Toward an optimal culling strategy for beef cattle herds

TOWARD AN OPTIMAL CULLING STRATEGY FOR BEEF CATTLE HERDS

by

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ABSTRACT

Alternatives available to the ranch manager for consideration in determining an optimal cattle culling strategy are discussed. A model is developed for analyzing culling strategy based on alternatives of keep or sell with or without pregnancy testing.

Relevant exogenous variables include initial herd composition, animal weights, market prices, death and reproductive rates, and closing inventory values. Endogenous variables include the future herd composition, subsequent inventory values, and control variables specifying decision options.

Culling strategy is analyzed using a single stage (one period) linear programming model. Sensitivity of the optimal culling strategy to pregnancy rates and the ratio of the price of cows to yearling steers is discussed. The value of pregnancy test information is estimated to be relatively low for pregnancy rates ranging from .6 to .95. It is predicted that the most benefit results from pregnancy testing of older, heavier cows without calves. Results are found to be consistent with a management program that does not arbitrarily sell breeding cows before the age of 10 and does not retain steer calves for sale as yearlings.

CHAPTER I

INTRODUCTION

The Problem and Previous Work

Culling strategy may be an important factor influencing the productivity of a cattle herd. Determining the influence that a particular strategy has upon future income, however, can be difficult to ascertain. Some work of this nature has been completed in the past. Faris (1960) used marginal cost analysis to determine an optimal replacement pattern. Benson (1976) used a steady state equilibrium model to determine an optimal culling age for beef cows. Dynamic programming was used by Jenkins and Halter (1963) and Redman and Kuo (1969) to determine optimal replacement decisions in dairy cattle herds. Rogers (1971) used a multi-stage linear programming model to determine optimal replacement decisions in a beef cattle herd but failed to adequately reveal his methodology. A review of the literature failed to reveal a satisfactory theoretical framework to deal with pregnancy testing in a beef cattle herd where replacements are limited to animals raised, as is typical on Arizona ranches. Also no attempt was found to use a model to test the sensitivity of optimal culling strategy to variation in biological coefficients such as the pregnancy rate.

A culling strategy is defined here as an exhaustive set of keep or sell decisions. These decisions ultimately determine the herd composition of a ranch. In determining a best culling strategy a ranch manager's decisions are influenced by expected market prices, ranch cash flow requirements, range and weather conditions, as well as the biological potential of the animals in the herd. These influences upon culling strategy are considered below.

Expected Market Prices

Variation among market prices make it most profitable to sell a particular class of animals at one point in time and another class of animals at another point in time. However, variation in the class of animals sold is not the only effect of market price fluctuation. As sale prices decrease, reduction of the herd size may be implemented in an attempt to maintain net cash flows and maximize net incomes. Ranch managers may later expand the size of the herd so as to benefit from expected improved market prices.

Expansion and reduction of cattle numbers in the face of changing market prices on a national scale has come to be known as the "cattle cycle." The "cattle cycle" repeats roughly every 10-12 years (Ensminger, 1976). As cattle numbers increase, the sale price of animals tends to decrease as classical supply and demand theory would predict. In face of downward trending prices ranch managers tend to reduce the size of their cow herds. When herd reduction becomes a general trend on a nationwide scale market prices recede rapidly. As cattle numbers decline, the sale price for cattle begins to rise, encouraging ranchers to again expand their herds. The cycle is then complete.

Price fluctuations can be expected to have a profound effect upon culling strategy. Faced with upward trending market prices, the ranch manager may place high value upon reproductive capacity so as to be able to expand his herd as quickly as possible. Faced with downward trending prices he may place greater value upon animals with rapid weight gain capacity so as to reap the advantages of relatively high market prices.

Cash Flow Requirements

After tax cash income requirements may have a substantial effect upon culling strategy. Cash requirements may necessitate alteration of culling decisions so as to continue operation of the ranch vis-à-vis financial obligations. Cash demands may even require that the rancher sell part or all of his herd.

Culling strategy may require adjustment in the face of rising input prices. Cash may be required in order to pay for necessary inputs that rise in price or to take advantage of price reductions.

Weather and Range Conditions

Weather and range conditions can be extremely important in determining incomes realized. Weather and range variation suggest that the rancher may wish to plan his culling strategy so as to allow flexibility in dealing with varying feed availability.

Cull Decisions

Culling decisions are frequently based on the probability of pregnancy of a cow and the expected weight gain and quality of her calf. Pregnancy testing is the only reliable indicator of pregnancy for an individual cow. However, the general health and condition of the cattle along with the range or feed conditions during the breeding season may indicate the general pregnancy rate for the entire herd. The ranch manager must decide the degree of accuracy required in pregnancy determination in order to meet his production goals.

The weight and quality of a calf is determined by its genetic potential and the environment in which it is raised. Some factors which cause differences among weights are age and sex of calf, milk production of the cow, and the quality and availability of feed. Past performance of progeny from a cow along with her own weight, particularly when she was a calf, is an indication of how her future progeny will perform. Accurate records of past performance increase the ranch manager's ability to predict future performance.

Decision Simplification and Generalization

For the purpose of developing a framework for analyzing the decisions faced by the ranch manager, some simplification of the problem that confronts him is advantageous. Normally a ranch manager evaluates each animal on an individual basis. The decision framework suggested in this report deals with animals by age group rather than as individuals and assumes that a rancher estimates the mean potential for each age group of cattle in terms of weight gain, calf quality, pregnancy rate, death rate, probability of weaning a calf, and probability of a calf surviving in case of the death of its mother.

The pregnancy decision faced by the rancher may be summarized as whether to keep or sell an individual animal and what criteria to use in making this decision. A prime consideration is whether improving the quality of information available to him through pregnancy testing is worth the effort and cost. A decision diagram that graphically illustrates the simplified decision process with and without pregnancy testing is illustrated in Figure 1. This diagram depicts three decisions confronted by the ranch manager; whether to pregnancy test, whether to keep or sell a particular animal, and finally how to replace the animal if the decision to sell is made.

Chapter Outline

The nature of the culling strategy decision facing the ranch manager has been explored in Chapter I. A theoretical optimization model is specified in Chapter II with the capacity of solving over a planning horizon for control variables representing the culling decisions. A specific ranch management situation is specified in Chapter III and a sensitivity analysis using a limited form of the theoretical model is performed. The sensitivity analysis shows the effect of variation in the pregnancy rate and relative prices upon optimal culling strategy and maximum net income. Chapter IV reports the results of this analysis and suggests possible further modification of the model. The purpose of Appendices A and B is to give the reader a detailed understanding of the model specified in Chapter III. Appendix A includes a generalized tableau of the linear programming model. Appendix B is a detailed specification of all revenue equations

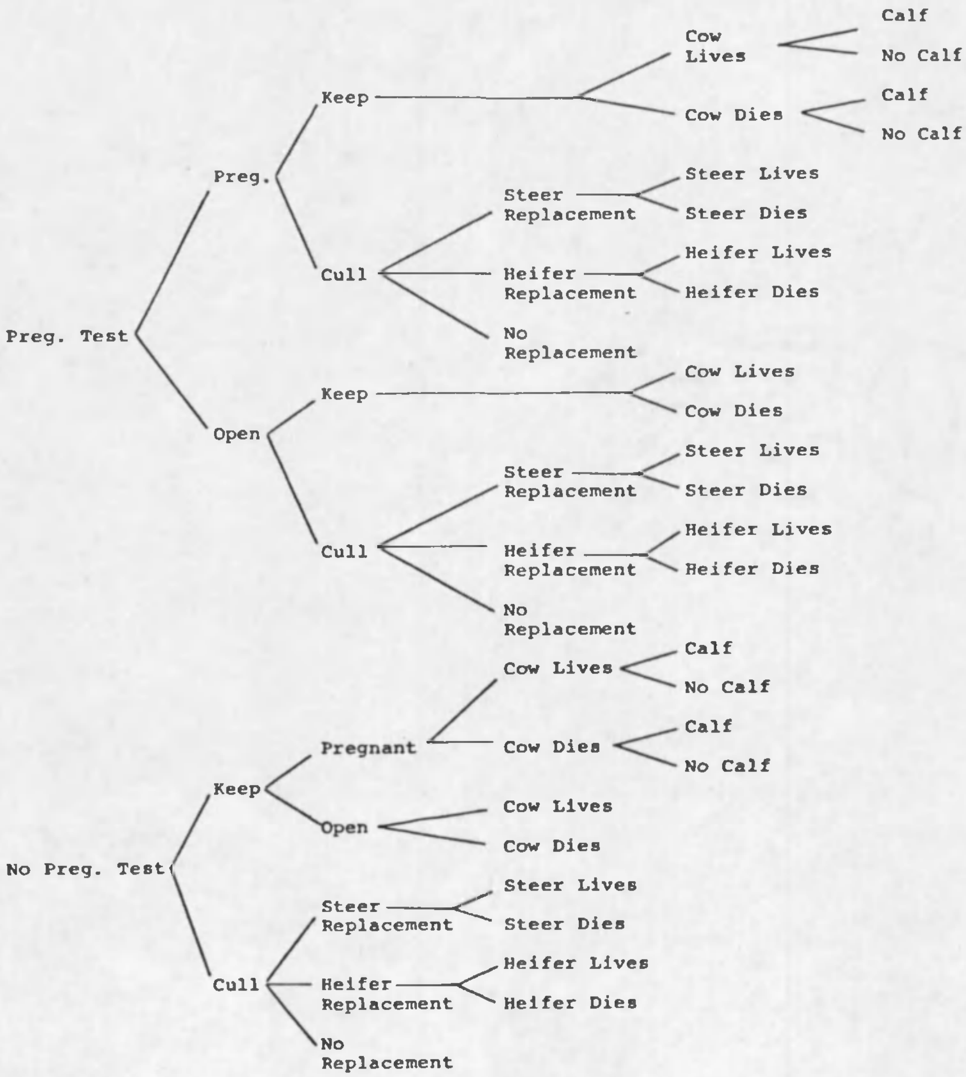

Figure 1. Decision Diagram

used in the linear programming model, a complete list of definition of variables used in these equations, and a verbal explanation of the components of one of the revenue equations.

CHAPTER II

THE THEORETICAL MODEL

Overview

A theoretical framework will be employed to analyze the culling strategy decisions faced by the ranch manager. This framework takes the form of a dynamic optimization model where the objective function is the maximization of the present value of revenue above variable costs. A discrete case of optimal control theory is used to derive the necessary conditions for maximization.

A number of assumptions are employed in the formation of the model. An initial herd composition is assumed where the animals in the herd are divided into a finite set of animal classes by age and any other characteristics distinguishable at culling time. Animals within a class are assumed to have the following identical characteristics: current and future sale prices, weights, pregnancy rates, weaning rates, death rates, and inventory values.

Each animal class is assigned appropriate control variables in the theoretical model representing possible decisions open to the ranch manager. For clarity it is assumed the culling decisions are made at the fall roundup. Transition equations are specified making the animals available at a given fall roundup a function of the animals available at the preceding fall roundup and culling decisions made with regard to these animals. This function is best described as the number of animals of a given animal class treated with a particular decision reaching each animal class in the following year.

Revenue or inventory values are assigned to each animal class treated with a specific decision. The objective function is maximized subject to a number of constraints. These constraints are: a non-negativity constraint that requires the number of animals treated with a particular cull decision to be non-negative, a constraint that limits the number of animals treated with the various cull decisions to the number of animals available for treatment, and a feed availability constraint that limits the number of animals kept by the carrying capacity of the range available to the ranch. The objective function is further limited by the transition equation discussed above.

Necessary conditions for maximization of the objective function can be determined by specifying the problem in Hamiltonian form. Necessary conditions are that the derivative of the Hamiltonian equation with respect to a given decision variable must equal zero and the derivative of the Hamiltonian with respect to a given animal class be equal to the inventory value of an animal in that animal class.

As an aid to the reader the notational procedure followed is briefly summarized:

1. Standard matrix notation is utilized although multiplication is indicated by a dot.
2. Capital Roman and Greek symbols are used to represent vectors and matrices with the exception that the unitary vector is represented by i. If used in subscripts or demensions these

symbols represent the subscript of the final element in a vector.

3. Lower case Roman or Greek symbols are used to represent individual elements or elements in a vector or matrix.
4. Elements subscripted with j and h relate to an animal class.
5. Elements subscripted with k relate to a decision variable.
6. Starred equations are key structural equations in the model. Starred variables are exogenous to the model.

The Model

Define $x_j(t)$ as the number of animals of the jth animal class in the herd at time t.

Define X(t) as a 1xJ row vector of the number of animals in the herd in each of J classes at time t

where $$X(t) = [x_1(t), \ldots, x_j(t), \ldots, x_J(t)]. \tag{II.1}$$

Define $z_{jk}(t)$ as the proportion of the animals in the jth class treated with the kth decision at time t. Let K_j be the number of possible decisions that apply to the jth class of animals.

Define $Z_j(t)$ as a $1xK_j$ row vector of the proportion of the jth class of animals treated with each of the K_j possible decisions at time t

where $$Z_j(t) = [z_{j1}(t), \ldots, z_{jk}(t), \ldots, z_{jK_j}(t)]. \tag{II.2}$$

Define K as the total number of possible decisions over all animal classes

where $$K = \sum_{j=1}^{J} K_j. \qquad \text{(II.3)}$$

Define $d(Z_j(t))$ as a JxK diagonal matrix with the row vector $Z_j(t)$ along the diagonal and zero elements off the diagonal as follows:

$$d(Z_j(t)) = \begin{bmatrix} Z_1(t) & & & & & & 0 \\ & \cdot & & & & & \\ & & \cdot & & & & \\ & & & Z_j(t) & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ 0 & & & & & & Z_J(t) \end{bmatrix} \qquad \text{(II.4)}$$

Define P_{jhk} as the number of animals reaching the hth animal class in time t+1 for each animal in the jth class treated with the kth decision in time t.

Define P_{jh} as a $1xK_j$ row vector of the number of animals reaching the hth animal class in time t+1 for each animal in the jth class at time t

where $$P_{jh} = [p_{jh1}, \ldots, p_{jhk}, \ldots, p_{jhK_j}]. \qquad \text{(II.5)}$$

Define P'_{jh} as the transpose of P_{jh}.

Define P as a KxJ matrix

where

$$P = \begin{bmatrix} P'_{11}, \ldots, P'_{1j}, \ldots, P'_{1J} \\ P'_{j1}, \ldots, P'_{jj}, \ldots, P'_{jJ} \\ P'_{J1}, \ldots, P'_{Jj}, \ldots, P'_{JJ} \end{bmatrix}. \quad \text{(II.6)}$$

Note that each element P'_{jh} is a K_j x 1 column vector.

Transition equations stating the number of animals at time t+1 as a function of the number of animals at t and their treatment may be expressed as

$$X(t+1) = X(t) \cdot d(Z_j(t)) \cdot P. \quad \text{(II.7)*}$$

Define $r_{jk}(t)$ as the expected net return above variable cost realized at time t from an animal in the jth class treated with the kth decision in time t.

Define $R_j(t)$ as a $1xK_j$ row vector of expected net returns above variable cost

where $$R_j(t) = [r_{j1}(t), \ldots, r_{jk}(t), \ldots, r_{jK_j}(t)]. \quad \text{(II.8)}$$

Define R(t) as a 1xK row vector of R_j's

where $$R(t) = [R_1(t), \ldots, R_j(t), \ldots, R_J(t)]. \quad \text{(II.9)}$$

Define $\lambda_j(t)$ as the inventory value of animals in the jth class at time t.

Define $\Lambda(t)$ as a row vector of $\lambda_j(t)$'s

where $$\Lambda(t) = [\lambda_1(t), \ldots, \lambda_j(t), \ldots, \lambda_J(t)]. \quad \text{(II.10)}$$

Define IPD(t) as the GNP price deflator at time t.

Define $\beta(t) = \frac{1}{(1+r)}^{t}$ as the present value, value at time 0, of $1 received at time t where r represents the interest or discount rate.

Assuming the objective of the ranch manager is the maximization of the present value of expected net returns above variable cost over a planning horizon of t=0 to t=T, the objective function of the model can be specified as:

$$V = \sum_{t=0}^{T-1} IPD(t) \cdot \beta(t) \cdot X(t) \cdot d(Z_j(t)) \cdot R'(t) + \Lambda(T) \cdot X'(T) \qquad \text{(II.11)*}$$

Maximization of (II.11) is subject to 3 constraints.

One of the constraints is a non-negativity constraint limiting the decision control variables to zero or positive values

$$X(t) \cdot d(Z_j(t)) \geq 0. \qquad \text{(II.12)*}$$

A second constraint limits the number of animals of each animal class treated with all relevant decisions to the number of animals available in that class.

Define i as the unit row vector

where $$i = [1, 1, 1, 1, \ldots, 1]. \qquad \text{(II.13)}$$

Then

$$X'(t) \geq d(x_j(t)) \cdot d(Z_j(t)) \cdot i'. \qquad \text{(II.14)*}$$

Feed availability also constrains the objective function.

Define M(t) as the carrying capacity of the range in year t.

Define a_{jk} as the yearly forage required for an animal of the jth animal class treated with the kth decision.

Define A_j as a $1xK_j$ row vector of elements a_{jk}

where $$A_j = [a_{j1}, \ldots, a_{jk}, \ldots, a_{jK_j}]. \qquad \text{(II.15)}$$

Define A as a 1xK row vector of A_j's

where $$A = [A_1, \ldots, A_j, \ldots, A_J]. \qquad \text{(II.16)}$$

The feed availability constraint can be stated so that the number of animals treated with decisions that require utilization of forage is limited to the carrying capacity at time t

where $$M(t) \geq X(t) \cdot d(Z_j(t)) \cdot A'. \qquad \text{(II.17)*}$$

Define X* as the initial herd composition (t=0)

where $$X(0) = X^* \qquad \text{(II.18)*}$$

Define Λ^* as the inventory value for animals in the herd at time T

where $$\Lambda(T) = \Lambda^*. \qquad \text{(II.19)*}$$

In summary the problem of selecting an optimal culling strategy is one of maximizing the objective function equation (II.11) subject to the transition equations (II.7) as well as constraint equations (II.12), (II.14), (II.17) given the initial herd composition (II.18) and inventory values at time t, equation (II.19). The model is summarized below.

Maximize

$$V = \sum_{t=0}^{T-1} IPD(t)\cdot\beta(t)\cdot X(t)\cdot d(Z_j(t))\cdot R'(t) + \Lambda(T)\cdot X'(T). \qquad \text{(II.11)*}$$

Subject to

$$X(t+1) = X(t)\cdot d(Z_j(t))\cdot P, \qquad \text{(II.7)*}$$

$$X(t)\cdot d(Z_j(t)) \geq 0, \qquad \text{(II.12)*}$$

$$X'(t) \geq d(x_j(t))\cdot d(Z_j(t))\cdot i, \qquad \text{(II.14)*}$$

$$M(t) \geq X(t)\cdot d(Z_j(t))\cdot A', \qquad \text{(II.17)*}$$

$$X(0) = X^*, \text{ and} \qquad \text{(II.18)*}$$

$$\Lambda(T) = \Lambda^*. \qquad \text{(II.19)*}$$

Necessary conditions for maximization of V can be derived by stating the problem in Hamiltonian form (Bryson and Ho, 1969).

Define υ_{jk} as a Lagrange multiplier and define T_j as a 1xK row vector of υ_{jk}'s

where $$T_j = [\upsilon_{j1}, \upsilon_{j2}, \ldots, \upsilon_{jK}, \ldots, \upsilon_{jK_j}]. \qquad \text{(II.20)}$$

Define T as a 1xK_j row vector of T_j's

where $$T = [T_1, T_2, T_j, \ldots, T_J]. \qquad \text{(II.21)}$$

Define ϕ_j as a Lagrange multiplier.

Define Φ as a 1xJ row vector of ϕ_j's

where $$\Phi = [\phi_1, \phi_2, \ldots, \phi_j, \ldots, \phi_J]. \qquad \text{(II.22)}$$

Define ω as a scaler Lagrange multiplier.

The Hamiltonian equation can be expressed as

$$\mathcal{H} = IPD(t)\cdot X(t)\cdot d(Z_j(t))\cdot R'(t)$$

$$- IPD(t+1)\cdot\beta(t)\cdot[(X(t+1)-X(t)\cdot d(Z_j(t))\cdot P)\cdot\Lambda'(t+1)]$$

$$+ X(t)\cdot d(Z_j(t))\cdot T'(t)+\Phi(t)\cdot(X'(t)-d(x_j(t)))\cdot d(Z_j(t))\cdot i'$$

$$+ (M(t)-X(t)\cdot d(Z_j(t))\cdot A')\cdot\omega(t). \qquad (II.23)*$$

Necessary conditions for the maximization of V include the following requirements (Bryson and Ho, 1969)

$$\frac{\partial\mathcal{H}}{\partial z_{jk}(t)} = 0 \text{ for all j, k, and t and} \qquad (II.24)*$$

$$\frac{\partial\mathcal{H}}{\partial x_j(t)} = \lambda_j(t) \text{ for all j and t.} \qquad (II.25)*$$

These necessary conditions are subject to the following constraint qualifications

$$\lambda_j(t), \upsilon_{jk}(t), \phi_j(t), \omega(t) \geq 0 \qquad (II.26)$$

$$X(t)\cdot d(Z_j(t))\cdot T'(t) = 0, \qquad (II.27)$$

$$\Phi(t)\cdot(X'(t)-d(x_j(t))\cdot d(Z_j(t))\cdot i') = 0, \text{ and} \qquad (II.28)$$

$$(M(t)-X(t)\cdot d(Z_j(t))\cdot A')\cdot\omega(t) = 0 \qquad (II.29)$$

for all j, k, and t

Solving for the necessary conditions $\frac{\partial\mathcal{H}}{\partial z_{jk}} = 0$:

$$\frac{\partial \mathcal{H}}{\partial z_{jk}(t)} = IPD(t)\cdot x_j \cdot r_{jk} + IPD(t+1)\cdot\beta(t)\cdot[x_j(t)\cdot \sum_{h=1}^{J} P_{jhk}\cdot\lambda_h(t+1)]$$

$$+ x_j(t)\cdot\upsilon_{jk}(t) - \phi_j(t)\cdot x_j(t)\cdot a_{jk}\cdot\omega(t) = 0. \qquad \text{(II.30)}$$

Dividing through by x_j results in the following equation

$$IPD(t)\cdot r_{jk} + IPD(t+1)\cdot\beta(t)\cdot[\sum_{h=1}^{J} P_{jhk}\cdot\lambda_h(t+1)] - \upsilon_{jk}(t)$$

$$- \phi_j(t) + a_{jk}\cdot\omega(t) = 0. \qquad \text{(II.31)}$$

Interpretation of the components of the necessary conditions (II.31) is informative for an interior point, i.e.,

where $z_{jk}(t) > 0$ and $\upsilon_{jk}(t) = 0.$ (II.32)

$IPD(t)\cdot r_{jk}$ is the revenue recovered at time t from an animal of the x_jth animal class treated with the z_{jk}th decision at time t. $IPD(t+1)\cdot\beta(t)\cdot \sum_{h=1}^{J} P_{jhk}\cdot\lambda_h(t+1)$ is the present value of the expected inventory value of an animal in the x_jth animal class treated with the z_{jk}th decision at time t.

$\phi_j(t)$ is the marginal value of an animal in the jth animal class at time t.

$a_{jk}\cdot\omega(t)$ is the marginal value of the forage required by an animal in the jth animal class treated with the kth decision.

Equation (II.31) states that the current revenue plus expected inventory value of an animal in the jth animal class should be equal to the marginal value of an animal in the jth animal class.

The necessary condition (II.25) is

$$\frac{\partial \mathcal{H}}{\partial x_j(t)} = IPD(t) \cdot Z_j(t) \cdot R_j'(t) + IPD(t+1) \cdot \beta(t) Z_j(t) \sum_{h=1}^{J} P_{jh}' \cdot \Lambda_h(t+1)$$

$$+ Z_j(t) \cdot T_j'(t) + \phi_j(t) \cdot (1 - \sum_{k=1}^{K_j} z_{jk}(t) - Z_j(t) \cdot A' \cdot \omega(t)) = \lambda_j(t) \qquad (II.33)^*$$

For $r_{jk}(t) > 0$. All animals in each class will be treated by one of the available decisions and it follows that

$$1 - \sum_{k=1}^{K_j} z_{jk}(t) = 0. \qquad (II.34)$$

Therefore (II.33) can be expressed as the following equation:

$$\frac{\partial \mathcal{H}}{\partial x_j(t)} = IPD(t) \cdot Z_j(t) \cdot R_j'(t)$$

$$+ IPD(t+1) \cdot \beta(t) [Z_j(t) \sum_{h=1}^{J} P_{jh}' \cdot \Lambda_h(t+1)]$$

$$+ Z_j(t) T_j'(t) - Z_j(t) A_j' \cdot \omega(t) = \lambda_j(t) \qquad (II.35)$$

Interpretation of Equation (II.35) follows.

$IPD(t) \cdot Z_j(t) \cdot R_j'(t)$ is the revenue received at time t from all decisions used in the jth animal class at time t. $IPD(t+1) \cdot \beta(t) \cdot Z_j(t) \sum_{h=1}^{J} P_{jh}' \cdot \Lambda_h(t+1)$ is the weighted average present value expected inventory value per head of the animals in the x_jth animal class at time t. $IPD(t+1) \cdot \beta(t) \cdot Z_j(t) \cdot A_j' \cdot \omega(t)$ is the weighted average value of the forage required by the animals in the jth animal class at time t.

Under the conditions of constraint qualification (II.27) for $x_j > 0$, $\upsilon_{jk}(t)$ is zero when $z_{jk}(t)$ is positive and $\upsilon_{jk}(t)$ is positive when $z_{jk}(t)$ is zero and therefore $Z_j(t)T'(t) = 0$. $\lambda_j(t)$ is the marginal value of an animal of the jth animal class at time t.

Equation (II.35) states that the marginal value of an animal in the jth animal class must be equal to: the weighted average revenue from all of the animals in the jth animal class, plus the weighted average expected inventory value of the animals of this class less the marginal value of weighted average forage required for the animals of this class.

The system of equations (II.7), (II.18), (II.19), (II.30), and (II.33) represents $J\cdot(T+1) + J\cdot T+K\cdot T$ equations in $J\cdot(T+1)+J\cdot T+K\cdot T$ unknowns $x_j(t)$, $z_{jk}(t)$, and $\lambda_j(t)$.

CHAPTER III

EMPIRICAL SPECIFICATION AND SENSITIVITY ANALYSIS

The model presented in Chapter II will now be used as the theoretical framework for specifying an empirical model. Most of the biological parameters used in this report are based on data and experience gathered from the Arsenic Tubs ranch. The Arsenic Tubs ranch is owned by the Apache Indian tribe of San Carlos, Arizona.

Since 1957 the San Carlos Apaches have worked cooperatively with The University of Arizona on range beef cattle production and breeding projects. A detailed description of Arsenic Tubs ranch and ranching operations is presented by Taylor (1967).

Assumptions

Five assumptions specifying the ranching situation are: (1) the ranch raises all its own replacements, (2) the rancher has the option of pregnancy testing, (3) cows first calve at 3 years of age, (4) all cows are sold by age 10, and (5) bull calves are marketed either as weaning steer calves or as yearling steers.

Animal Classes

According to United States Department of Agriculture Beef Improvement Federation (1976), many performance traits of the cow and calf are related to the age of the cow. Since cows are sold at least by age 10 the model inventories animals by age groups. Separate classes are assigned for females (cows) from age 1-10.

Cows tend to gain weight steadily until age 7. This gain is followed by roughly 3 years of relative stability and then a gradual weight loss (Benson, 1976). However, animals that have undergone the stress of pregnancy and lactation during a given year weigh less than animals that have not (Marshall, Parker, and Dinkel, 1976). As a result cows from age 3-10 are also categorized according to lactation status.

Weaning weights are influenced by age of calf, age of dam, and sex of calf (United States Department of Agriculture Beef Improvement Federation, 1976). Therefore, calves are classified by sex and age of dam, and it is assumed that the average age of calves in these categories are equal. Dogie calves, calves that survive the death of their mothers, are all assumed to be of the same relatively low weight. All yearling steers are assumed to reach the same weight by the time they are sold regardless of age of dam. Therefore there is only one class of dogie calves and one class of yearling steers specified.

In summary, cows are assigned to 18 different classes distinguishing them by age and whether they have weaned a calf in the past year. Calves make up 16 classes distinguishing them by age of dam and sex. Dogie calves make up one class as do yearling steers. Assumed weights of these 36 classes of animal at fall roundup are shown in Table III.1.

Decision Classes

It is assumed that the rancher always chooses to sell dogie calves, yearling steers, and 10 year old cows. The options to keep or

Table III.1. Animal Weights at Fall Roundup[a]

Age of Dam	Dogie[b] Calves	Heifer[c] Calves	Steer[c] Calves	Age of Animal	Yearling[b] Steers	Cows[d] with Calf at Side	Cows[d] without Calf at Side
1	--	--	--	1	730	--	700
2	--	--	--	2	--	--	830
3	330	374	410	3	--	795	954
4	330	392	430	4	--	885	1,062
5	330	410	450	5	--	955	1,146
6	330	410	450	6	--	975	1,170
7	330	410	450	7	--	1,000	1,200
8	330	410	450	8	--	1,000	1,200
9	330	410	450	9	--	1,000	1,200
10	330	410	450	10	--	1,000	1,200

[a]Animal weights are given in this table in the following manner: cow and yearling steer weights are given by age of the animal. Dogie calf weights and steer and heifer calf weights are given by age of dam.

[b]Dogie calf weight and yearling steer weight are based on a survey of faculty of the Department of Animal Science, The University of Arizona (Benson et al., 1978-1979).

[c]Weaning weight by age of dam categories are derived from average calf weights (Itulya, 1978) and adjusted to United States Department of Agriculture Beef Improvement Federation (1976) correction factors.

[d]Cow weights are established by using the growth pattern determined by Benson (1976). However, those weights are adjusted downward to reflect generally lighter cow weights found in the San Carlos herd (Benson et al., 1978-1979).

sell all other animals are available except for the relatively light calves from 3 and 4 year old cows which are assumed to be sold unless there are not enough replacement heifer or steer calves available from other classes.

The use of pregnancy testing is a possible option for all cows from 2-9 years. The following decisions are thus open to the rancher:

1. Keep an animal without pregnancy testing.
2. Sell an animal without pregnancy testing.
3. Pregnancy test an animal and keep her if she is pregnant or sell her if she is open.
4. Pregnancy test an animal and keep her if she is open or sell her if she is pregnant.

Note that the options to pregnancy test and keep or sell without regard to the pregnancy test information are not considered here since pregnancy testing and then ignoring the pregnancy test results would never be included in an optimal strategy for the present model.

In summary there are 94 possible decision alternatives altogether. Selling is the only possible decision for dogie calves, yearling steers, 10 year old cows and heifer and bull calves from 3 and 4 year old cows. There are two decisions possible for heifer and bull calves of cows 5-10 years old and yearling heifers. There are four decisions possible for 2-9 year old cows without calves and 3-9 year old cows with calves.

As stated previously, a number of characteristics are influenced by age of dam and age of animal. Although pregnancy rates would be expected to fall into this category (Benson et al., 1978-1979), data from the San Carlos herd (Itulya, 1978) show no significant differences in pregnancy rates for cows aged 3-10. It is assumed here that the dogie calf's ability to survive the death of its mother is similarly not correlated with age of dam. Three year old first calf heifers and animals 7-9 years old suffer a higher mortality rate than older animals (Benson et al., 1978-1979). A further assumption made in this report is that an animal found pregnant in a pregnancy test always delivers a calf. The weaning rate of pregnant cows is thus the probability that their calves live to fall roundup. Weaning rates tend to be stable for animals from 3-9 years old while they are somewhat lower for older animals (Itulya, 1978). Itulya's San Carlos data seem to contradict the work of Laster et al. (1973) and Smith, Laster, and Gregory (1976) as to the survivability of first calf heifers. Table III.2 summarizes the probabilities of animals of each animal class surviving until the following fall roundup.

The Planning Horizon

The length of the planning horizon may be critical to making optimal cull decisions since prices follow a cycle. In order to test the sensitivity of cull decisions to price fluctuations, prices during a 12 year cattle cycle are useful.

Table III.2. Animal Survival Rates[a]

Age of Dam	Pr(Dogie Calf Lives)[b]	Pr(Calf Lives)[b]	Age of Animal	Pr(Yearling Steer Lives)[b]	Pr(Cow Lives)[b]
1	--	--	1	.98	.98
2	.67	--	2	--	.97
3	.67	.97	3	--	.98
4	.67	.97	4	--	.98
5	.67	.97	5	--	.98
6	.67	.97	6	--	.98
7	.67	.97	7	--	.97
8	.67	.97	8	--	.97
9	.67	.97	9	--	.96
10	.67	.96	10	--	--

[a]Animal survival rates rounded to the nearest one hundredth are given in this table in the following manner: coefficients indicate the probability that an animal survives until the following fall roundup as represented by the abbreviation Pr(animal lives). Survival rates are indicated by age of dam for dogie calves and other calves, and by age for yearling steers and cows.

[b]Survival rates are based on a survey of faculty in the Department of Animal Science, The University of Arizona (Benson et al., 1978-1979).

Analysis Objective

The objective of the analysis is to determine optimal culling strategies. Under normal circumstances the culling strategy is defined by the level of culling (selling) in each animal class during a given year. It is further defined by the use of pregnancy testing in each cow class. The result of the optimal culling strategy is the herd composition for the coming year and the sale activities which maximize revenue above variable costs over a planning horizon. The optimal culling strategy is dependent on the initial herd composition, the cost of pregnancy testing, pregnancy, death and weaning rates, and current and expected market prices.

Size of the Model

Referring to the model in Chapter II an X vector containing 36 animal classes, a Z vector containing 94 decision classes, and a 12 year planning horizon have so far been specified. Since dogie calves, yearling steers, 10 year old cows and calves of 3 and 4 year old cows are always sold, the number of animal classes is reduced at 28. Also, 8 of 94 possible decisions need not be considered since they represent assumed selling actions. Thus 86 decision classes remain.

Using the equation $J\cdot(T+1) + J\cdot T+K\cdot T$ presented at the end of Chapter II in evaluating the number of variables and equations where J represents the number of animal classes, K represents the number of decision classes, and T represents the length of the planning horizon, a set of 1,732 variables and equations are required to specify the model.

The size of the model provides incentive to explore alternate methods for accomplishing the above stated objectives. A linear programming model can be used to complete a single stage sensitivity analysis since the use of the linear programming model is justifiable upon mathematical grounds as follows:

Linear Program Justification

Define u_{jk} as the number of animals of the x_jth animal class treated with the z_{jk}th decision at time t.

Define U(t) as a $1xK_j$ row vector of $u_{jk}(t)$'s

where
$$U(t) = [u_{11}(t), \ldots, u_{jk}(t), \ldots, u_{JK}(t)] = X(t)\cdot d(Z_j(t))$$

$$= [x_1(t)\cdot z_{11}(t), \ldots, x_j(t)\cdot z_{jk}(t), \ldots, x_J(t)\cdot z_{JK}(t)]. \quad \text{(III.1)}$$

The Hamiltonian equation (II.23) may be rewritten as follows:

$$\mathcal{H} = IPD(t)\cdot U(t)\cdot R'(t) - IPD(t+1)\cdot\beta(t)\cdot[(X(t+1)-u(t)\cdot P)\cdot\Lambda'(t+1)]$$

$$+ U(t)\cdot T'(t) + \Phi\cdot(X'(t) - d(U_j(t)\cdot I'))$$

$$+ (M(t) - U(t)\cdot A')\cdot\omega(t) \quad \text{(III.2)}$$

Note that
$$\frac{\partial\mathcal{H}}{\partial Z} = \frac{\partial\mathcal{H}}{\partial U}\cdot\frac{\partial U}{\partial Z} = \frac{\partial\mathcal{H}}{\partial U}\cdot X \quad \text{(III.3)}$$

setting $\frac{\partial\mathcal{H}}{\partial U} = 0$ will maximize $\mathcal{H}$ with respect to U and satisfy the necessary conditions (II.24). Since $\mathcal{H}$ is linear in U, maximizing $\mathcal{H}$ can be stated as a linear programming model as follows:

Maximize
$$V = U(t)\cdot(R'(t)\cdot IPD(t) + IPD(t+1)\cdot\beta(t)\cdot P\cdot\Lambda'(t+1)) \quad \text{(III.4)}$$

subject to

$$U_{jk}(t) \geq 0 \text{ for all } j \text{ and } k, \tag{III.5}$$

$$U_j(t)I'(t) \leq x_j(t) \text{ for all } j, \tag{III.6}$$

$$M(t) \geq U(t)A', \tag{III.7}$$

$$X(o) = X^*, \text{ and} \tag{III.8}$$

$$\Lambda(T) = \Lambda^*. \tag{III.9}$$

A corresponding abbreviated example of a single stage linear programming tableau is presented in Appendix A. In order to empirically specify a model of this kind the linear program has a number of requirements. Objective function coefficients based upon weights, survival rates, pregnancy rates, prices at t and inventory values at t+1 for animals of each animal class are presented in Appendix B.

Specification of the objective function coefficients used in this report follows.

Objective Function Coefficients

Animal weights (Table III.1) and survival rates (Table III.2) have been presented earlier in this chapter. Market values of animals in each animal class are specified in Tables III.3 and III.4. Table III.3 specifies the slaughter grade assumed for animals in each animal class. Table III.4 specifies the market price per pound for each of the assumed slaughter grades for the year 1966-1977 (Federal-State Market News Service, 1978).

Sensitivity analysis may require the use of market prices for different years in order to represent the rises and falls of the cattle

Table III.3. Slaughter Grade Value of Animal Classes

Animal Class	Valued as
Cows 5-10	canner cows
Cows 2-4	utility cows
Yearling Heifer	choice yearling heifer
Yearling Steer	choice yearling steer
Heifer calf	choice heifer calf
Steer calf	choice steer calf

Table III.4. Slaughter Cattle Prices in Dollars per Pound Live Weight[a]

	Canner Cow	Utility Cow	Choice Heifer Calf	Choice Steer Calf	Choice Yearling Heifer	Choice Yearling Steer
1966	.1325	.1725	.2287	.2794	.2122	.2418
1967	.1362	.1730	.2318	.2806	.2125	.2538
1968	.1495	.1790	.2410	.2925	.2290	.2660
1969	.1645	.2010	.2840	.3315	.2685	.3060
1970	.1781	.2075	.2994	.3712	.2800	.3250
1971	.1850	.2181	.3250	.3988	.2938	.3488
1972	.2218	.2581	.4300	.5103	.3925	.4410
1973	.2894	.3285	.5050	.6305	.4660	.5295
1974	.1625	.2090	.2455	.2935	.2475	.3038
1975	.1485	.2700	.2352	.3368	.2550	.3605
1976	.1812	.2412	.2924	.3878	.2900	.3668
1977	.2112	.2638	.3528	.4553	.3425	.4138

[a]Prices at Stockton, California (Federal-State Market News Service, 1978).

cycle. Price deflators are thus specified so as to be able to evaluate and compare the revenue to the ranch under different market price conditions. The price deflators used are reported in Table III.5 as are Gross National Product deflators for consumer goods 1966-1977 (United States Department of Commerce, 1976).

Table III.5. Implicit Price Deflators for Consumer Goods 1966-1977[a]

1966	1.261
1967	1.230
1968	1.182
1969	1.130
1970	1.081
1971	1.035
1972	1.000
1973	.948
1974	.858
1975	.790
1976	.751
1977	.711

[a]United States Department of Commerce (1976).

The model requires the specification of an initial herd composition. Table III.6 shows the initial herd composition assumed for this report. The number of animals is based upon the survival rates specified in Table III.2 rounded to nearest whole number of animals. The number of calves is based upon these data with a 0.8 pregnancy rate of all mature cows assumed. The number of animals in each of the calf classes is rounded to the nearest whole number of animals.

Table III.6. Initial Herd Composition[a]

Age of Dam	Heifer[c] Calves	Steer[c] Calves	Cow Age	Cows with[b] Calves at Side	Cows without[b] Calves at Side
1	--	--	1	--	10
2	--	--	2	--	10
3	4	4	3	8	2
4	4	4	4	8	2
5	4	4	5	8	2
6	4	4	6	8	2
7	4	4	7	8	2
8	4	4	8	7	2
9	4	4	9	7	2
10	3	3	10	7	2
Total	31	31		61	36

[a]Initial herd composition is specified in this table as follows: the number of calves are designated by age of dam and the number of cows by age of animal. These numbers specify a herd composition immediately following a fall roundup and previous to any culling.

[b]The number of cows is based upon a 100 cow herd with survival rates specified by Table III.2 and rounded to the nearest whole number of animals.

[c]The number of calves is based upon the survival rates as specified in Table III.2 and a 0.8 pregnancy rate of all mature cow classes. The number of calves in each class is rounded to the nearest whole animal.

Two additional considerations for specification of the linear programming model are opportunity discounting rates and inventory values. In the following example opportunity discounting is calculated from an assumed interest rate of 8%. Assigning inventory values for animal classes is more difficult.

Inventory values should be based upon the expected cash flows resulting from keeping an animal. Appropriate inventory values may be determined by solving the theoretical model specified in Chapter II. The sensitivity of the model to changing inventory values can, however, be measured using the linear programming model specified above. The equations assigning inventory values are reported in Appendix B. Inventory values representing the value of breeding stock are initially determined by adding a premium to market values for slaughter animals. These premiums are altered so that the herd composition remains stable when running the model without pregnancy test options and assuming the pregnancy rate is 0.8, the normal pregnancy rate for the Arsenic Tubs herd. The resulting premiums are reported in Table III.7. The feed requirement of any animal kept is assumed to be 1 as shown in Table III.8. The feed requirement of an animal sold is assumed to be zero.

Sensitivity Analysis

It is possible to use the linear programming model that has been specified to test the sensitivity of the optimal culling strategy to a wide range of factors affecting cattle ranch decision making. An

Table III.7. Breeding Cattle Inventory Premiums[a]

Cow Age	Added Value per Head[b]
1	$ 25
2	150
3	150
4	150
5	150
6	75
7	75
8	75
9	60
10	50

[a]This table shows the premiums added to market values in establishing inventory values for breeding animals.

[b]Inventory values are specified by age of animals.

Table III.8. Animal Unit Feed Requirement by Animal and Decision Class

	Animal Unit Feed Requirement
Cow with calf	
Keep	1
Sell	0
Preg. Test Keep Preg.	Pr (Preg. (i))
Preg. Test Keep Open	(1-Pr(Preg. (i)))
Heifer yearling	
Keep	1
Sell	0
Cow without calf	
Keep	1
Sell	0
Preg. Test Keep Preg.	Pr. Preg. (i)
Preg. Test Keep Open	(1-Pr(Preg. (i)))
Heifer calf	
Keep	1
Sell	0
Bull calf	
Keep	1
Sell	0

example of testing the sensitivity of an optimal strategy to the pregnancy rate is reported in Chapter IV.

In summary, the linear programming model specified is a single stage (one time period) control problem. An optimal time path could be determined by solving a sequence of linear programming problems where the herd composition is revised each year based upon the transition equations (II.7) and where the inventory values of the animals are updated based upon the equations (II.25). The determination of an optimal time path over a cattle cycle for the particular herd composition assumed would be instructional but the primary focus of this report is upon the sensitivity of the culling strategy to the expected pregnancy rate.

The activities selected in the linear programming model are dependent upon ratios of net revenues and thus are dependent upon price ratios. The sensitivity of the linear programming model to pregnancy rate is tested under two price conditions. The 1968 market prices are used for years t=0 and t=1 in one case. Under 1968 price conditions the value of canner cow animals was high relative to yearling steer prices. The 1975 market prices are used for years t=0 and t=1 in a second case. Canner cow prices were low in 1975 relative to yearling steer prices. The price ratios of canner cows to yearling steers for the years 1966-1977 are shown in Table III.9. The results of the linear programming runs are reported in Chapter IV.

Table III.9. Price Ratio of Canner Cows to Yearling Steers

Year	Ratio[a]
1966	.5497
1967	.5224
1968	.5620
1969	.5375
1970	.5480
1971	.5303
1972	.5029
1973	.5465
1974	.5348
1975	.4119
1976	.4940
1977	.5103

[a]Derived from prices recorded in Table III.4.

CHAPTER IV

RESULTS

Optimal Strategies

The linear programming model is tested for sensitivity to pregnancy rates and price ratios. The initial herd composition given in Table III.5 is the approximate number of animals that would be expected in a 100 cow herd with a .8 pregnancy rate. The sensitivity analysis simulates a ranch with a normal .8 pregnancy rate that experiences either a fall (.6), a rise (.95), or a continuation (.8) of the normal pregnancy rate in a year when market prices of mature cows are relatively high (1968) and relatively low (1975) with respect to yearling steer prices. The price premiums for breeding stock are kept constant throughout the sensitivity analysis assuming the expected value of future production remains unchanged.

The model was first run using only keep or sell options without pregnancy testing. The results of these runs are presented in Table IV.1. The model is stable under all conditions except at the .6 pregnancy rate for 1975 prices when least productive cows in the herd, 8 and 9 year old cows without calves, are sold and heifers saved to replace them.

Selling 8 and 9 year old cows and keeping additional heifer calves at the .6 pregnancy rate with 1975 prices results from a change in the relative value of cows to heifer calves. Note that the price

Table IV.1. Optimal Culling Strategies without Pregnancy Testing[a]

		1968 Prices			1975 Prices		
		Pregnancy Rate					
Class & Age[b]	Initial Number of Animals	.6	.8	.95	.6	.8	.95
Cows with calf at side							
3	8	K	K	K	K	K	K
4	8	K	K	K	K	K	K
5	8	K	K	K	K	K	K
6	8	K	K	K	K	K	K
7	8	K	K	K	K	K	K
8	7	K	K	K	K	K	K
9	7	K	K	K	K	K	K
Cows without calf at side							
1	10	K	K	K	K	K	K
2	10	K	K	K	K	K	K
3	2	K	K	K	K	K	K
4	2	K	K	K	K	K	K
5	2	K	K	K	K	K	K
6	2	K	K	K	K	K	K
7	2	K	K	K	K	K	K
8	2	K	K	K	S	K	K
9	2	K	K	K	S	K	K

Table IV.1.--Continued

Class & Age[b]	Initial Number of Animals	1968 Prices			1975 Prices		
		Pregnancy Rate					
		.6	.8	.95	.6	.8	.95
Heifer calves							
3	4	S	S	S	S	S	S
4	4	S	S	S	S	S	S
5-10	23	K(12) S(11)	K(12) S(11)	K(12) S(11)	K(16) S(7)	K(12) S(11)	K(12) S(11)
Bull calves							
3	4	S	S	S	S	S	S
4	4	S	S	S	S	S	S
5-10	23	S	S	S	S	S	S

[a]K = keep; S = sell. If part of the animals in a particular class are kept and part sold, the number so treated is indicated in parentheses.

[b]Classes of cows are indicated by age of animal; classes of calves are indicated by age of dam.

premiums for breeding stock are held constant. The relative value of cows to heifer calves thus changes as a result of the lower price level in 1975, the lower pregnancy rate of cows, as well as the lower ratio of the price per pound of cows to the price per pound of heifers.

Table IV.2 contains the results of the sensitivity analysis with pregnancy test activities included. More pregnancy testing is done at lower pregnancy rates, with more pregnancy testing occurring at each pregnancy level with 1975 prices than with 1968 prices. The strategy to pregnancy test and keep the open animals and sell the pregnant ones never enters the solution. All pregnancy testing is done with the idea of keeping the pregnant animals.

The pregnancy testing of cows with calf at side drops radically as pregnancy rates increase. The .6 pregnancy level with 1975 prices shows 5 year olds being pregnancy tested while 6 year olds are not. Penalty costs for bringing the keep strategy into solution for the 5 year olds, however, is only $.056 per cow while the penalty cost for bringing the pregnancy test strategy for the 6 year olds into solution is $.283 per cow. This inversion is thus not significant. The results of pregnancy testing and culling the open cows with calf at side is keeping all available heifers and some of the steer calves. Relaxation of the constraint to sell the heifer calves of 3 and 4 year olds, results in selling the steers and keeping the heifer calves.

Pregnancy testing of cows without calf at side remains stable under 1975 price conditions with all animals 4 years and older being tested. The 1968 price conditions produce slightly less stability with 5 year olds and older being pregnancy tested under .6 and .8

Table IV.2. Optimal Culling Strategies with Pregnancy Testing[a]

Class & Age[b]	Initial Number of Animals	1968 Prices			1975 Prices		
		Pregnancy Rate					
		.6	.8	.95	.6	.8	.95
Cows with calf at side							
3	8	K	K	K	K	K	K
4	8	K	K	K	K	K	K
5	8	K	K	K	PTKP	K	K
6	8	K	K	K	K	K	K
7	8	K	K	K	PTKP	PTKP	K
8	7	PTKP	K	K	PTKP	PTKP	K
9	7	PTKP	PTKP	K	PTKP	PTKP	PTKP
Cows without calf at side							
1	10	K	K	K	K	K	K
2	10	K	K	K	K	K	K
3	2	K	K	K	K	K	K
4	2	K	K	K	PTKP	PTKP	PTKP
5	2	PTKP	PTKP	K	PTKP	PTKP	PTKP
6	2	PTKP	PTKP	K	PTKP	PTKP	PTKP
7	2	PTKP	PTKP	PTKP	PTKP	PTKP	PTKP
8	2	PTKP	PTKP	PTKP	PTKP	PTKP	PTKP
9	2	PTKP	PTKP	PTKP	PTKP	PTKP	PTKP

Table IV.2.--Continued

Class & Age[b]	Initial Number of Animals	1968 Prices			1975 Prices		
		Pregnancy Rate					
		.6	.8	.95	.6	.8	.95
Heifer calves							
3	4	S	S	S	S	S	S
4	4	S	S	S	S	S	S
5-10	23	K(21.6) S(1.4)	K(15.4) S(7.6)	K(12.3) S(10.7)	K	K(18.8) S(4.2)	K(12.95) S(10.05)
Bull calves							
3	4	S	S	S	S	S	S
4	4	S	S	S	S	S	S
5-10	23	S	S	S	K(5.8) S(17.2)	S	S

[a]K = keep; S = sell; PTKP = pregnancy test keep pregnant animals sell open ones. If part of the animals in a particular class are kept and part sold, the number so treated is indicated in parentheses.

[b]Classes of cows are indicated by age of animal; classes of calves are indicated by age of dam.

pregnancy rates. The .95 pregnancy rate results in only 7-9 year old cows without calf at side being pregnancy tested.

The decision to keep or sell heifer calves of 5-10 year old dams is sensitive to pregnancy rates with more being kept as pregnancy rates drop (see Table IV.2). Steer calves on the other hand are sold in all cases except at 1975 prices with a .6 pregnancy rate. Relaxation of the constraint to sell heifer calves from 3 and 4 year olds in this solution results in keeping the heifers and selling all steer calves.

Value of Pregnancy Testing

An estimation of the value of pregnancy testing is possible by comparing the optimal solution values when including the pregnancy test options to their counterpart excluding pregnancy test options. This estimate is not completely accurate because the inventory values are expected to increase when pregnancy testing is practiced. Estimation of inventory values awaits solution of a multiperiod model.

Table IV.3 shows objective function values for all pregnancy rate levels for 1968 and 1975. It also shows the difference between the objective function with pregnancy testing and without pregnancy testing. The cost of pregnancy testing, $1 per pregnancy test, is then added to the value to give the herd solution for value of pregnancy testing information. This number is divided by the number of animals tested giving the value of information per animal pregnancy tested.

Observation of Table IV.3 shows that the value of pregnancy testing increases under both 1968 and 1975 prices as pregnancy rates fall. Herd values of pregnancy testing are consistently higher under

Table IV.3. Value of Pregnancy Test Information

	1968 Prices			1975 Prices		
Pregnancy Rate	.6	.8	.95	.6	.8	.95
Objective Function Value Pregnancy Tested	$39831.10	$41115.60	$42084.77	$28405.22	$29248.34	$29885.25
Objective Function Value Not Pregnancy Tested	39673.87	41048.29	42079.09	28157.61	29128.47	29875.03
Difference	$ 157.23	$ 67.31	$ 5.68	$ 247.61	$ 119.87	$ 10.22
Number Pregnancy Tests	24	17	6	42	34	19
Herd Value[a] Pregnancy Test	$ 181.23	$ 84.31	$ 11.68	$ 289.61	$ 153.87	$ 29.22
Value per Cow Pregnancy Tested	$ 7.55	$ 4.96	$ 1.95	$ 6.90	$ 4.53	$ 1.54

[a]This value excludes the cost of pregnancy tests.

1975 price conditions but value per cow tested is consistently higher under 1968 price conditions. The value of pregnancy testing under all conditions appears relatively small when compared to the objective function values. Pregnancy testing on the other hand in each case considered increases net returns assuming the current $1 per pregnancy test is the only cost. If opportunity costs for labor are high it is expected that ranchers will not profit from the decision to pregnancy test. These results are consistent with data collected from East Central Arizona cattle ranches by the author in August 1977 showing a 44% use of pregnancy testing. Sixteen ranchers were interviewed from a list provided by the Extension Service.

Suggested Additional Research

Ultimate success of this model rests upon the ability to solve the multi-stage form of the model efficiently. Additional difficulties of animal classification also remain. Animal scientists consulted have had difficulties estimating generalized weight difference between cows that have lactated and calved and those that have not (Benson et al., 1978-1979).

Further work is also required to take into account the stochastic nature of range and weather conditions and their effects upon the biological productivity of a herd.

The model has not as yet been used to test for the sensitivity of optimal culling strategy to a number of variables. Slight alteration of the model such as including the options to buy pregnant heifers could well provide a significantly different view of the value of pregnancy test information.

Summary

An optimization model has been created and tested for sensitivity to changes in pregnancy rate with considerable price variations. The results of this sensitivity analysis suggest that the value of pregnancy test information is relatively low. Under the assumptions of the model, pregnancy testing older heavier animals without calves is most beneficial. The optimal solutions are consistent with a management program that does not arbitrarily sell breeding cows before the age of 10 and does not retain steer calves for sale as yearlings. Possibilities for further development of the model, as well as further uses of it in its current form have been suggested.

APPENDIX A

GENERALIZED LINEAR PROGRAMMING TABLEAU

	B	KCci	SCci	PTKPCci	PTKOCci	KCi	SCi	PTKPCi	PTKOCi	KHoi	SHoi	KBoi	SBoi
XCci	$\geq$	1	1	1	1								
XCi	$\geq$					1	1	1	1				
XHoi	$\geq$									1	1		
XBoi	$\geq$											1	1
AU	100 $\geq$	1	0	Pr(Preg)	(1-Pr(Preg))	1	0	(Pr(Preg))	(1-Pr(Preg))	1	0	1	0
Rev		See Appendix B for Revenue Equations											

Definition of Symbols Used in the Linear Program Tableau

AU	= Animal units of feed available.
B	= B column of linear program tableau.
KBoi	= The decision to keep the bull calf of a dam of i years.
KCi	= The decision to keep a cow of i years without calf at side.
KCci	= The decision to keep a cow of i years with calf at side.
KHoi	= The decision to keep the heifer calf of a dam of i years.
(1-Pr(Preg))	= The probability that a cow is not pregnant.
Pr(Preg)	= The probability that a cow is pregnant.
PTKoCi	= The decision to pregnancy-test a cow of i years without calf at side and keep her if open and sell her if pregnant.
PTKoCci	= The decision to pregnancy test a cow of i years with calf at side and keep her if open and sell her if pregnant.
PTKPCi	= The decision to pregnancy test a cow of i years without calf at side and keep her if pregnant and sell her if open.
PTKPCci	= The decision to pregnancy test a cow of i years with calf at side and keep her if pregnant and sell her if open.
Rev	= The present value expected revenue from a decision.
SBoi	= The decision to sell the bull calf of a dam of i years.
SCi	= The decision to sell a cow of i years without calf at side.
SCci	= The decision to sell a cow of i years with calf at side.
SHoi	= The decision to sell the heifer calf of a dam of i years.
XBoi	= Initial number of bull calves, i specifies age of dam.
XCi	= Initial number of cows without calf at side, i specifies age.

XCci = Initial number of cows with calf at side, i specifies age.

XHoi = Initial number of heifer calves, i specifies age of dam.

APPENDIX B

REVENUE EQUATIONS

Definition of Variables Used in Revenue Equations

i	= Age.
IBo	= Inventory value bull calf.
IC	= Inventory value of a cow without calf at side.
ICc	= Inventory value of a cow with calf at side.
IDc	= Inventory value of a dogie calf.
IHo	= Inventory value of a heifer calf.
IPD	= Implicit price deflator.
IS	= Inventory value of a yearling steer.
M	= Multiplier used to alter TVC or TVCc value = 1.0.
PPBO	= Market price per pound for a bull calf.
PPC	= Market price per pound for a cull cow.
PPHo	= Market price per pound for a heifer calf.
PPS	= Market price per pound for a yearling steer.
PPT	= Cost for a pregnancy test per head.
Pr(i,preg)	= Probability that a cow of age i is pregnant.
Pr(i, lives)	= Probability that an animal of age i survives to age i+1.
RBo	= Revenue from the sale of a bull calf.
RC	= Revenue from the sale of a cow without calf at side.
RCc	= Revenue from the sale of a cow with calf at side.
RDc	= Revenue from the sale of a dogie calf.

RHo	=	Revenue from the sale of a heifer calf.
RKBo	=	Expected revenue from keeping a bull calf.
RKC	=	Expected revenue from keeping a cow without calf at side.
RKCc	=	Expected revenue from keeping a cow with calf at side.
RKHo	=	Expected revenue from keeping a heifer calf.
RPTkoC	=	Expected revenue from pregnancy testing and keeping a cow without calf at side if she is open and selling her if she is pregnant.
RPTKoCc	=	Expected revenue from pregnancy testing and keeping a cow with calf at side if she is open and selling her if she is pregnant.
RPTKPC	=	Expected revenue from pregnancy testing a cow without calf at side and keeping her if she is pregnant and selling her if she is open.
RPTKPCc	=	Expected revenue from pregnancy testing a cow with calf at side and keeping her if she is pregnant and selling her if she is open.
RS	=	Revenue from selling a yearling steer.
RSBo	=	Revenue from selling a bull calf.
RSC	=	Revenue from selling a cow that has not weaned a calf.
RSCc	=	Revenue from selling a cow that has weaned a calf.
RSHo	=	Revenue from selling a heifer calf.
t	=	Designates a particular year.
TVC	=	Value above market price of breeding animals per head without calf at side.
TVCc	=	Value above market price of breeding animals per head with calf at side.
VC	=	Variable cost of a pregnant cow (zero).
VS	=	Variable cost of a yearling steer (zero).
VWc	=	Variable cost of a weaned calf (zero).

wtBo = Weight of a bull calf.

wtC = Weight of a cow without calf at side.

wtCc = Weight of a cow with calf at side.

wtHo = Weight of a heifer calf.

wtS = Weight of a yearling steer.

$\beta(t) = \frac{1}{(1+r)t}$ = discounting factor.

Definition of Complex Variables Used in Revenue Equations

$IBo(i+1,t+1) = IPD(t+1)\cdot\beta(1)\cdot\lambda Bo(i+1,t+1)-VWc(i)$

$\lambda Bo(i+1,t+1) = wtBo(i+1)\cdot PPBo(t+1)$

$IC(i+1,t+1) = IPD(t+1)\cdot\beta(1)\cdot\lambda C(i+1,t+1)$

$\lambda C(i+1,t+1) = wtC(i+1,t+1)\cdot PPC(i+1,t+1)+TVC(i,t+1)\cdot M$

$ICc(i+1,t+1) = IPD(t+1)\cdot\beta(1)\cdot\lambda Cc(i+1,t+1)$

$\lambda Cc(i+1,t+1) = wtCc(i+1,t+1)\cdot PPC(i+1,t+1)+TVCc(i,t+1)\cdot M$

$IDc(t+1) = IPD(t)\cdot\beta(1)\cdot(wtDc)\cdot 1/2\cdot(PPHo(i+1,t+1)+PPBo(i+1,t+1))$

$IHo(i+1,t+1) = IPD(t+1)\cdot\beta(1)\cdot\lambda Ho(i+1,t+1)-VWc(i)$

$\lambda Ho(i+1,t+1) = wtHo(i+1)\cdot PPHo(t+1)$

$IS(1,t+1) = IPD(t+1)\cdot\beta(1)\cdot wtS(1)\cdot PPS(1,t+1)-VS(i)$

$RBo(i,t) = IPD(t)\cdot wtBo(i)\cdot PPBo(t)$

$RC(i,t) = IPD(t)\cdot wtC(i)\cdot PPC(i,t)$

$RCc(i,t) = IPD(t)\cdot wtCc(i)\cdot PPC(i,t)$

$RHo(i,t) = IPD(t)\cdot wtHo(i)\cdot PPHo(t)$

$RS(1,t) = IPD(t)\cdot wtS(1)\cdot PPS(1,t)$

Revenue Equations

$RKBo(i,t) = Pr(i, \text{steer lives})\cdot IS(1,t+1)$

$RSBo(i,t) = RBo(i,t)$

$$RKC(i,t) = Pr(i,Preg)\cdot Pr(i,cow\ lives)\cdot Pr(i,calf\ lives)\cdot (ICc(i+1,t+1)-VC(i))+(1-Pr(i,Preg))\cdot Pr(i,cow\ lives)\cdot (IC(i+1,t+1))+Pr(i,Preg)\cdot Pr(i,cow\ lives)\cdot (1-Pr(i,calf\ lives)\cdot [IC(i+1,t+1)-VC(i)]+Pr(i,Preg)\cdot (1-Pr(i,cow\ lives))\cdot Pr(i,Dogie\ calf\ lives)\cdot [IDc(t+1)-VC(i)]+Pr(i,Preg)\cdot Pr(i,cow\ lives)\cdot Pr(i,calf\ lives)\cdot 1/2[IHo(i+1,t+1)+IBO(i+1,t+1)]$$

$$RPTKPC(i,t) = Pr(i,Preg)\cdot Pr(i,cow\ lives)\cdot Pr(i,calf\ lives)\cdot [ICc(i+1,t+1)-VC(i)]+Pr(i,Preg)\cdot Pr(i,cow\ lives)\cdot (1-Pr(i,calf\ lives)\cdot [IC(i+1,t+1)-VC(i)]+Pr(i,Preg)\cdot (1-Pr(i,cow\ lives))\cdot Pr(i,Dogie\ calf\ lives)\cdot [(IDC(t+1)-VC(i))]+(1-Pr(i,Preg))\cdot RC(i,t)+Pr(i,Preg)\cdot Pr(i,cow\ lives)\cdot Pr(i,calf\ lives)\cdot 1/2\cdot [IHo(i+1,t+1)+IBo(i+1,t+1)]-PPT$$

$$RPTKOC(i,t) = Pr(i,Preg)\cdot RC(i,t)+(1-Pr(i,Preg))\cdot Pr(i,cow\ lives)\cdot IC(i+1,t+1)-PPT$$

$$RSC(i,t) = RC(i,t)$$

$$RPTKPCc(i,t) = Pr(i,Preg)\cdot Pr(i,cow\ lives)\cdot Pr(i,calf\ lives)\cdot [ICc(i+1,t+1)-VC(i)]+Pr(i,Preg)\cdot Pr(i,cow\ lives)\cdot (1-Pr(i,calf\ lives))\cdot [IC(i+1,t+1)-VC(i)]+Pr(i,Preg)\cdot (1-Pr(i,cow\ lives))\cdot Pr(i,Dogie\ calf\ lives)\cdot [IDc(t+1)-VC(i)]+(1-Pr(i,Preg))\cdot RCc(i,t)+Pr(i,Preg)\cdot Pr(i,cow\ lives)\cdot Pr(i,calf\ lives)\cdot 1/2\cdot [IHo(i+1,t+1)+IBo(i+1,t+1)]-PPT$$

$$RPTKOCc(i,t) = Pr(i,Preg)\cdot RCc(i,t)+(1-Pr(i,Preg))\cdot Pr(i,cow\ lives)\cdot IC(i+1,t+1)-PPT$$

$$RSCc(i,t) = RCc(i,t)$$

$$RSDc(i,t) = RDc(i,t)$$

$$RKHo(i,t) = Pr(i,cow\ lives)\cdot IC(1,t+1)$$

$$RSHo(i,t) = RHo(i,t)$$

Verbal Explanation of a Revenue Equation

The revenue equations are perhaps less difficult to comprehend with some verbal explanation. The revenue equation used here as an example is the equation for the expected net revenue from pregnancy testing a cow that has weaned a calf, keeping her if she is pregnant and selling her if she is open.

Line 1 $RPTKPCc(i,t) =$

Line 2 $Pr(i,Preg)\cdot Pr(i,cow\ lives)\cdot Pr(i,calf\ lives)\cdot$
$[ICc(i+1,t+1)-VC(i)]$

Line 3 $+Pr(i,Preg)\cdot Pr(i,cow\ lives)\cdot (1-Pr(i,calf\ lives))\cdot$
$[IC(i+1,t+1)-VC(i)]$

Line 4 $+Pr(i,Preg)\cdot (1-Pr(i,cow\ lives))\cdot Pr(i,Dogie\ calf\ lives)\cdot$
$[IDc(t+1)-VC(i)]$

Line 5 $+(1-Pr(i,Preg))\cdot RCc(i,t)$

Line 6 $+Pr(i,Preg)\cdot Pr(i,cow\ lives)\cdot Pr(i,calf\ lives)\cdot 1/2\cdot$
$[IHo(i+1,t+1)+IBo(i+1,t+1)]$

Line 7 $-PPT$

Line 1 states that the variable that we are concerned with is the expected net revenue from pregnancy testing a cow of i years with calf at side and keeping her if pregnant and selling her if open.

Line 2 represents the expected net revenue from this animal if she lives and weans a calf. This line is made up of the probabilities that she is pregnant, that she lives and that she has a calf that lives, multiplied by the inventory value of an animal that has weaned a calf minus the variable costs of keeping a pregnant cow.

Line 3 represents the expected net revenue of this animal if she is pregnant and lives but does not wean a calf. This line is made up of the probabilities that she is pregnant, that she lives and that she has a calf that dies, multiplied by the inventory value of an animal that has not weaned a calf minus the variable costs of keeping a pregnant cow.

Line 4 represents the expected net revenue of an animal that is pregnant and that dies, but has a calf that lives. It is made up of the probabilities that the cow is pregnant, that she dies, but she has a dogie calf that lives, multiplied by the inventory value of a dogie calf minus the variable costs of keeping a pregnant cow.

Line 5 represents the expected net revenue if this animal is not pregnant and thus immediately sold. It is made up of the probabilities that the animal is not pregnant multiplied by the revenue realized from selling a cow of this animal class.

Line 6 represents the expected net revenue from this cow's calf. It is made up of the probabilities that she is pregnant, that she lives, that she weans a calf multiplied by one half the inventory value of a heifer calf plus one half the value of the inventory value of a steer calf.

Line 7 represents the cost of pregnancy testing per head.

In summary, the expected net revenue from a cow with calf at side if she is pregnancy tested and kept if she is pregnant and sold if she is open is equal to: The expected net revenue of her living and weaning a calf, plus the expected net revenue of her living but not weaning a calf, plus the expected net revenue if she dies but a dogie calf survives, plus the expected net revenue if she is not pregnant and thus sold, plus the expected net revenue from her calf if she weans one, minus the cost of pregnancy testing.

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