57	13.14	19.70	26.27	32.84	39.40
Appalachia	2.76	5.53	8.29	11.06	13.82	16.58
Cornbelt	1.53	3.06	4.59	6.12	7.64	9.17
Northeast	-0.85	-1.71	-2.56	-3.41	-4.26	-5.11
Pacific	1.09	2.19	3.28	4.38	5.47	6.57
Southern Plains	0.39	0.79	1.19	1.59	1.98	2.38
Upper Midwest	4.97	9.94	14.91	19.89	24.86	29.83

Table 6.7c. Percent Change in the Quantity of Veterinary Services With Elasticity of Demand for Milk Equal to -0.6

Region	Percent Increase In Milk Output					
	5	10	15	20	25	30
USA	4.38	8.76	13.14	17.51	21.89	26.27
Appalachia	1.84	3.69	5.53	7.37	9.21	11.06
Cornbelt	1.02	2.04	3.06	4.08	5.10	6.12
Northeast	-0.57	-1.14	-1.71	-2.27	-2.84	-3.41
Pacific	0.73	1.46	2.19	2.92	3.65	4.38
Southern Plains	0.26	0.53	0.79	1.06	1.32	1.59
Upper Midwest	3.31	6.63	9.94	13.26	16.57	19.89

This forecast uses comparative statics. It is assumed that only the price of milk changes and any change in other netput quantities will not affect market prices. However, if these changes in turn affect their respective market prices then the actual outcome will occur at a different point along the profit function surface.

Also, it has been arbitrarily assumed that the technological change will shift milk supply and feed demand in a parallel fashion. This is equivalent to rotating the estimated profit function on the feed and milk axes while holding all second order terms constant. In actuality the new profit function can assume any shape and elasticities will change.

Again, many of the results are impossible because, several cross price elasticities are questionable. For example, a percentage increase for cull cows in the U.S. is smaller than for any region. It is possible that regions not included, responded with very small changes in quantity. However, these are typically lower producing regions. Intuitively, one would not expect much impact from these regions.

One possibility for the questionable elasticities is that price supports placed milk price on an upward trend increasing milk production throughout the data period. To accurately estimate the technology set in any given period a range of prices is needed. However, due to technological change or changes in resource endowments a dairy farmer's technology set will change over time. Therefore, the profit maximizing quantities chosen for a given price vector may fall along a series of different technology sets. It is true that the trend variable will pick up any linear changes that are unexplained by prices. However, if price supports have steadily increased the blend price relative to other goods, then it is absurd to assume the trend variable will account for technological change over time.

This study applied partial adjustments to a system of net supplies. Results suggested immediate adjustment for most netputs. Similar to this project is a dynamic model proposed by Epstien and implemented by Howard and Shumway. The advantage of this model over the dynamic framework is it's ease in application.

Finally table 6.8 summarizes the most likely result of the introduction of bST as dictated by this study. The percentage change in output is given with a ten percent increase in milk supply, -0.4 demand elasticity and a ten percent increase in feed demand.

Table 6.8. Percent Change in Netputs Given a 10% Increase in Milk Output and an Elasticity of Demand for Milk Equal to 0.4

Region	Cull Cows	Feed	Labor	Veterinarian Services
USA	-2.6	2.7	16.0	13.1
Appalachia	1.0	3.7	2.9	5.5
Cornbelt	-1.5	0.8	6.4	3.0
Northeast	0.4	-2.5	-3.6	-1.7
Pacific	0.2	-8.5	-4.4	2.2
Southern Plains	-0.4	0.7	-13.8	0.8
Upper Midwest	0.79	-0.96	2.66	9.9

Impact of bST on Dairy Farmers

In conclusion, one can examine the winners and losers to bST. The nation ends up using more labor for dairy production. This is intuitively possible since the hormone may need to be injected on a daily basis. Also, the incidence of disease may rise requiring better management by herds men. Finally, record keeping will increase with daily administrations and increased output. Three regions, Cornbelt, Southern Plains and Northwest use less labor and fewer cows. This can occur if there is a large jump in production due to bST and a small increase in the quantity of milk demanded. Confusingly, the Pacific and Southwest decreases labor but adds to the number of cows. This goes against one's intuition since bST increases production per cow.

Kalter suggested that unemployment may jump with rapid adoption. Our partial adjustment suggests immediate adoption, however only three regions use less labor. This study contradicts the assumption that bST is like other labor saving technologies. All regions will not lose by employing less labor. In fact, depending on the revenue earned by dairies and type of linkages to dairies certain rural areas in the Upper-Midwest, Cornbelt and Appalachia may benefit with higher tax revenue and increased business activity.

The number of cull cows increase in Appalachia, Northeast, Pacific and Upper Midwest. The increased rate of culling could imply a long term trend in decreasing the stock of cows. Or, it could imply a faster depreciation rate for heifers. For example, the study summarized by Fallert argues that first lactation heifers may produce more milk than older cows. That implies dairies would have incentive to cull at an earlier age. Regardless, the increased rate of culling increases the velocity of transactions for local communities and increases revenue given constant prices and local money supply. If the demand for culled heifers is inelastic and price decreases then revenue will also decrease. The number of culled cows decrease in the United States, Northeast and Southern Plains. Since partial adjustments were assumed at unity, a decreased rate of culling implies an accumulation of stock in the short and long run.

The quantity of feed demanded decreases in three regions the Cornbelt, Upper-Midwest and Pacific. Less feed demanded can occur if fewer cows are employed. The number of cull cows increase in each of these regions which can imply fewer cows. These dairies benefit in terms of less cost on feed. If farmers in the Northeast and Upper-Midwest grow their own feed, then additional revenue may be earned with land no longer needed for dairy production. However, land prices may fall due to the decrease in opportunity cost.

Several regions increase in the quantity of feed demanded. These dairies "lose" in terms of higher feed costs. The Southern Plains also earns less revenue from cull cows, which could mean this area stands the least to benefit. Whether dairies can meet the squeeze in profits depends on the the government price supports and remaining netput revenue. If government holds price such that movement along the demand curve is elastic then dairies will gain revenue. If the change in price is inelastic then dairies lose revenue and have to look elsewhere to maintain profits.

For veterinary services costs increase for all regions except the Northeast. This falls in line with the current controversy over bST. The FDA is concerned about whether bST will increase the occurrence of disease in heifers. It is interesting to note that this study associates lower milk price with more veterinary services. One must be leery since milk price has steadily risen for over ten years.

This study lays doubt to concerns about bST increasing the size of dairies. The partial adjustment coefficient was typically insignificantly different from one. Which implies complete adjustment to the new technology during the first period. It is doubtful that dairies can significantly expand in size after one year. If these results hold true, concerns raised in Goldschmidt's hypothesis do not apply to bovine somatotropin.

Whether rural areas benefit or suffer due to less revenue from dairy farmers depends on cost and revenue from all netputs. Revenue earned from milk production will decrease if the price supported demand for milk is inelastic. Costs and revenue from the remaining netputs vary from region to region. So the probability of survival and change in profits for dairies will vary from region to region. Also, even if dairies go out of business the final impact upon the community is uncertain. Dairies must be a major source of revenue for a local community to feel any impact by bovine somatotropin.

APPENDIX A

Proof of Hotelling's lemma(Beattie and Taylor)

claim,

$$\frac{d\Pi(P)}{dp_i} = \phi_i(P) \qquad i = 1,\ldots,n$$

proof,

$$\frac{d\Pi(P)}{dp_i} = \frac{\Sigma_j p_j \cdot \phi_j(P)}{dp_i} + \phi i(P) \qquad i \neq j$$

by definition,

$$v_i = \phi_i(P) \text{ and } F(V) = 0$$

thus,

$$F(\phi_1(P),\phi_2(P),\ldots,\phi_n(P)) = 0$$

this implies,

$$\frac{\Sigma_j\, dF(V)}{dv_j} \cdot \frac{d\phi_j(P)}{dp_i} = 0 \qquad \text{(a)}$$

By FOC,

$$\frac{dF(V)}{d(v_i)} = -\frac{1}{k}p_i \qquad i = 1,\ldots,n \text{ (b)}$$

where k is the Lagrange multiplier
substitute (b) into (a),[9]

$$-\frac{1}{k}\Sigma_j p_j \frac{d\phi_j(P)}{dp_i} = 0$$

$$\Sigma_j p_j \frac{d\phi_j(P)}{dp_i} = 0$$

so,

$$\frac{d\Pi(P)}{dp_i} = p_j \cdot \frac{\Sigma_j d\phi_j(P)}{dp_i} + \phi i(P)$$

$$= \phi_i(P)$$

9/ This also states that supply and demand equations are homogeneous of degree zero. Young's theorem and Hotelling's lemma imply that $d\Pi(P)/dp_i dp_j = d\phi_i/dp_j = d\phi_j/dp_i$. Euler's equation is obtained by substituting the last term into (b).

APPENDIX B

Conditions for Profit function.

1. Weak Monotonicity - profits are non-decreasing (increasing) in net-output (input) prices.

proof - Let V and V' represent two profit maximizing bundles for P and P'. Also, assume $P' \geq P$, the $\geq$ symbol means that $p'_i \geq p_i$ for all $i=1,\ldots,n$. Now look at an increase in output prices It follows that $P' \cdot V' \geq P' \cdot V$ and $P' \cdot V \geq P \cdot V$. The two inequalities imply $P \cdot V \geq P' \cdot V'$. Likewise, an increase in net-input prices will not increase profits i.e. $P' \cdot V' \leq P \cdot V$.

2. Linear Homogeneity - the dual function is homogeneous of degree one in all prices i.e. $\Pi(tP)=t\Pi(P)$.

proof - The property argues that the same netput bundle, V, maximizes profits at the price vector P and a proportionally larger set of prices tP. Suppose this were not true. Suppose V does not maximize profits at tP. Instead let V' be the profit maximizing bundle at tP. This implies,

$$(tP)V' > (tP)V$$
$$P \cdot V' > P \cdot V$$

But we already showed that the profit maximizing bundle is unique. So the second equation V'>V can not hold. Therefore multiplying prices by a constant will not change the profit maximizing netput bundle. So, homogeneity of degree one must hold.

3. Convexity - The profit function is convex upwards(downwards) for output(input) prices.

proof - Let $P''=(1+t)P''-tP'=P+t(P-P')$, (P,V) and (P',V') be two profit maximizing bundles. Also let V" be the profit maximizing bundle at P" such that:

$$\Pi(P'')=P''V''=(1+t)PV''-tP'V'' \qquad (a)$$

However, points P' and P do not have to maximize profits at V i.e.

$$\Pi(P) \geq PV'' \qquad (b)$$

and

$$\Pi(P') \geq P'V'' \qquad (c)$$

putting a, b and c into equation form, we get:

$$\Pi(P'') \leq (1+t)\Pi(P) - t\Pi(P')$$

4. Continuity - The profit function is continuous in prices.

proof - Varian notes that all concave functions are continuous at all points in the domain of P.

APPENDIX C

Regression Results For Each Region

United States

Number of Iterations - 6
-2 lnLikelihood - 228.306
Sum of Squares Error - 50.060

Parameter Estimates

Coefficient	Value	Standard Error	T-Stat ($H_o: \gamma=0$)	Prob(γ_{ij}>t)
$\gamma_c=(1-k_c)$	-0.483	0.079	-6.093	0.000
$\gamma_f=(1-k_f)$	-0.181	0.241	-0.753	0.455
$\gamma_l=(1-k_l)$	-0.010	0.119	-0.081	0.936
$\gamma_m=(1-k_m)$	-0.135	0.172	-0.783	0.437
$\gamma v=(1-k_v)$	-0.075	0.139	-0.542	0.591
$\gamma_{1,c}$	0.899	0.052	17.377	0.000
$\gamma_{c,c}$	0.001	0.000	6.840	0.000
$\gamma_{c,f}$	0.048	0.010	4.762	0.000
$\gamma_{c,l}$	0.002	0.000	4.100	0.000
$\gamma_{c,v}$	0.002	0.000	5.272	0.000
$\gamma_{c,t}$	8.864E-05	0.000	0.757	0.452
$\gamma_{1,f}$	-86.916	17.077	-5.090	0.000
$\gamma_{f,f}$	4.486	12.880	0.348	0.729
$\gamma_{f,l}$	0.390	0.058	6.712	0.000
$\gamma_{f,v}$	0.015	0.000	4.100	0.000
$\gamma_{f,t}$	-1.018	0.308	-3.306	0.002
$\gamma_{1,l}$	-0.839	0.084	-9.990	0.000
$\gamma_{l,l}$	0.009	0.002	4.191	0.000
$\gamma_{l,v}$	-0.004	0.002	-2.578	0.013
$\gamma_{l,t}$	-0.001	0.001	-0.829	0.411
$\gamma_{1,v}$	-0.839	0.084	-9.990	0.000
$\gamma_{v,v}$	0.009	0.002	4.191	0.000
$\gamma_{v,t}$	-0.001	0.001	-2.112	0.040
$\gamma_{1,m}$	120.806	17.583	6.871	0.000
$\gamma_{m,t}$	2.404	0.400	6.014	0.000

Appalachia

Number of Iterations -9
-2 lnLikelihood - 326.033
Sum of Squares Error - 50.000

Parameter Estimates

Coefficient	Value	Standard Error	T-Stat ($H_o: \gamma=0$)	Prob(γ_{ij}>t)
$\gamma_c=(1-k_c)$	-0.064	0.408	-0.157	0.876
$\gamma_f=(1-k_f)$	0.166	0.227	0.732	0.468
$\gamma_l=(1-k_l)$	-0.399	0.154	-2.589	0.013
$\gamma_m=(1-k_m)$	0.046	0.112	-0.410	0.684
$\gamma_v=(1-k_v)$	-0.132	0.164	-0.804	0.425
$\gamma_{1,c}$	1.043	0.349	2.8785	0.004
$\gamma_{c,c}$	-0.001	0.001	-0.830	0.411
$\gamma_{c,f}$	-0.057	0.063	-0.909	0.368
$\gamma_{c,l}$	-2.402E-05	0.000	-0.840	0.405
$\gamma_{c,v}$	2.477E-05	0.000	2.248	0.029
$\gamma_{c,t}$	0.000	0.001	0.466	0.643
$\gamma_{1,f}$	-100.943	23.667	-4.265	0.000
$\gamma_{f,f}$	26.599	17.449	1.524	0.134
$\gamma_{f,l}$	-0.003	0.002	-1.863	0.068
$\gamma_{f,v}$	0.000	0.000	0.306	0.761
$\gamma_{f,t}$	0.190	0.374	-0.508	0.613
$\gamma_{1,l}$	-0.017	0.003	-5.906	0.000
$\gamma_{l,l}$	9.138E-06	0.000	5.912	0.000
$\gamma_{v,l}$	-2.977E-06	0.000	-3.652	0.001
$\gamma_{l,t}$	-8.536E-05	0.000	-3.657	0.001
$\gamma_{1,v}$	-0.005	0.001	-6.303	0.000
$\gamma_{v,v}$	3.601E-06	0.000	4.167	0.000
$\gamma_{v,t}$	-3.825E-05	0.000	-2.885	0.006
$\gamma_{1,m}$	118.738	12.738	9.321	0.000
$\gamma_{m,t}$	2.079	0.326	6.377	0.000

Cornbelt

Number of Iterations - 20
-2 lnLikelihood - 313.876
Sum of Squares Error - 50.000

Parameter Estimates

Coefficient	Value	Standard Error	T-Stat (H_o: τ=0)	Prob(γ_{ij}>t)
$\gamma_c=(1-k_c)$	-0.385	0.171	-2.257	0.028
$\gamma_f=(1-k_f)$	0.113	0.321	0.353	0.726
$\gamma_l=(1-k_l)$	-0.274	0.157	-1.744	0.087
$\gamma_v=(1-k_v)$	-0.530	0.299	-1.769	0.087
$\gamma_m=(1-k_m)$	-0.018	0.340	-0.052	0.958
$\gamma_{1,c}$	0.943	0.116	8.120	0.000
$\gamma_{c,c}$	0.001	0.000	2.343	0.023
$\gamma_{c,f}$	0.006	0.020	0.304	0.763
$\gamma_{c,v}$	3.291E-05	0.000	2.343	0.023
$\gamma_{c,l}$	1.934E-05	0.000	1.774	0.082
$\gamma_{c,t}$	-0.000	0.000	-0.969	0.337
$\gamma_{1,f}$	-78.847	26.798	-2.943	0.005
$\gamma_{f,f}$	8.492	14.038	0.605	0.548
$\gamma_{f,v}$	0.000	0.001	-0.969	0.337
$\gamma_{f,l}$	-0.003	0.002	-1.663	0.103
$\gamma_{f,t}$	0.578	0.460	-1.257	0.214
$\gamma_{1,l}$	-0.009	0.001	-7.721	0.000
$\gamma_{l,l}$	4.337E-06	0.000	4.479	0.000
$\gamma_{l,t}$	-8.476E-05	0.000	-3.735	0.001
$\gamma_{l,v}$	-7.517E-07	0.000	-1.379	0.174
$\gamma_{1,v}$	-0.005	0.001	-5.195	0.000
$\gamma_{v,v}$	8.721E-07	0.000	1.0134	0.316
$\gamma_{v,t}$	-3.150E-05	0.000	-2.505	0.016
$\gamma_{1,m}$	107.925	34.456	3.132	0.003
$\gamma_{m,t}$	2.678	0.939	2.853	0.006

Northeast

Number of Iterations - 19
-2 lnLikelihood - 373.004
Sum of Squares Error - 50.000

Parameter Estimates

Coefficient	Value	Standard Error	T-Stat ($H_o: \gamma=0$)	Prob(γ_{ij}>t)
$\gamma_c=(1-k_c)$	-1.085	0.272	-3.987	0.000
$\gamma_f=(1-k_f)$	-0.832	0.355	-2.343	0.023
$\gamma_l=(1-k_l)$	-0.158	0.222	-0.709	0.482
$\gamma_m=(1-k_m)$	-0.443	0.222	-2.001	0.051
$\gamma_v=(1-k_v)$	-0.046	0.118	-0.389	0.699
$\gamma_{1,c}$	1.009	0.130	7.794	0.000
$\gamma_{c,c}$	0.0003	0.000	1.451	0.153
$\gamma_{c,f}$	0.005	0.009	0.508	0.614
$\gamma_{c,v}$	2.030E-05	0.000	4.088	0.000
$\gamma_{c,l}$	-5.488E-05	0.000	-5.028	0.000
$\gamma_{c,t}$	0.001	0.000	-4.997	0.000
$\gamma_{1,f}$	-63.307	12.670	-4.997	0.000
$\gamma_{f,f}$	-11.278	6.441	-1.751	0.086
$\gamma_{f,v}$	-0.001	0.000	5.422	0.000
$\gamma_{f,l}$	0.001	0.002	0.432	0.668
$\gamma_{f,t}$	-0.104	0.101	-1.024	0.311
$\gamma_{1,l}$	-0.010	0.002	-3.986	0.000
$\gamma_{l,l}$	-1.416E-06	0.000	-0.885	0.380
$\gamma_{l,v}$	-5.380E-07	0.000	-0.798	0.429
$\gamma_{l,t}$	-0.0002	0.000	-4.531	0.000
$\gamma_{1,v}$	-0.003	0.000	-7.789	0.000
$\gamma_{v,v}$	6.664E-07	0.000	1.466	0.149
$\gamma_{v,t}$	-3.734E-05	0.000	-5.530	0.000
$\gamma_{1,m}$	112.439	17.583	6.395	0.000
$\gamma_{m,t}$	2.608	0.428	6.095	0.000

Pacific

Number of Iterations - 7
-2 lnLikelihood - 257.305
Sum of Squares Error -50.121

Parameter Estimates

Coefficient	Value	Standard Error	T-Stat ($H_o: \gamma=0$)	Prob(γ_{ij}>t)
$\gamma_c=(1-k_c)$	-0.4060.330	-1.230	0.225	
$\gamma_f=(1-k_f)$	-0.340	0.164	-2.069	0.044
$\gamma_l=(1-k_l)$	-0.289	0.148	-1.955	0.056
$\gamma_m=(1-k_m)$	0.216	0.470	0.460	0.648
$\gamma v=(1-k_v)$	0.044	0.302	0.145	0.885
$\gamma_{1,c}$	0.982	0.234	4.195	0.000
$\gamma_{c,c}$	0.0003	0.000	0.906	0.369
$\gamma_{c,f}$	0.028	0.018	1.508	0.138
$\gamma_{c,v}$	2.409E-05	0.000	1.239	0.221
$\gamma_{c,l}$	3.837E-05	0.000	-1.539	0.130
$\gamma_{c,t}$	0.001	0.001	1.438	0.157
$\gamma_{1,f}$	-167.487	20.703	-8.090	0.000
$\gamma_{f,f}$	58.242	20.174	2.887	0.006
$\gamma_{f,v}$	-0.000	0.002	-0.079	0.938
$\gamma_{f,l}$	0.008	0.003	2.650	0.011
$\gamma_{f,t}$	-1.065	0.487	-2.189	0.033
$\gamma_{1,l}$	-0.014	0.003	-5.242	0.000
$\gamma_{l,l}$	-2.290E-06	0.000	-0.787	0.435
$\gamma_{l,v}$	-7.181E-07	0.000	-0.378	0.707
$\gamma_{l,t}$	0.0001	0.000	1.057	0.296
$\gamma_{1,v}$	-0.004	0.002	-1.577	0.121
$\gamma_{v,v}$	2.443E-07	0.000	0.141	0.888
$\gamma_{v,t}$	2.030E-05	0.000	0.434	0.666
$\gamma_{1,m}$	187.971	100.743	1.866	0.068
$\gamma_{m,t}$	3.902	2.059	1.895	0.064

Southern Plains

Number of Iterations - 7
-2 lnLikelihood - 323.551
Sum of Squares Error - 50.002

Parameter Estimates

Coefficient	Value	Standard Error	T-Stat ($H_o:\gamma=0$)	Prob(γ_{ij}>t)
$\gamma_c=(1-k_c)$	-0.832	0.154	-5.400	0.000
$\gamma_f=(1-k_f)$	-0.323	0.281	-1.149	0.256
$\gamma_l=(1-k_l)$	-0.109	0.279	-0.391	0.698
$\gamma_m=(1-k_m)$	0.058	0.274	0.212	0.833
$\gamma_v=(1-k_v)$	-0.173	0.123	-1.415	0.163
$\gamma_{1,c}$	1.005	0.087	11.608	0.000
$\gamma_{c,c}$	-6.264E-05	0.000	-0.138	0.891
$\gamma_{c,f}$	0.021	0.014	1.469	0.148
$\gamma_{c,v}$	-2.485E-05	0.000	-3.712	0.000
$\gamma_{c,l}$	-8.704	0.000	-0.533	0.597
$\gamma_{c,t}$	0.001	0.000	3.591	0.001
$\gamma_{1,f}$	-94.511	24.086	-3.924	0.000
$\gamma_{f,f}$	-17.017	20.284	-0.839	0.506
$\gamma_{f,v}$	0.0003	0.000	1.251	0.220
$\gamma_{f,l}$	0.007	0.004	3.591	0.001
$\gamma_{f,t}$	-1.829	0.400	-3.658	0.001
$\gamma_{1,l}$	-0.019	0.004	-4.530	0.000
$\gamma_{l,l}$	6.936E-07	0.000	2.072	0.044
$\gamma_{l,v}$	-2.526E-06	0.000	-6.764	0.000
$\gamma_{l,t}$	-0.0001	0.000	-1.358	0.177
$\gamma_{1,v}$	-0.003	0.001	-5.153	0.000
$\gamma_{v,v}$	2.510E-06	0.000	8.911	0.000
$\gamma_{v,t}$	3.276E-06	0.000	0.392	0.697
$\gamma_{1,m}$	111.269	31.327	3.552	0.001
$\gamma_{m,t}$	2.833	0.885	3.203	0.002

Upper-Midwest

Number of Iterations - 9
-2 lnLikelihood - -351.808
Sum of Squares Error - 50.001

Parameter Estimates

Coefficient	Value	Standard Error	T-Stat (H_o:γ=0)	Prob(γ_{ij}>t)
$\gamma_c=(1-k_c)$	-0.414	0.300	-1.381	0.173
$\gamma_f=(1-k_f)$	-0.095	0.255	-0.372	0.712
$\gamma_l=(1-k_l)$	-0.278	0.251	-1.108	0.273
$\gamma_m=(1-k_m)$	-0.343	0.312	-1.100	0.277
$\gamma_v=(1-k_v)$	-0.546	0.170	-3.206	0.002
$\gamma_{1,c}$	0.961	0.200	4.803	0.000
$\gamma_{c,c}$	0.0003	0.000	1.173	0.246
$\gamma_{c,f}$	0.042	0.014	3.028	0.004
$\gamma_{c,v}$	-1.633E-05	0.000	-2.528	0.015
$\gamma_{c,l}$	2.509E-05	0.000	1.906	0.062
$\gamma_{c,t}$	0.001	0.000	3.739	0.001
$\gamma_{1,f}$	-75.256	18.259	-4.122	0.000
$\gamma_{f,f}$	9.784	9.864	0.992	0.326
$\gamma_{f,l}$	0.000	0.001	0.170	0.865
$\gamma_{f,v}$	-0.001	0.001	-1.602	0.116
$\gamma_{f,t}$	-1.056	0.353	-2.994	0.004
$\gamma_{1,l}$	-0.008	0.002	-4.626	0.000
$\gamma_{l,l}$	3.185E-07	0.000	0.268	0.789
$\gamma_{l,v}$	-1.582E-06	0.000	-4.354	0.000
$\gamma_{l,t}$	5.972E-05	0.000	1.656	0.104
$\gamma_{1,v}$	-0.002	0.000	-6.097	0.000
$\gamma_{v,v}$	4.095E-07	0.000	0.600	0.551
$\gamma_{v,t}$	-1.513E-06	0.000	-1.292	0.202
$\gamma_{1,m}$	109.223	24.398	4.477	0.000
$\gamma_{m,t}$	2.547	0.695	3.554	0.001

LIST OF REFERENCES

Amemiya, Takeshi, Advanced Econometrics, Cambridge, Mass: University Press, 1985

Anderson, Richard G. and Jerry G. Thursby, "Confidence Intervals For Elasticity Estimators in Translog Models", Review of Economics and Statistics, 1986

Bauman, Dale E., "Effect of Growth Hormone on Growth Rates and Mammary Development of Ruminants." Journal of Dairy Science, Vol. 65, Supp. 1, 1982

Beattie, Bruce R. and Robert C. Taylor, The Economics of Production, John Wiley and Sons Inc., 1985

Betts, Carolyn, Costs of Producing Milk, 1975-1984, U.S. Department of Agriculture, Agricultural Economic Report Number 569

Bines, J. A. and I. C. Hart, "Metabolic Limits to Milk Production, Especially Roles of Growth Hormone and Insulin." Journal of Dairy Science, Vol. 65, No. 8, 1982

Binswanger, Hans P., "A Cost Function Approach to the Measurement of Elasticities of Factor Demand and Elasticities of Substitution.' American Journal of Agricultural Economics, May 74

Brumby, Peter J. and John Hancock, "The Galactopoetic Role of Growth Hormone in Dairy Cattle."New Zealand Journal Of Science and Technology, February 1955

Buxton, Boyd M., Tom McGuckin, Richard Selley and Gayle Willet, Milk Production A Four-State Earnings Comparison.

Caves, D.W., and L.R. Christensen, "Global Properties of Flexible Functional Forms." American Economic Review 70(1980): 442-52

Cochrane, W.W., Development of American Agriculture, University of Minnesota Press, 1979, pp.378-395

Conneman, George J., "Historical Perspective On the Adoption of New Technologies on Dairy Farms." National DHIC Farmers' Biotechnology Conference, March 10, 1987, Philadelphia Pennsylvania

Dahlgran, Roger A., "A Synthesis of Microeconomic Duality Theory and Distributed Lag Modeling With Implications For U.S. Dairy Policy." North Central Journal Of Agricultural Economics, Vol. 7, No. 1, January, 1985

Diewert, W. E., "Applications of Duality Theory.", Frontiers of Quantitative Economics, Vol. 2, Edited by M. Intrilligator and D. Kendrick, Amsterdam, North-Holland, 1974

Eppard, Phillip J. and Dale E. Bauman, The Effect of Growth Hormone on Performance of Lactating Dairy Cows," Proceedings of the Cornell Nutrition Conference for Feed Manufacturers, October 31, 1984

Epstien, Larry G., "Duality Theory and Functional Forms for Dynamic Factor Demands," Review of Economic Studies, 48(1981)

Fallert, Richard A., Tom McGuckin, Carolyn Betts, and Gary Bruner, bST and the Dairy Industry, A National, Regional and Farm-level Analysis, U.S. Department of Agriculture. Agricultural Economics Report Number 579. October 1987

Fenchel, W., "On Conjugate Convex Functions", Canadian Journal of Mathematics

Gallant, Ronald A., "The Fourier Flexible Form." American Journal of Agricultural Economics, Vol. 66, No. 2, May, 1984

Goldschmidt, W. R., As You Sow: Three Studies in the Social Consequences of Agribusiness, Montclair NJ: Allenheld Osmun and Co., 1978

Gorman, W.M., "Measuring the Quantities of Fixed Factors", Value, Capital and Growth: Papers in Honour of Sir John Hicks, Chicago: Aldine.

Guilkey, David K., "Alternative Tests For a First Order-Vector Autoregressive Error Specification, Journal of Econometrics, 2(1974), 95-104

Hallberg, M.C. and Robert Parsons, Who Will Gain and Who Will Lose From Production Technologies in the Dairy Industry, Staff Paper #104, Pennsylvania State University, January 1986

Hanoch, G., "Generation of New Production Functions Through Duality", An Econometric Approach to Production Theory, Amsterdam, North Holland Press

Howard, Wayne H., Shumway, Richard C., A Comparison of Two Functional Forms in Dynamic Analysis, Working Paper wp87/6, Department of Agricultural Economics and Business, University of Guelph, July, 87

Huy, Birgit, G. Joachim Elterich and Conrado M. Gempesaw II, Analysis of the Regional Structure of Dairy Production in the United States, A Profit Function Approach, Agricultural Experiment Station Report No. 472, University of Delaware, December, 1987

Jacobsen, S.E., "Production Correspondences", Econometrica, 1970, Volume 38, 754-770

Johnston, J., Econometric Methods, Third Edition, McGraw-Hill Book Company, 1984

Kalter, Robert J., et. al., Biotechnology and the Dairy Industry: Production Costs, Commercial Potential, and the Economic Impact of the Bovine Growth Hormone, Cornell Staff Paper, AE Research 85-20

_______________, "The New Biotech Agriculture: Unforeseen Economic Consequences.", Issues In Science and Technology, Fall 1985

Knutson, Ronald, J.B. Penn and William T. Boehm, Agricultural and Food Policy, Englewood Cliffs, New Jersey, Prentice Hall Inc., 1983

Lau, Lawrence J., "A Characterization of the Normalized Restricted Profit Function." Journal of Economic Theory, Vol. 12, 1976

_______________, "Comments on Applications of Duality Theory", Frontiers of Quantitative Economics, vol.2, ed. M.D. Intrilligator and D. A. Kendrick, Amsterdam: North-Holland Publishing Co., 1974

Lopez, Ramon, "Structural Implications of a Class of Flexible Functional Forms for Profit Functions," International Economic Review, Vol.26, 1985

Machlin, L.J., "effect of Growth Hormone on Milk Production and Feed Utilization in Dairy Cows," Journal of Dairy Science, Vol. 56, No. 5, 1973

Magrath, William B. and Loren W. Tauer, New York Milk Supply with bovine Growth Hormone, Cornell Agricultural Economics Staff Paper No. 86-15, June 1986

McFadden, D.L.,"Cost, Revenue and Profit Functions", An Econometric Approach to Production Theory, Amsterdam: North-Holland Press

Norton, George W., "Why Do We Need New Technologies In Times of Agricultural Surpluses?", IR-6 Information Report No. 87-2, Virginia Polytechnic Institute and State University, May 1987

Offut, Susan E. and Fred Kuchler, "Issues and Developments in Biotechnology: What's an Economist To Do?" Agricultural Economics Research, Vol. 39, No. 1, Winter 1987

Peel, C.J., Dale E. Bauman, Ronald C. Gorewit and Charles J. Sniffen, "Effect of Exogenous Growth Hormone on Lactational Performance in High Yielding Dairy Cows," Journal of Nutrition, Vol. 111, No. 9, 1981

Pindyck, Robert S., Daniel L. Rubinfeld, Econometric Models and Economic Forecasts, McGraw-Hill Book Company, 1981

Rockafeller, R.T., Convex Analysis, Princeton, Princeton University Press, 1970

Russell, Robert R. and Maurice Wilkinson, Microeconomics, A Synthesis of Modern and Neoclassical Theory, John Wiley and Sons, 1979

Shephard, R.W., Theory of Cost and Production Functions, Princeton, Princeton University Press, 1953

Shumway, Richard C., "Supply, Demand, and Technology in a Multiproduct Industry: Texas Field Crops." American Journal of Agricultural Economics, 1983

Theil, Henri, Principles of Econometrics, John Wiley and Sons Inc., 1971

Thompson, Gary, "Choice of Flexible Functional Forms: Review and Appraisal," Western Journal of Agricultural Economics, 1988

U.S. Department of Agriculture, Agricultural Prices, Annual Summary, 1975 to 1985

U.S. Department of Agriculture, Agricultural Statistics, 1975 to 1985

U.S. Department of Labor, Employment and Wages Annual Survey, 1978 to 1985., Bulletin 2215

Uzawa, H., "Duality Principles in the Theory of Cost and Production", International Economic Review, Volume 5, 216-220

Varian, Hal R., Microeconomic Analysis, W. W. Norton and Company, 1984

Weaver, Robert D., "Caveats on the Application of the Fourier Flexible Form: Discussion", American Journal of Agricultural Economics, May 1984

White, H. "Using Least Squares to Approximate Unknown Regression Functions." International Economic Review 21(1980):149-170

Wohlgenant, M.K., "Conceptual and Functional Form Issues in Estimating Demand Elasticities for Food." American Journal of Agricultural Economics, May, 1984

Young, Douglas L., Ron C. Mittelhammer, Ahmad Rostamizadeh and David W. Holland, Duality Theory and Applied Production Economics Research: A Pedagogical Treatise, Research Bulletin 0962, Washington State University, 1985