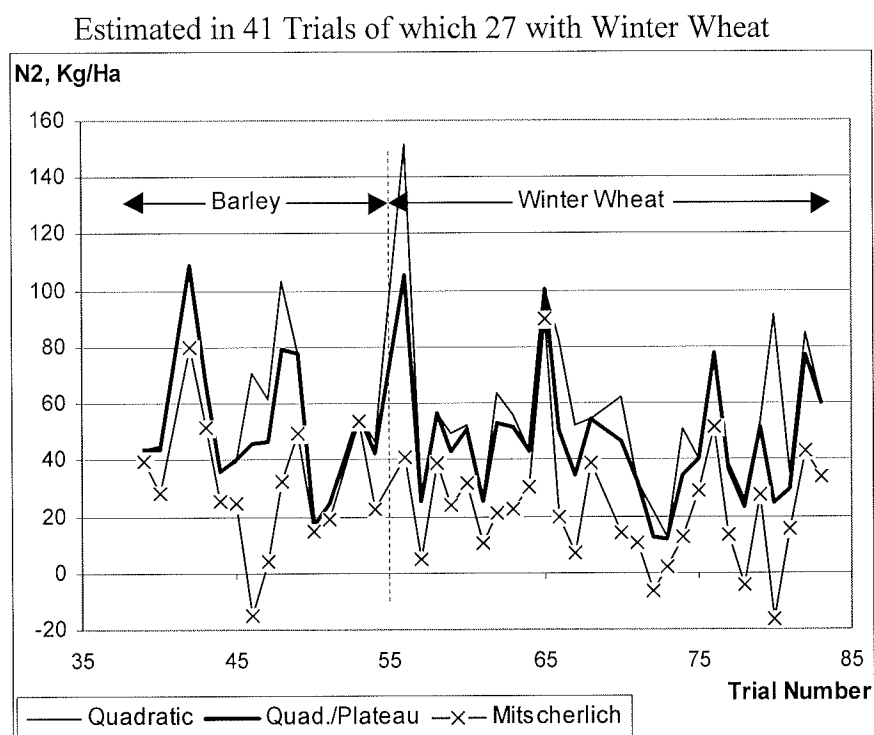


## 6.2 Climatic Variation and Nitrogen Mineralization

### 6.2.1 Empirical Basis

The study was based on cereal field trials at different locations in different years. By selection only trials from a certain type of clay soil were included. The trial sites had not been treated with manure in previous years, and the previous season crops were cereals. The idea was to use only data where the potential mineralization of nitrogen could be expected to be relatively uniform between trial sites. Nevertheless, varying environmental conditions, such as soil temperature and moisture content, influence  $N_2$  at individual trial sites in individual years, see Section 2.2.2. Figure 6.2 shows substantial variation in  $N_2$  estimates between trials. Only estimates based on quadratic, quadratic with plateau from parabola's top, and Mitscherlich functions were included in the comparison. Those three functions – and especially the quadratic with plateau – were found well behaved based on goodness-of-fit results and theoretical considerations of our “first generation” models (i.e. Chapter 5 results). Recall that the mineralization estimates were based on extrapolation of the fitted functions outside the data domain, which is a cause of general uncertainty. To avoid possible impacts associated with differences between crops, the following analyses were restricted to 27 winter wheat trials as the largest sub sample of 41 trials for which both an  $N_2$ -estimate and climate data were available, see Section 4.2.1. Figure 6.2 shows that the quadratic function yields relatively high  $N_2$ -estimates. Mitscherlich results in lower estimates – in four cases even negative net mineralization – while quadratic with plateau estimates are in the middle. With few exceptions the three data sets exhibit the same fluctuation pattern between trials. The  $N_2$ -

**Figure 6.2 Estimated Mineralization during Growing Season, Kg N/Ha**



series estimated using the quadratic function appears more volatile than the two others do. Average and dispersion statistics in Table 6.1 confirm these visual impressions. The standard deviation is about 20–30 kg N/ha in the three series. A difference of 100 kg or more is found between the lowest and highest estimate. The coefficient of correlation between N<sub>2</sub>-series as estimated by the quadratic with plateau on one side and the quadratic and the Mitscherlich on the other is about 0.85. For the two latter series the correlation figure is 0.5.

Data on four climate indicators were available as weekly averages for nineteen weeks during the growing season, see Section 4.2.3 and Appendix A, Table A.2. Climate measurements are available for 44 subdivisions of Denmark (grids), averaging 30x30

kilometers (about 18x18 miles). Each of the examined field trials was identified with its relevant grid and year. The climate observation for a grid was taken to represent the

**Table 6.1 N<sub>2</sub> Estimates in Cereal Trials with Climate Data, Kg N/Ha**

	Quadratic	Quad./Plateau	Mitscherlich
All 41 Trials:			
Average	56.2	48.3	25.3
Standard Deviation	28.0	23.3	22.0
27 W. Wheat:			
Average	55.8	46.4	22.5
Standard Deviation	29.0	23.2	21.2

situation at trial sites located in that grid. Precipitation (P) and potential evaporation (E) are in millimeters of water per area unit accumulated per week. P is based on measurements while the E-data are model estimates of evaporation from a grass covered area unit given actual air temperature, force of wind, and air moisture conditions. In the absence of actual soil temperature measurements available information about average weekly air temperature (T) and weekly solar radiation (R) were used. Air temperature influences soil temperature by convection but no information was available about exactly how air and soil temperatures are correlated. Solar radiation influences air temperature and thereby also soil temperature. A more direct impact occurs when radiation is absorbed by soil, especially when plant cover is sparse in the beginning of the growing season.

Weekly data on the climate variables were averaged over the whole growing season for each pertinent trial site and year. To allow for model flexibility and for the possibility of varying impacts in different stages of the growth process, averages of

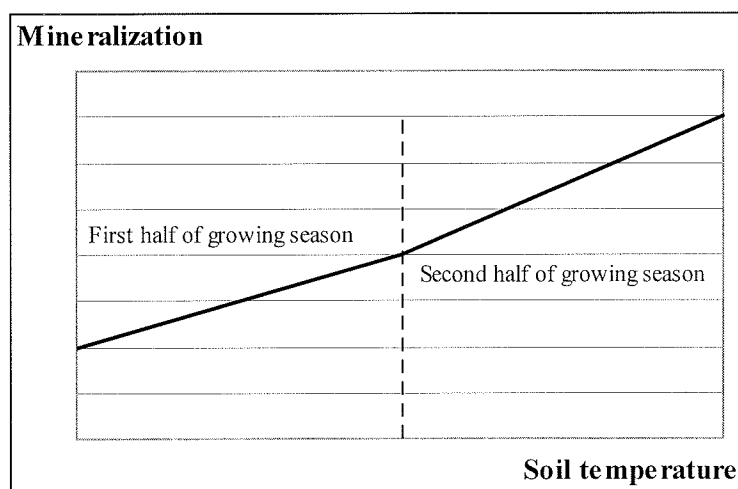
climate data were also prepared for the first and second half of the growing season as explained in Section 4.2.3. A simplification would be to let a new variable,  $W = P + E$ , represent net water balance in soil. The value of  $W$  will normally be negative during the whole growing season, because rainfall can not keep up with disappearance of soil water via evaporation.  $W$  therefore indicates weekly reduction of soil water, which at the end of the winter period normally equals full water holding capacity of the soil. In the absence of directly measured soil temperature and soil moisture these climate variables were used as the best available indicators. It is clear that the adequacy of some indicators can be questioned, especially the proxies for soil temperature. Because of the interrelationship between the climate indicators one can also expect problems with multicollinearity. An exploration of the independent variable series and a variable selection strategy was therefore part of the model fitting procedure.

### **6.2.2 Estimation Models**

Soil moisture and soil temperatures are known to influence mineralization of nitrogen, see Section 2.2.1. During the fall and winter months, weather conditions also influence run off and leaching of nitrogen to ground water. Therefore, the amount of plant available nitrogen at the beginning of the growing season,  $N_1$ , may vary depending on temperature and precipitation during the preceding winter period. It was assumed that the effect of pre-growing season climate is captured by the measurement of  $N_1$ , so that only climate data for the growing season was needed in the analyses.

Information in Section 2.2.1 suggests that mineralization is linearly dependent upon changes in soil moisture. At extreme conditions, dramatic, non-linear effects can occur, e.g. when soil water is below a critical minimum during drought situations, or when flooding and total saturation of soil causes scarcity of oxygen. It is believed that extreme situations like the ones mentioned above are not pertinent to the selected trial sites. According to information exhibited in Section 2.2.1, soil temperatures over a normal range, influence mineralization linearly, perhaps with a tendency toward convexity. A quasi-convex relationship was fabricated by specifying soil temperature indicators for both first half and second half of the growing season, see Figure 6.3. Temperature is higher in the second half of the period, see Appendix A, Table A.2. A higher coefficient estimate for the temperature variable in this period can therefore capture a possible convexity tendency even if a basic linear model structure is maintained. Likewise, a possible change in the impact of solar radiation on soil temperature as foliage coverage increases over the season can be captured in the seasonal

**Figure 6.3 Mineralization and Soil Temperature**



decomposition of the data. The same decomposition also allows for approximating a possible non-linear relationship between N2 and soil water.

Notation used in the estimation is illustrated below on the basis of linear models. With all-season averages of weekly climate data the model has the following structure and notation:

$$(6.1) \text{ N2}_i = \alpha + \beta_1 P_i + \beta_2 E_i + \beta_3 T_i + \beta_4 R_i + \varepsilon_i$$

where N2 represents mineralization of nitrogen during the growing season as estimated with three different functional forms,  $i$  denotes trial number in the sample of 27 winter wheat trials, and P, E, T, and R are the climate variables as defined in the previous section. With subdivision of the growing season the equation to be estimated takes the following form:

$$(6.2) \text{ N2}_i = \alpha + \beta_1 P_{ia} + \beta_2 P_{ib} + \beta_3 E_{ia} + \beta_4 E_{ib} + \beta_5 T_{ia} + \beta_6 T_{ib} + \beta_7 R_{ia} + \beta_8 R_{ib} + \varepsilon_i$$

where subscript a denotes first half and b the second half of the growing season. An exploratory analysis of variables helped to determine, whether P and E should be retained instead of W. Inclusion of multiplicative combinations of variables to detect possible interaction between variables was abandoned, because exploratory calculations showed little effect, which could not justify the increased complexity of the model. Nevertheless, the potential number of different model specifications was large. Model specification was hampered by little a priori knowledge about the relevance of each climate variable, about interaction between climate variables, and about concrete functional relationship under field conditions between mineralization and climate variables. The performance of alternative model specifications with varying numbers and combinations of independent

variables were compared. It is of course not ideal to let data influence model specification. On the other hand, a logical model structure could only be specified with substantial uncertainty given the available climate indicators. The exploratory examination of various alternatives was used as a means of finding a balanced solution for the model specification.

### 6.2.3 Evaluation of Possible Model Specifications

As one could expect, there is a strong correlation indicating multicollinearity between the available climate data series, see Table 6.2. Multicollinearity tends to inflate the variance of the parameter estimates, which makes estimation and interpretation of such estimates difficult. A general evaluation of the climate variables as made on the basis of the total climate database, which has 5,016 observations (44 locations, 19 weeks in the growing season, and 6 years). No correlation was found between precipitation and

**Table 6.2 Coefficients of Correlation between Climate Indicators**

Variables	Total Climate Base, 1993-1998		Wheat Trial Sites
	All 5,016 Observ.	264 All-Season Avg.	27 All-Season Avg.
P*T	0.01 <sup>a</sup>	-0.08 <sup>a</sup>	-0.17 <sup>a</sup>
E*R	0.94	0.87	0.84
E*T	0.84	0.72	0.82
T*R	0.72	0.58	0.76
P*R	-0.32	-0.55	-0.33 <sup>b</sup>
P*E	-0.26	-0.51	-0.44 <sup>c</sup>

Note: P=precipitation, E=potential evaporation, T=air temperature, and R=solar radiation.

a) Coefficient not significantly different from 0. b) Significant at 90% level.

c) Significant at 98% level. All other coefficients highly significant (over 99.9%).

air temperature, whereas an expected strong positive correlation was observed between evaporation and radiation (0.94) and between evaporation and air temperature (0.84).

These figures indicate a high degree of multicollinearity between independent variables. A strong positive correlation (0.72) between air temperature and radiation was not surprising. Statistically significant negative correlation, although not strong, ( $-0.32$ ), was found between precipitation and radiation. This must be ascribed to overcast sky and reduced radiation when rainfall occurs during the daytime. A negative coefficient of the same size between precipitation and evaporation reflects that air moisture content is high and potential evaporation therefore relatively low during rainfall periods.

Aggregating the weekly data to all-season averages reduced the number of observations to 264. This did not cause a change of the correlation pattern, although slight changes of numerical correlation values occurred, cf. Table 6.2. The climate observations, which match the selected 27 winter wheat field trials, exhibited the same correlation pattern when averaged on an all-season basis. Also, when weekly data were averaged for first and second half of growing season, respectively, the correlation pattern was generally retained, see Table 6.3. A negative correlation ( $-0.4$ ) between precipitation and temperature was stronger than in Table 6.2 and the coefficient was found to be

**Table 6.3 Correlation between Climate Indicators, Half-Season Averages**

Variables	Observations at 27 Winter Wheat Trial Sites	
	First Half Average	Second Half Average
P*T	$-0.40^b$	$-0.41^b$
E*R	0.83	0.87
E*T	0.74	0.84
T*R	0.60	0.91
P*R	$-0.38^b$	$-0.51^c$
P*E	$-0.33^a$	$-0.65$

Note: Variable symbols as in Table 6.2. a) Significant at 90% level. b) Significant at 95% level. c) Significant at 99% level. All others over 99.9% level.



significantly different from zero. For the other variable combinations the correlation was stronger in the second half of the growing season. Statistically significant correlation coefficients were not found between variables representing the first and second half of the growing season, respectively.

An expected high correlation was found between W (P + E) and the other variables because of the close link between E and those variables, see Table 6.4.

**Table 6.4 Correlation between Climate Indicators, All- and Half-Season Avg.**

Variable	Observations at 27 Winter Wheat Trial Sites		
	All-Season Avg.	First Half Avg.	Second Half Avg.
T*R	0.76	0.60	0.92
T*W	-0.50 <sup>a</sup>	-0.64	-0.62
W*R	-0.61	-0.66	-0.70

Note: P and E added to form net water balance, W. a) Significant at 99% level, all other coefficients over 99.9% level.

In fact, highly significant coefficients were found between all pairs of variables, both for all-season and half-season averages of observations.

Calculation of the variance inflation factor (VIF) is one way of determining which variables are involved in multicollinearity between independent variables. For variable number  $i$  the VIF is defined as  $1/(1 - R_i^2)$ , where  $R_i^2$  is the coefficient of determination found by regressing independent variable number  $i$  on all other independent variables. The resulting factor shows by how much the variance of the corresponding parameter estimate is larger than it would have been if there were no multicollinearity. VIF factors are generally high for temperature, evaporation and radiation, which supports the tabled correlation figures in this section.

Multicollinearity suggests redundant model specification. It is not obvious which variables should be removed. Manipulation of the basic functions by deleting one or more variables results in biased estimates of the remaining parameters, while at the same time the variance of these estimates can be reduced. A trade-off between bias and variance can be approached by selecting the models with the lowest error MSE. MSE of the error term can be shown to reflect the combined variation due to bias and general variance (Kennedy 1998). Computer software is available for a variety of variable selection procedures, which were applied. All tested procedures resulted in almost identical results. In all cases it should be born in mind that automated variable selection is no substitute for proper model specification. It can be used only as an auxiliary approach when knowledge about functional relationship is incomplete and/or availability of statistical information is a limiting factor.

The basic models outlined in equation (6.1) and (6.2) were initially evaluated by regressing the three N2-series (estimated with the quadratic, the quadratic with plateau, and the Mitscherlich functions) on all the specified independent variables. This examination showed that half-season specification resulted in better  $R^2$ - and F-statistics than calculations based on all-season averages of climate variables, and that separate specification of P and E yielded better results than models with the W variable. Generally, F-statistics were low. Only with half-season variable specification of all four climate indicators was a significant F-statistic (at the 95 percent level) found in the version where N2 estimates were based on the quadratic function. Using the two other N2 series resulted in F-values at the 90 percent significance level. In this specification the

number of explanatory variables was 8, i.e. two higher than in the model variants with merged P and E, and the degrees of freedom were therefore correspondingly lower. Still, a significant reduction of error MSE was found compared with variants with merged P and E variables.

Based on several available variable selection procedures and a general evaluation of MSE values and changes in  $R^2$  by adding or deleting variables in the models, the following specifications were selected as the basis for a model prediction of N2 at the trial sites:

Quadratic	$N2_Q = f(P_a, E_a, R_a, E_b)$
Quadratic with plateau	$N2_{QP} = f(P_a, E_a, R_a, P_b, E_b)$
Mitscherlich	$N2_{MIT} = f(P_a, P_b, T_b, R_b)$

In two cases, best solutions included four of eight possible indicators. In the third case, five independent variables entered the model. Air temperature, T, did not enter the “optimal” model when regressions were based on N2 estimates from the quadratic and the quadratic with plateau. It should be recalled, however, that evaporation, E, itself is based on model estimates in which measured air temperature is an important factor. In the Mitscherlich version T entered the model but E did not. Rainfall, P, and solar radiation, R, appeared in all models. The combination of variables representing first half and second half of growing season, denoted a and b respectively, did not exhibit a clear pattern when comparing the three alternatives. First-half variables dominate the quadratic versions and second-half variables dominate the Mitscherlich version. No obvious

explanation can be offered about the cause of these differences and the results signal additional evidence of basic specification problems within the given data limitations.

The results from fitting the prediction models with the selected specification of independent variables are summarized in Table 6.5. The  $R^2$  values are only slightly lower

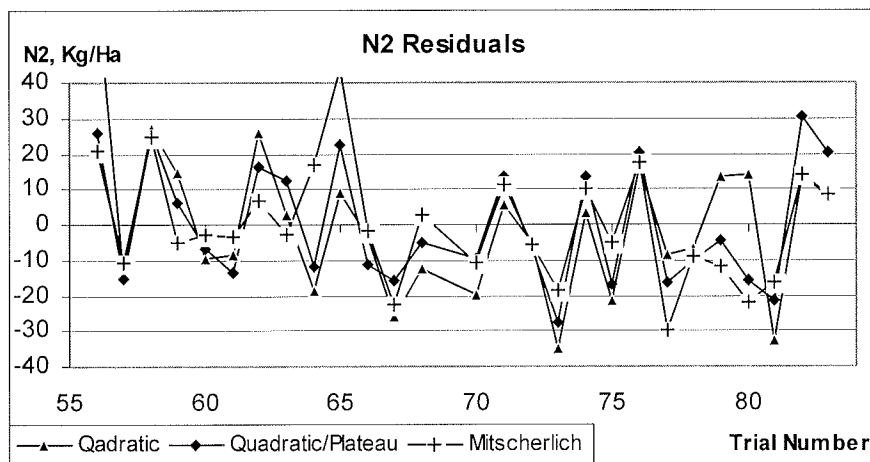
**Table 6.5 Prediction Results,  $N_2 = f$  (Selected Climate Variables)**

	N2 estimated on the basis of		
	Quadratic	Quadratic/Plateau	Mitscherlich
$R^2$	0.46	0.45	0.39
MSE	534	368	326
F-statistics	4.76	3.38	3.44
F signif. level	0.01	0.02	0.02
Parameter est. ( $\hat{\beta}$ ):			
$P_a$	5.5	7.4	3.2
$E_a$	31.8	29.3	-
$T_a$	-	-	-
$R_a$	-4.8	-3.8	-
$P_b$	-	-4.0	-1.7
$E_b$	-7.6	-12.5	-
$T_b$	-	-	23.1
$R_b$	-	-	-3.2

compared with application of the models where all 8 variables were retained, but a reduction of MSE-values takes place, especially in the case where  $N_2$  is estimated on the basis of the quadratic with plateau. F-statistics show that at least one of the  $\hat{\beta}$  estimates is different from zero. The t-statistics for the parameter estimates indicate that the null-hypothesis can be rejected for all regression coefficients, with the exception of  $P_a$  and  $P_b$  in the Mitscherlich version. However, with the data-driven variable selection the test-statistics can not be interpreted with the same certainty as in a non-manipulated model.

The signs of the regression coefficients indicate that rainfall above “normal” has a positive impact on mineralization in the first part of the growing season. Surprisingly, the calculated impact is negative in the second half. Evaporation shows the hypothesized effect in the second half but the positive coefficient in the first half is puzzling. It is possible that the evaporation figure, which is strongly correlated with temperature and radiation, is overshadowed by and reveals a stronger positive influence from soil temperature. The same reasoning may apply to an unexpected negative influence from radiation. Temperature, which was only selected in the Mitscherlich version, exhibits the expected coefficient sign. The signs of selected variable coefficients were consistent with the signs in the basic analysis, where all eight variables were forced into the models.

**Figure 6.4 Difference between Observed and Predicted N2-Values  
27 Winter Wheat Trials**



Note: Recall that “observed” N2-data are by-products of fitting curves to trial data.

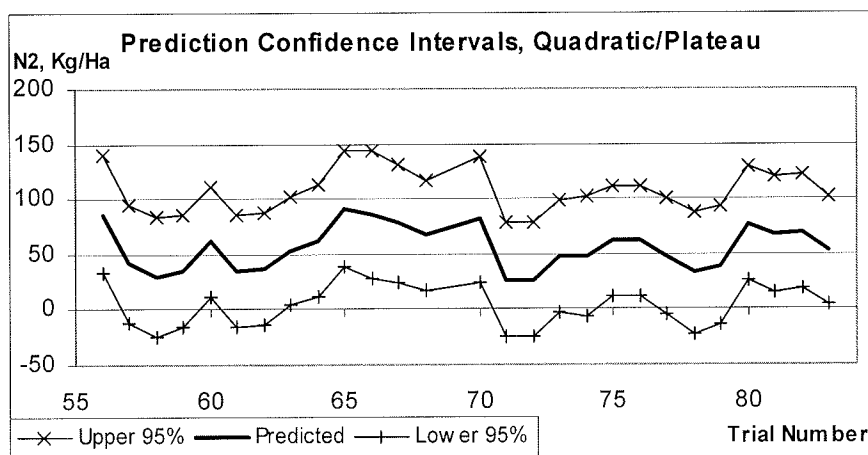
One could argue that the apparent inconsistencies between hypothesis and actual parameter signing are less important if the predictive power of the models is large. As

already shown, the F-statistics were not overwhelming and the  $R^2$  values were modest.

This is underlined by the graph in Figure 6.4. Deviations in the order  $\pm 30$  kg nitrogen per ha between observed and predicted N<sub>2</sub> values are significant in relation to average figures of 25 – 55 kg per ha, see Table 6.1.

Residuals are also important in relation to evaluated fertilizer application of around 200 kg per ha, see Table 5.6. A procedure for in-season correction of fertilizer application is the natural goal of establishing N<sub>2</sub> predictions, but a prediction model with sufficient prediction power and certainty can obviously not be based on the available data input. The uncertainty is further illustrated in Figure 6.5, which exhibits a significant width of the 95 percent confidence interval for predictions based on available climate indicators using N<sub>2</sub> observations established by fitting the quadratic with plateau to trial data.

**Figure 6.5 Confidence Intervals in Predictions of N<sub>2</sub>**  
27 Winter Wheat Trials



General uncertainty is also associated with the “observed” data on the dependent N<sub>2</sub>-variable, which is based on estimated NS as the N-axis intercept obtained by fitting different functions to the field trial yield/NF data. It is also a source of uncertainty that climate variables, which are established for local areas, could deviate to some extent from actual climatic conditions prevailing on the particular trial sites.

### **6.3 Conclusion**

It seems safe to conclude that the climate data available in this study did not allow sufficiently precise N<sub>2</sub> predictions. Collinearity between available independent variable indicators was one obstacle. Proper model specification was another. Direct measurement of soil temperature would probably represent an improvement, but it is very likely that other indicators than climate variables need to be considered in the model specification. The practical possibility of NF corrections would be better if early season climate indicators dominated the predictions. Whether in-season correction of fertilizer application is feasible depends on timeliness of supplying nitrogen during the season in accordance with estimated needs. Possible damage to crops by late fertilizer application is another factor to be considered.

## Chapter 7

### **Some Economic and Environmental Aspects of Fertilization**

Field experiments and response research aim at improving the foundation for decision making at the farm level. The objective is to determine a proper functional relationship for yield response to nutrient application and to establish the economically optimal level of nutrient application. Over the last couple of decades response research has also been brought to bear in connection with establishing environmental legislation about fertilization norms, etc. So far, response research has not rendered a clear-cut picture and a range of different functional forms is still the subject of scientists' serious considerations.

In this study the quadratic function with plateau was found to combine a good fit to observed data with good ability to produce credible estimates of optimal nitrogen application and mineralization of soil nitrogen. However, other functions also rendered reasonable results. In this chapter a comparison is made among six functions that were fitted to pooled data from the sample of 27 winter wheat trials, see Table 7.1. The two polynomial functions – the quadratic and the cubic – have been among the most popular formulas in applied response research over the last four to five decades. The two classical functions – the LRP-Liebig and the Mitscherlich – have attracted renewed interest during the 1980s and 1990s together with the plateau version of the quadratic function. The Cobb-Douglas function with plateau, which was included in this study, displayed good behavior in many respects. Various modifications of the selected models might improve their performance. This possibility was not pursued in this study.



In terms of  $R^2$  and MSE the six functions perform almost equally well regarding their representation of observed data. Figure 7.1 shows that the predicted response curves – with the LRP-Liebig as the most pronounced exception – are very close over the domain of applied nitrogen, which is 0 – 250 kg N per ha. The six functions all have yield axis intercept of about 40 hkg grain per ha, which represents the estimated output generated on the basis of soil nitrogen alone.

**Table 7.1 Functions Fitted to 27 Pooled Winter Wheat Trials**

	QUA	CUB	MIT	QP1	CDP	LRP
$\alpha$	40.33211 (1.0745)	39.4665 (1.1595)	89.97863 (1.7690)	39.94183 (1.1070)		40.54867 (1.0913)
$\beta_1$	0.408789 (0.0202)	0.48768 (0.0449)	50.68628 (1.8534)	0.42809 (0.0247)	14.13107 (3.6792)	0.335862 (0.0169)
$\beta_2$	-0.00092 (0.00008)	-0.00178 (0.00044)	0.010669 (0.00103)	-0.00102 (0.00011)	0.339502 (0.0495)	
$\beta_3$		2.3E-6 (1.17E-6)			20.64997 (7.4843)	
$N_{\max}^*$	222.6			210.2	176.7 (11.6938)	127.7 (4.8738)
$R^2$	0.60	0.60	0.60	0.60	0.60	0.59
MSE	178.5	177.8	177.8	178.1	178.5	181.3
$N_{\text{opt}2}^*$	200.2	214.1	241.4	190.0	176.7	127.7
$Y_{\text{opt}2}^{**}$	85.4	84.8	86.1	84.5	85.0	83.4

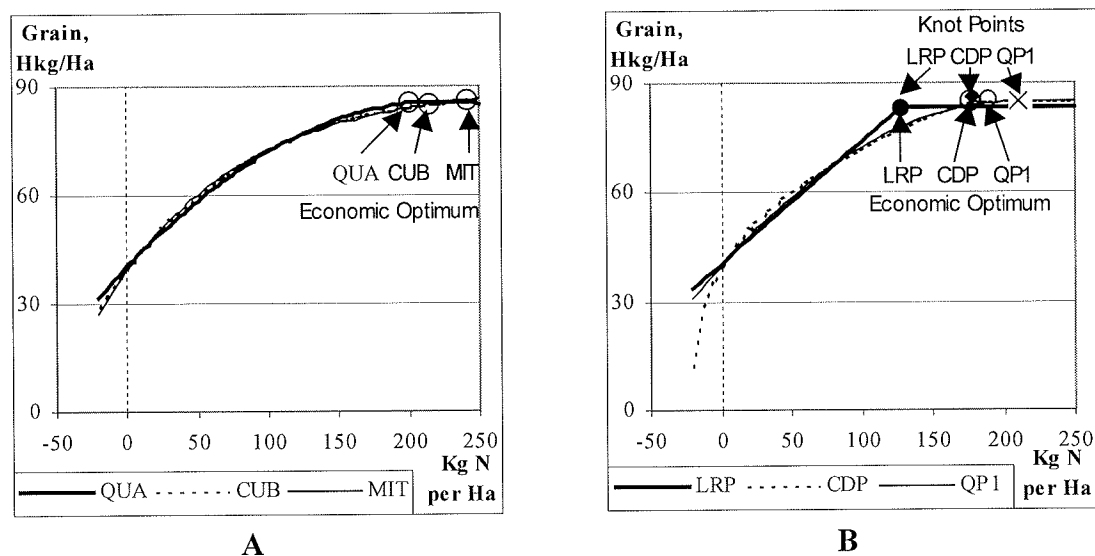
Note: Standard errors in parentheses. All parameter estimates significant at 95% level. Results may deviate in comparison with Chapter 5 and Appendix B where parameter estimates etc. were obtained as averages – corrected for outliers – of results from fitting the functions to each individual trial. However, differences are modest except for the cubic and the Mitscherlich functions, for which the outlier corrections were significant. The fitted cubic function has no stationary values and reveals a concave Stage II segment up to the inflection point at a N-value of 258 kg/ha.

\* = kg per ha, \*\* = hkg per ha.

## 7.1 Optimal Nitrogen Application

The estimated optimal nitrogen application – i.e. the amount of fertilizer that maximizes the value of grain output per hectare minus the value of applied fertilizer evaluated at prevailing prices after full implementation of the 1993 policy reform in the EU – varies from 128 in the LRP-Liebig to 241 kg per ha in the Mitscherlich function. The grain yield corresponding with optimal nitrogen application is estimated at 83.4 hkg per ha using the LRP-Liebig function and from about 85 to 86 hkg per ha with the five

**Figure 7.1 Grain Response to Nitrogen Based on Different Functional Forms**



other formulas. For the six functions the estimated net profit at optimum ranges from about 6,450 to about 6,650 Kr. per ha.

The wide range of  $N_{opt}$ - estimates reported in different response research studies is particularly disturbing in a farm management context. Moreover, a given deviation in N-

application from the estimated  $N_{opt}$  has very different economic impacts for the individual functional forms. Table 7.2 illustrates the impact of undershooting or overshooting the optimal fertilizer amount by 20 kg per ha for each of the six functions in this study. This stipulated deviation corresponds roughly with ten percent of  $N_{opt}$  found in the different functions, excepting the LRP- Liebig. For the LRP-Liebig and the plateau version of Cobb-Douglas with  $N_{opt}$  at the knot point the calculated revenue loss from below-optimal

**Table 7.2 Estimated Economic Impact of Deviations from Optimal N-Application**  
Net Profit Change by  $\pm 20$  Kg Nitrogen Deviation from Estimated Optimum, Kr./Ha

	Optimal N-Application -20 Kg/Ha			Optimal N-Application +20 Kg/Ha		
	Fertilizer	Grain	Net Effect	Fertilizer	Grain	Net Effect
QUA	70.00	-101.22	-31.22	-70.00	38.78	-31.22
CUB	70.00	-81.92	-11.92	-70.00	61.21	-8.79
MIT	70.00	-78.03	-8.03	-70.00	63.04	-6.96
QP1	70.00	-104.62	-34.62	-70.00	35.38	-34.62
CDP	70.00	-257.37	-187.37	-70.00	0.00	-70.00
LRP	70.00	-570.97	-500.97	-70.00	0.00	-70.00

Note: Kr. per ha converts to \$US per acre by multiplying with 0.05 at an exchange rate of 1 \$US = 8 DKr.

fertilization is substantial and exceeds by far the lower fertilizer expenditure. When overshooting by 20 kg N per ha there is no yield increase and the economic net effect is therefore identical with the increased fertilizer expenditure. For the other functions the economic impacts of the same deviations from  $N_{opt}$  are much smaller. By coincidence, the profit decline is almost the same for the stipulated 20 kg positive and negative deviations from  $N_{opt}$ . For wider variations from optimum the negative economic effect of below-optimal N-application is significantly larger than the result of above-optimal use of fertilizer.

## 7.2 Safeguards against Unpredictable Variation in Available Soil-N

Annual variation in mineralization of soil nitrogen can cause deviations in the total supply of plant available nitrogen. This variation is beyond farmer's control. With fertilizer applied in accordance with the estimated optimal amount under normal mineralization conditions the economic impact of a  $\pm 20$  kg N variation in mineralization is the same as the yield effect in columns three and six of Table 7.2. Given the occurrence of such natural variation Boyd et al. (1970) found that it would be prudent to increase the nitrogen application and insure against unpredictable changes in yield. The penalty from over-fertilization is small compared with the loss from applying too little nitrogen especially if a function like the LRP-Liebig is considered the adequate basis for decision making. Cerrato and Blackmer (1990) challenged the idea that it is profitable to apply too much rather than too little nitrogen. Their argument was that growers under the influence of the notion of "insurance" fertilization may be tempted to base decision on a model, e.g. a quadratic, which in comparison with a more correct response model tends to overestimate the optimal input value.

Two different topics are being dealt with in this discussion: Choice of model and safeguards against unpredictable fluctuations in input optimum. Table 7.3 shows that the effect of "insurance" fertilization is very different depending upon which function is assumed to be the correct response model. In the example, the fertilizer application is increased to 20 kg nitrogen above normal optimum, which entails a cost of Kr. 70 per ha. If in a certain year total nitrogen supply is 20 kg below normal because of reduced mineralization the "insurance" strategy improves profit because the value of avoided

**Table 7.3 Effects on Profit of “Insurance” Fertilization, Kr. Per Ha**  
 Nitrogen Application Always 20 Kg Per Ha Above Estimated Normal Optimum

Mineralization Relative to Normal:	20 Kg/Ha Below Normal	Equal to Estimated Normal	20 Kg/Ha Above Normal
Quadratic	31.22	-31.22	-93.66
Cubic	11.92	-8.79	-20.12
Mitscherlich	8.03	-6.96	-19.08
Quadratic/Plateau	34.62	-34.62	-69.99
Cobb-Douglas/Plat.	187.37	-70.00	-70.00
LRP-Liebig	500.97	-70.00	-70.00

yield loss more than outweighs the extra fertilization costs. This is most evident for the two plateau models with optimum solution at the knot point, whereas the gain is more modest for the other models.

In a year with normal mineralization, the “insurance” strategy results in losses for all functional forms, because, by definition, above-optimal fertilizer application is unprofitable. For the plateau models with knot point optima the extra fertilizer cost is not partly offset by a yield increase as in the other models. If mineralization provides unpredicted extra 20 kg nitrogen supply the “insurance” strategy entails the same loss as in a normal year for the two models with knot point solutions. For the other models the loss is larger than in the situation with normal mineralization because of diminishing marginal yield. With the quadratic model the total loss even exceeds the extra fertilizer cost because a decline in total yield is revealed in the fertilizer nitrogen domain. In the quadratic plateau model the loss is close to the added fertilizer cost because the start of the plateau segment almost coincides with  $N_{opt}$ .

If the LRP-Liebig or the Cobb-Douglas plateau models are considered the correct functional forms there is good opportunity to apply an efficient safeguard strategy in the form of permanent over-fertilization compared with the estimated optimal N-application under normal mineralization conditions. On average, an “insurance” strategy will hardly be profitable if other types of response curves best represent the functional form. The core problem is to establish evidence that shows which of the models is more appropriate. This study leans towards the quadratic plateau model as a function that offers a good representation of grain response under conditions as those prevailing in Denmark, see Chapter 5.

Precise prediction of mineralization at a point of time in the growing season where corrective N-application is still possible would be a better alternative than constant over-fertilization. As indicated in Chapter 6, mineralization predictions may be difficult to establish. Available mineralized soil nitrogen (N1) at the beginning of the growing season can be estimated directly on the basis of soil sample analyses but mineralization during the growing season (N2) can not easily be measured or predicted.

### **7.3 Costs of Model Mis-Specification**

The results presented in Table 7.1 through 7.3 show that the choice of model is a matter of considerable economic importance. Fertilization according to optimum estimates using a wrong model is sub-optimal compared with the proper fertilization rate indicated by the correct model. Frank, Beattie and Embleton (1990) illustrated this by establishing costs of mis-specification. It was done by calculating net return resulting

from setting fertilization rates in the function assumed to be true at the optimal levels estimated in all the other functions they examined. By successively letting each function take the “true” function’s place all combinations of mis-specifications could be examined. Cerrato and Blackmer (1990) used the same approach to illustrate the consequences of incorrect decision making about fertilization rates.

The results of a similar comparison of the six models that are considered in the present chapter of this study are shown in Table 7.4. The data show that it costs from Kr. 319 to Kr. 496 per ha to base fertilization on calculated optimum in the LRP-Liebig if one of the other functions is the “true” model. In contrast, if the LRP-Liebig with its relatively low  $N_{opt}$  is the “true” model increased nitrogen application in accordance with other models’ optima could reduce profit by Kr. 172 to Kr. 398 per ha. With the quadratic with plateau as the preferred model significant mis-specification costs arise if nitrogen were applied based on optima found in the LRP-Liebig or in the Mitscherlich. The costs of incorrectly choosing the quadratic, the cubic or the plateau version of the Cobb-Douglas over the correct quadratic with plateau would be relatively modest according to Table 7.4

**Table 7.4 Calculated Economic Losses by Incorrect Selection of Response Model**  
Kroner per Ha

“True” Function	Pofit Change when applying Optimal N-Application based on:					
	QUA	CUB	MIT	QP1	CDP	LRP
Quadratic	0	-15	-132	-8	-43	-410
Cubic	-6	0	-15	-18	-46	-319
Mitscherlich	-37	-15	0	-60	-100	-377
Quad./Plateau 1	-9	-49	-144	0	-15	-336
Cobb-Doug./Plat.	-82	-131	-226	-46	0	-496
LRP-Liebig	-254	-302	-398	-218	-172	0

The potential economic impacts of incorrect model choice would be less dramatic if the LRP-Liebig were ruled out as a possible decision making basis.

It should be noted that there is no data symmetry around the matrix diagonal in Table 7.4. This follows from the fact that economic impacts of deviating from the “true” model’s optimum are different for positive and negative deviations of the same size. The information in Table 7.4 does not solve the basic problem of choosing the correct functional form. However, the table shows that cost of mis-specification reduces to relatively small amounts if the choice of model can be narrowed down to a few models with nitrogen optima within a limited range.

#### **7.4 Environmental Aspects of Fertilization**

The member states of The European Union must abide with common regulatory measures aiming at a reduction of nitrate pollution of the aquatic environment. The basic legislation was introduced in the so-called Nitrate Directive in 1991 (Rude and Frederiksen 1994). Danish legislation, which took its beginning with the start of the 1980s, has been adjusted to satisfy both common EU provisions and additional national environmental requirements. An extensive and elaborate set of rules now in force approaches the nitrate pollution problem from many different angles (Environmental Group of Danish Farmers’ Union and Danish Family Farmers’ Association, 1999). The rules range from provisions about minimum norms for manure storage facilities, to a ban on manure spreading during fall and winter months, to maximum livestock numbers in relation to farmed area.



Farm manure and slurry, which make up a significant part of total nitrogen supply, represent the greatest risk in the form of nitrogen leach to the aquatic environment and ammonium evaporation. This follows from the fact that manure must be spread before planting or before start of the growing season. There is a potential risk of nitrogen leaching in periods outside the actual growing season, when plant's nitrogen absorption capacity is limited and soil water movement is downward under normal climatic conditions in Denmark. A core element in the Danish nitrogen legislation is therefore the prescription of minimum utilization rates for nitrogen content in applied manure.

Timely field application of manure, i.e. application within a restricted period of the year, is an important instrument to avoid loss of nitrogen. This entails significant investments in storage capacity. Sufficient storage capacity normally corresponds to at least nine months' supply of manure.

Farmers are obliged annually to submit crop rotation and fertilization plans that prove the satisfaction of environmental provisions, specifically that the total supply/use nitrogen balance does not imply over-fertilization. Failure to submit plans exposes farmers to a nitrogen tax of Kr. 5 per kg N, which compares with a market price of about Kr. 3.5 per kg. If prescribed maximum norms for nitrogen application to fields are exceeded farmers incur significant fines.

Maximum fertilization norms apply to all farms, including farms that do not use livestock manure as input in crop production. Maximum norms are specified for all common crops and types of soil (Plantedirektoratet 1999). Section 5.1.3 explained that

the norms are set at 90 percent of optimal nitrogen rates derived from fitting the cubic function to field trials from the same source as used in this thesis. Table 5.7 and 5.8 showed that for winter wheat the estimated optimal nitrogen application based on the cubic function is substantially lower than the optimum found by fitting the Mitscherlich and quadratic functions, and higher than the rates found by fitting the Cobb-Douglas with plateau and the LRP-Liebig. The optimal N-rate based on the cubic function is close to the optimal rate that was derived from the quadratic with plateau, which all things considered is the preferred function according to this study. The farm profit reduction caused by a ten percent below-optimal nitrogen application to winter wheat was estimated at about Kr. 105 per ha in Table 7.2 using the quadratic with plateau as the basis of evaluation and at Kr. 82 per ha when derived from the cubic function. Surprisingly, the cubic functional form seems to be a reasonable basis for establishing nitrogen application norms even if the cubic function appeared less adequate from other points of view, see Chapter 5.

Whether the ten percent reduction of optimal N-application is well justified from an economic point of view is another question, the reply to which depends on a host of factors. Two general questions are of main importance for a proper evaluation of the issue: One concerns the actual impact on loss of nitrogen to the aquatic environment by changes in fertilization intensity. The other deals with the externalities associated with nitrogen pollution. The second question can hardly be assessed with a high degree of accuracy. It would therefore be difficult to assess benefits accruing from reduced pollution and compare them with the loss of farm income due to the corresponding sub-

optimal fertilization. Therefore it is difficult to fine tune rules if general cost/benefit considerations were the only basis for political decisions about the reduction of agriculture's use of plant nutrients.

Danish agriculture's total use of nitrogen in fertilizer and manure decreased by almost a quarter from 635,000 tons in the beginning of the 1980s to about 500,000 tons at the end of the 1990s (Danish Farmers' Union 1999a). Half of the decrease reflects a lower need caused by a general decline in agricultural area over the period and the introduction of set aside programs in the EU agricultural policy. The other half can be ascribed to the effect of environmental legislation. The lower use of nitrogen was achieved via lower fertilizer application whereas the annual nitrogen supply from livestock manure was relatively stable over the period. This reflects the fact that the utilization rate for nitrogen in manure has improved considerably as a result of more timely field application over the year. Although it is difficult to measure exactly, there is no doubt that the political reduction goals for nitrogen run-off and nitrogen leaching during the winter period have been achieved to a high degree.

The first question about the link between nitrogen pollution and fertilization intensity is generally disputed. A simple but plausible argument is that any deviations from standard application rates entail corresponding changes in loss of nitrogen to the environment. A more refined approach to the problem takes into consideration that changes in nitrogen application up to a maximum yield level are associated with changes in plant absorption of nitrogen. The nitrogen utilization rate at different application levels is exactly what estimation of response functions is all about.

However, the problem is even more complex. Chapter 2 cited studies showing that plants to a certain extent buffer nitrogen supply by storing nitrogen in leaves and stems and also in the kernels, the weight of which equals measured yield. The buffering effect in the form of higher N-concentration in both straw and grain increases when unmeasured changes in straw yield is also taken into account. Even if a yield plateau in terms of total weight of grain has been reached this plant buffering system has the effect that inorganic nitrogen concentration in the soil does not increase in line with nitrogen application in excess of the maximum yield. Similarly, denitrification of inorganic soil nitrogen (a reverse mineralization) plays a buffering role so that leaching labile inorganic nitrogen concentration in the soil (predominantly nitrate ions) does not change in line with N-application within certain limits.

In their extensive study based on four long-term winter wheat field experiments Raun and Johnson (1995) found that the plant-soil buffering of inorganic nitrogen was significant. Annual nitrogen fertilization rates within N-requirement for maximum yield plus 25 kg N per ha did not increase inorganic N-accumulation in soil. This indicates that modest N-application deviations from  $N_{\max}$  – intentional as well as unintentional – entail no great environmental risk. In this light the mandatory lowering of N-application norms by 10 percent in relation to  $N_{\text{opt}}$  may not in itself contribute significantly to a reduction of N-pollution of the aquatic environment. It seems obvious, however, that to the extent the lower maximum norm for total nitrogen use entails an increased utilization rate for manure nitrogen there will also be a positive environmental effect.

## Chapter 8

### Summary and Conclusions

Mathematical description of crop response to applied nutrients was the topic of numerous studies over the last century. At the beginning of the 1900s, Mitscherlich formulated his law of diminishing output. Economists eagerly embraced its principle as being in concert with marginal value theory. It was also found consistent with practical field experience and the general conception of physical and chemical processes. Mitscherlich's asymptotic growth theory was founded on statistical analyses of data from a large number of field experiments and it represented a clear break with Justus von Liebig's law of the minimum, which since the middle of the 19<sup>th</sup> century had been the generally accepted response theory. von Liebig did not express his response law in mathematical terms. His theory was generally taken to imply response linearity, non-substitution between input factors, and the existence of maximum yield. He stated that yield of a crop is governed by quantity changes of the most scarce input factor. As the minimum factor is increased the yield will go up proportionally until the supply of another factor becomes limiting. If a non-minimum factor is increased or decreased, the yield is not affected.

Heady and co-authors' work in the middle of the 20<sup>th</sup> century gave rise to the widespread use of polynomial models in production research and they are still popular as the basis for extension and farm management decisions. Factor substitution was considered a general possibility in these concave everywhere-differentiable models. Later studies pointed out that the easy-to-apply polynomial models could result in above-

optimum fertilization norms. Not least the studies by Paris and co-authors in the 1980s and the 1990s revived certain qualities embedded in the classical response models, especially the concept of a yield maximum extending over a plateau and non-substitution between nutrients. Combinations of “pure” models have attracted a great deal of interest over the last two decades. A quadratic polynomial combined with a plateau is one of these hybrid models, and modifications of the logistic Mitscherlich function with complex forms of the independent variable in the exponent is another.

In the present one-product one-factor study of cereal response to nitrogen a comparison was made between ten different functional forms, including some of those promoted in previous studies. Support for assessing the adequacy of various mathematical response expressions was sought in literature on the plant physiological mechanisms that determine how nitrogen influences cereal yield. Different studies indicate that the number of head-bearing straws per plant, kernels per head, and kernel weight are all positively correlated with the amount of available nitrogen. Some of these key factors may manifest as linear yield response, but in combination they tend to suggest a curvilinear, concave relationship. Some studies find that a maximum cereal yield occurs when total leaf area exceeds 6-9 times the planted area. Beyond that level additional leaves will shadow existing leaves, causing the photosynthetic activity of the shadowed leaves to cease. The plants are able to store available nitrogen, which can not be utilized in the photosynthetic assimilation process. A yield plateau of a certain extent after maximum is reached is therefore a plausible possibility. At even higher N-application levels yield losses may occur as an indirect result of pests caused by insects and fungi in

lush crops. Also lodging of lush crops may reduce yields because a bent straw restricts water supply to upper parts of a cereal plant.

Ten functions were fitted to data from 84 Danish cereal field trials at clay soil sites where no manure had been applied in previous years and where preceding crop was cereal. The actual analytical results should not be taken to represent response for other crops or under other conditions than those specified above. However, the functions' goodness-of-fit position relative to each other, taken together with their basic mathematical structures, may yield information of more general value. The mean squared error (MSE) was applied as the most important criterion of evaluating the functions' goodness-of-fit to the trial data. No single functional form was found clearly superior to the others on this basis. The quadratic polynomial and quadratic with plateau had the highest number of first and second placements in a ranking of all functions according to the MSE value. The LRP-Liebig and the Cobb-Douglas functions scored low in this ranking. The average MSE values of all 84 trial data fits were not significantly different for curvilinear plateau models and polynomials, but the average MSE was high for the LRP-Liebig and different from some of the other functions. A strictly linear growth pattern, which was tested for comparison, was clearly inferior to the selected functional forms.

Besides the statistical fit to observed data the functions were also evaluated on the basis of how well they complied with hypothesized response patterns based on agronomic knowledge. In this overall assessment the quadratic function with plateau attracted much attention. Some of the other functions with as good a statistical fit were ruled out because

of unrealistic values for yield and profit optimizing N-application or because of implausible nitrogen axis intercepts. The popular and very flexible cubic polynomial is one example of a function, which should be applied with caution because it often behaves poorly outside the observed nitrogen domain.

The estimated N-axis intercept was hypothesized to represent the supply of mineralized nitrogen from soil resources of organic nitrogen compounds. It was one of the criteria in the evaluation of the functions' general behavior. Soil nitrogen plays a significant role in total nitrogen supply. Estimation of nitrogen mineralization was therefore also one of the general goals of this study. Total nitrogen uptake in crops can be divided in that derived from applied fertilizer and that from the soil. It was assumed that fertilizer nitrogen and mineralized soil nitrogen are equally accessible to growing plants and that each has the same nutritional value. The estimation was based on the condition that yield would be zero if no nitrogen were available. Provided that the functional form is representative over the total domain of nitrogen supply, in particular over the low N-level unobserved portion of the domain, the total supply of plant available soil nitrogen can be assessed on the basis of the intersection between the response curve and the nitrogen axis. Available soil nitrogen was measured in soil samples at the beginning of the growing season for about half of the examined field trials. Thus, it was possible to derive further mineralization of soil nitrogen during the growing.

The Cobb-Douglas and the square root specifications resulted in a large number of negative in-season mineralization estimates, which was not likely to occur on the pertinent trial sites. This was an important basis for deeming the two specifications



inadequate as reflections of the “true” response relationship. The linear specification in the LRP-Liebig resulted in relatively high mineralization estimates. The quadratic function tended to yield somewhat higher mineralization estimates than its plateau version. Estimates based on the cubic function were in the same range as those based on quadratic specifications. However, in a number of cases the cubic function did not reveal an intercept with the nitrogen axis to the left of the fertilizer nitrogen domain. All things considered – statistical fit, estimates of mineralization and of yield and profit maximizing nitrogen application – the quadratic plateau version was found to give the most adequate description of yield response to nitrogen in the examined Danish field trials.

It would improve the estimation of the response function if precise information about the importance of soil nitrogen were available. Likewise, it would be important for farm management decisions on optimal fertilizer application to avail of reliable predictions of in-season mineralization, especially if such predictions could be made in time for in-season corrective nitrogen application. Agronomic knowledge indicates that mineralization increases with soil temperature and soil moisture content during the growing season. Certain climate indicators – precipitation, potential evaporation, air temperature, and solar radiation – were available for almost half of the field trial localities. It was examined whether between-trials variation in the estimated mineralization data resulting from curve fitting could be reproduced in a model with the available climate indicators as independent variables. Strong collinearity and the lack of a proper soil temperature indicator rendered this part of the analysis inconclusive.

With prevailing output and input prices estimates of profit maximizing nitrogen application differed significantly among the examined response functions. Moreover, a given deviation in nitrogen supply from the estimated optimum situation had very different economic impacts for the individual functional forms. The costs of undershooting the optimal nitrogen supply were found to be especially high when calculated on the basis of the LRP-Liebig and the plateau version of the Cobb-Douglas functions with  $N_{opt}$  at the knot point. The calculated yield revenue loss exceeded by far the lower nitrogen expenditure. When overshooting the optimum in those two models there was no yield increase to offset higher fertilizer costs. For the other functions the economic impact of the same deviations in nitrogen application were found to be much smaller.

This information indicates that if, for example, the LRP-Liebig were the “true” model there would be good opportunity to apply an efficient safeguard strategy against unpredicted changes in soil nitrogen supply in the form of a certain permanent over-fertilization. On average, an “insurance” strategy would hardly be profitable if, say, a quadratic specification were considered the appropriate model.

The choice of model is a matter of considerable economic importance. Fertilization according to estimated optimum using a wrong model is sub-optimal compared with proper fertilization rates indicated by the correct model. The costs of model mis-specification were found particularly high when the LRP-Liebig was compared with other models examined in this study. Mis-specification costs reduce to

relatively small amounts if model choice can be narrowed down to curvilinear models with nitrogen optima within a limited range.

Environmental aspects of fertilization have attracted increasing attention over the last decades. Leaching of nitrogen to ground water is a main concern. Intensive use of livestock manure represents an important problem because it must be applied before the actual growing season when plant's nitrogen absorption capacity is limited and when soil water movement is downward. A core element in Danish environmental legislation involves prescription of minimum utilization rates for nitrogen content in applied manure. Timely field application within restricted periods is an important instrument to avoid loss of nitrogen to the aquatic environment. Mandatory maximum fertilization norms apply to all farms, also to farms that do not use livestock manure. The norms, which are specified for all crops, different soil types, are set at ninety percent of economically optimal nitrogen application derived from fitting the cubic function to field trial data from the same source as used in this thesis.

Whether the ten- percent reduction of optimal N-application is well justified from an economic point of view depends on the actual loss of nitrogen to the aquatic environment by changes in fertilization intensity and on the externalities associated with nitrogen pollution of water. In this context it is interesting to note that several studies indicate a certain plant-soil buffering capacity. It means that labile inorganic nitrogen concentration in the soil does not change in line with N-application within certain limits. In such circumstances, modest overshooting – intentional or unintentional – from yield maximizing fertilization entails no great environmental risk. In this light the mandatory

lowering of N-application by ten percent in relation to  $N_{opt}$  may not in itself contribute significantly to reduction of N-pollution. It seems obvious, however, that to the extent the lower maximum norm for total nitrogen use entails an increased utilization rate for manure nitrogen there will also be a positive environmental effect.

This study of yield response to nitrogen reveals some areas of further research, which could improve the possibilities of establishing more robust response estimates. Some areas relate to the design of field trials while others pertain to the response models. Unpredictable climate induced variation in mineralization of soil nitrogen among otherwise homogeneous trial sites is a cause of uncertainty. Below normal mineralization may have the result that  $N_{max}$  and  $N_{opt}$  are not revealed within the upper bound of the applied fertilizer nitrogen domain or in its vicinity. It is important that the highest nitrogen application level can reveal the existence of yield maximum/plateau under varying climatic conditions.

Another trial design question relates to the lower end of the nitrogen application scale, where additional fertilizer application levels could improve estimation of the curvature of the response function enabling a more precise estimation of mineralization under field conditions.

Some authors suggest that horizontal movement of labile plant accessible nitrogen between trial plots with different levels of nitrogen application could be significant. If that is the case adequate changes of trial design should be considered to prevent false inferences from the field trials about the slope of the true response curve, yield maximizing and optimizing nitrogen amounts.

Variations in nitrogen mineralization, which has an obvious impact on the model estimation of the yield/fertilization relationship, is linked with water and temperature conditions in the soil. A series of field experiments placed on trial sites with the same soil characteristics and with direct monitoring of soil water and soil temperature could improve the basis for satisfactory model prediction of mineralization. Likewise, trials placed on fallow soil with measurements of plant available nitrogen at the beginning and end of the growing season of available soil-N could provide some guidance regarding the extent of mineralization and its dependency on environmental factors.

Improved knowledge about mineralization would improve the possibilities of establishing an extended yield-nitrogen-climate model. It could be assumed that climate induced variation in nitrogen mineralization in itself does not influence the general yield/nitrogen relationship because nitrogen supplied from different sources – applied fertilizer or mineralized soil nitrogen – have the same accessibility and nutritional value for the growing plants. It could further be assumed that climatic factors – not necessarily the same as those influencing mineralization – have an impact on the yield/nitrogen productivity. Given sufficient information, an extended model could estimate yield as a function of total N-supply and climatic factors where total N-supply could be established as the sum of mineralization estimates and applied nitrogen. Fine-tuning of nitrogen application in accordance with soil-N fluctuations attracts attention in both a farm profit and an environmental context. The feasibility of an extended decision model depends on both the costs of acquiring the needed additional knowledge and the possibilities of corrective N-applications during the growing season.

## Appendix A

### Data Used

#### A.1. Danish Field Trial Data

A total of 84 field trials were selected for the analysis. For each trial general information is stated in boxes in Appendix Table A.1:

Trial:	Identification number 1-84
Year:	Year of trial 1987-1998
GridNo:	Geographic location identification number 1-44
Crop:	Name of crop: SBAR = spring barley WBAR = winter barley WWHE = winter wheat
Nmin:	Plant-available nitrogen (mineralized) at beginning of growing season based on soil sample, kilogram (kg) per hectare (ha)

The headings of the table columns have the following meaning:

Yield	Hectokilogram (hkg = 100 kg) per hectare
Nappl	Applied fertilizer nitrogen, kilogram (kg) per ha
RepliNo	Replication number in the trial

Metric measures are used throughout the thesis. For information:

1 kg = 2.2046 lbs.
1 ha = 2.469 acres
1 hkg cereals per ha = 1.49 bushels of wheat 1.86 bushels of barley 2.79 bushels of oats
1kg nitrogen per ha = 0.89 lbs. of nitrogen per acre

#### A.2. Climate Data

Climate information about weekly precipitation, potential evaporation, average temperature, and incoming radiation for 44 regions (grid numbers) in Denmark is available since 1993. Table A.2.1 shows averages of weekly observations by years and

grids in those combinations, which match the 41 trials numbers with location data. The information in the table is sorted by trial number. Climate data are calculated as averages for the whole growing season (calendar week 13 through 31), for first half of the season (week 13 through 22), and for the second half (week 23 through 31). Precipitation and potential evaporation have been added as net water balance.

The headings of the table columns with climate information have the following meaning:

Temperature	Average weekly temperature in center grades, °C
Radiation	Incident of solar radiation
Water balance	Difference between precipitation and potential evaporation, millimeters (mm) per area unit; 1 mm = 0.0394 inches

## APPENDIX A

## A.1. Danish Field Trial Data

Source: Landskontoret for Planteavl, Danish Farmers' Union and Danish Family Farmers' Association

	Yield	Nappl	RepliNo
Trial: 1	34.70	0	1
Year: 87	30.09	0	2
GridNo: 0	37.77	0	3
Crop: SBAR	37.46	0	4
Nmin: 0.00	35.62	0	5
	46.03	40	1
	45.09	40	2
	47.89	40	3
	53.80	40	4
	54.73	40	5
	51.41	80	1
	53.56	80	2
	56.03	80	3
	58.49	80	4
	59.72	80	5
	54.56	120	1
	57.51	120	2
	57.21	120	3
	58.39	120	4
	57.51	120	5
	56.07	160	1
	56.36	160	2
	57.23	160	3
	56.36	160	4
	59.54	160	5
Trial: 2	36.53	0	1
Year: 87	39.71	0	2
GridNo: 0	36.53	0	3
Crop: SBAR	31.76	0	4
Nmin: 0.00	34.94	0	5
	48.24	40	1
	49.84	40	2
	48.24	40	3
	45.02	40	4
	45.02	40	5
	49.84	80	1
	53.06	80	2
	46.63	80	3
	53.06	80	4
	51.45	80	5
	47.94	120	1
	47.94	120	2
	44.75	120	3
	49.54	120	4
	44.75	120	5
	52.41	160	1
	42.88	160	2
	46.06	160	3
	47.65	160	4
	44.47	160	5
Trial: 3	32.59	0	1
Year: 87	39.11	0	2
GridNo: 0	43.99	0	3
Crop: SBAR	40.74	0	4
Nmin: 0.00	45.68	40	1
	44.05	40	2
	47.31	40	3
	50.57	40	4
	49.29	80	1
	47.65	80	2
	52.58	80	3
	52.58	80	4
	54.03	120	1
	54.03	120	2
	52.39	120	3
	54.03	120	4
	50.69	160	1
	52.33	160	2
	52.33	160	3
	53.96	160	4
Trial: 4	29.79	0	1
Year: 88	36.44	0	2
GridNo: 0	34.54	0	3
Crop: SBAR	35.81	0	4
Nmin: 0.00	35.17	0	5
	45.77	40	1
	53.98	40	2
	48.30	40	3
	52.09	40	4
	45.77	40	5
	57.35	80	1
	62.74	80	2
	66.86	80	3
	57.67	80	4
	61.79	80	5
	66.14	120	1
	62.03	120	2
	68.04	120	3
	68.67	120	4
	65.83	120	5
	66.86	160	1
	61.79	160	2
	76.36	160	3
	70.34	160	4
	70.03	160	5
Trial: 5	32.94	0	1
Year: 88	30.19	0	2
GridNo: 0	30.19	0	3
Crop: SBAR	34.31	0	4
Nmin: 0.00	34.31	0	5
	44.02	40	1
	39.90	40	2
	38.52	40	3
	35.77	40	4
	41.27	40	5
	44.34	80	1
	41.57	80	2
	41.57	80	3
	40.19	80	4
	42.96	80	5
	47.12	120	1
	40.19	120	2
	40.19	120	3
	41.57	120	4
	40.19	120	5
	45.56	160	1
	42.80	160	2
	40.04	160	3
	41.42	160	4
	41.42	160	5
Trial: 6	38.84	0	1
Year: 89	32.63	0	2
GridNo: 0	32.63	0	3
Crop: SBAR	35.74	0	4
Nmin: 0.00	52.88	40	1
	46.66	40	2
	51.32	40	3
	49.77	40	4
	59.17	80	1
	54.50	80	2
	57.61	80	3
	59.17	80	4
	63.84	120	1
	60.73	120	2
	65.40	120	3
	62.28	120	4
	66.80	160	1
	62.14	160	2
	66.80	160	3
	63.69	160	4
Trial: 7	33.02	0	1
Year: 89	37.97	0	2
GridNo: 0	37.97	0	3
Crop: SBAR	41.27	0	4
Nmin: 0.00	52.83	40	1
	47.88	40	2
	46.23	40	3
	44.58	40	4
	52.83	80	1
	54.48	80	2
	51.18	80	3
	52.83	80	4
	54.48	120	1
	54.48	120	2
	52.83	120	3
	52.83	120	4
	54.48	160	1
	54.48	160	2
	51.18	160	3
	52.83	160	4
	47.88	200	1
	49.53	200	2
	49.53	200	3
	49.53	200	4
Trial: 8	35.00	0	1
Year: 89	36.32	0	2
GridNo: 0	33.02	0	3
Crop: SBAR	32.03	0	4
Nmin: 0.00	43.26	0	5
	38.02	40	1
	40.33	40	2
	40.33	40	3
	41.32	40	4
	49.59	40	5
	42.32	80	1
	41.65	80	2
	44.63	80	3
	41.98	80	4
	50.25	80	5
	44.68	120	1
	44.02	120	2
	43.69	120	3
	50.64	120	4
	54.61	120	5
	46.78	160	1
	46.78	160	2
	41.43	160	3
	52.46	160	4
	46.44	160	5
	45.56	200	1
	46.56	200	2
	39.24	200	3
	51.55	200	4
	48.22	200	5
Trial: 9	50.95	0	1
Year: 90	48.26	0	2
GridNo: 0	46.05	0	3
Crop: SBAR	53.40	0	4
Nmin: 0.00	67.94	40	1
	69.67	40	2
	60.28	40	3
	64.48	40	4
	73.58	80	1
	72.34	80	2
	70.61	80	3
	68.39	80	4
	71.67	120	1
	72.16	120	2
	71.91	120	3
	72.89	120	4
	68.71	160	1
	70.92	160	2
	68.46	160	3
	73.62	160	4



A.1. Danish Field Trials (continued)

	Yield	Nappl	RepliNo
Trial: 10	34.01	0	1
	37.25	0	2
Year: 90	32.39	0	3
	32.39	0	4
GridNo: 0	30.77	0	5
	50.15	40	1
Crop: SBAR	45.29	40	2
	46.91	40	3
Nmin: 0.00	46.91	40	4
	43.68	40	5
	56.20	80	1
	59.51	80	2
	59.51	80	3
	56.20	80	4
	54.55	80	5
	66.35	120	1
	61.38	120	2
	64.69	120	3
	59.72	120	4
	59.72	120	5
	61.52	160	1
	69.84	160	2
	64.85	160	3
	66.51	160	4
	61.52	160	5
Trial: 11	27.34	0	1
	27.83	0	2
Year: 88	27.83	0	3
	30.49	0	4
GridNo: 0	42.19	40	1
	42.19	40	2
Crop: SBAR	45.36	40	3
	49.02	40	4
Nmin: 0.00	53.03	80	1
	50.58	80	2
	57.67	80	3
	58.65	80	4
	60.20	120	1
	59.00	120	2
	64.54	120	3
	64.29	120	4
	62.14	160	1
	59.96	160	2
	65.52	160	3
	62.86	160	4
Trial: 12	44.43	0	1
	42.32	0	2
Year: 91	48.36	0	3
	43.83	0	4
GridNo: 0	46.55	0	5
	56.39	40	1
Crop: SBAR	54.25	40	2
	61.26	40	3
Nmin: 0.00	59.74	40	4
	55.17	40	5
	63.32	80	1
	61.19	80	2
	57.84	80	3
	64.24	80	4
	61.50	80	5
	64.54	120	1
	68.17	120	2
	66.66	120	3
	66.35	120	4
	63.63	120	5
	66.90	160	1
	65.68	160	2
	60.51	160	3
	63.25	160	4
	60.51	160	5

	Yield	Nappl	RepliNo
Trial: 13	36.77	0	1
	33.07	0	2
Year: 87	36.16	0	3
	39.86	0	4
GridNo: 0	43.26	0	5
	48.58	40	1
Crop: SBAR	45.17	40	2
	50.74	40	3
Nmin: 0.00	53.22	40	4
	54.77	40	5
	52.27	80	1
	51.96	80	2
	58.11	80	3
	56.88	80	4
	58.72	80	5
	53.19	120	1
	53.79	120	2
	57.44	120	3
	58.66	120	4
	58.35	120	5
	58.13	160	1
	57.83	160	2
	59.33	160	3
	58.43	160	4
	58.13	160	5
Trial: 14	28.34	0	1
	27.00	0	2
Year: 88	29.69	0	3
	31.04	0	4
GridNo: 0	29.69	0	5
	42.04	40	1
Crop: SBAR	40.68	40	2
	40.14	40	3
Nmin: 0.00	42.85	40	4
	44.75	40	5
	50.18	80	1
	48.82	80	2
	50.18	80	3
	44.75	80	4
	55.60	80	5
	54.05	120	1
	55.41	120	2
	56.22	120	3
	50.00	120	4
	56.76	120	5
	58.04	160	1
	58.04	160	2
	59.39	160	3
	54.80	160	4
	58.85	160	5
	54.80	200	1
	56.15	200	2
	53.45	200	3
	51.29	200	4
	58.04	200	5
Trial: 15	51.31	0	1
	45.50	0	2
Year: 88	42.97	0	3
	53.08	0	4
GridNo: 0	44.24	0	5
	57.01	40	1
Crop: SBAR	58.80	40	2
	58.29	40	3
Nmin: 0.00	62.63	40	4
	56.24	40	5
	63.91	80	1
	65.19	80	2
	66.47	80	3
	67.75	80	4
	65.19	80	5
	66.80	120	1
	66.80	120	2
	68.08	120	3
	70.14	120	4
	68.08	120	5
	66.72	160	1
	66.72	160	2
	69.28	160	3
	69.28	160	4
	69.28	160	5
	67.15	200	1
	67.66	200	2
	69.70	200	3
	67.15	200	4
	68.43	200	5

	Yield	Nappl	RepliNo
Trial: 16	28.05	0	1
	24.29	0	2
Year: 88	20.77	0	3
	24.29	0	4
GridNo: 0	23.76	0	5
	36.33	40	1
Crop: SBAR	37.22	40	2
	33.48	40	3
Nmin: 0.00	36.50	40	4
	35.61	40	5
	47.24	80	1
	48.13	80	2
	45.81	80	3
	49.92	80	4
	44.91	80	5
	54.82	120	1
	53.38	120	2
	55.53	120	3
	56.97	120	4
	55.71	120	5
	59.85	160	1
	57.36	160	2
	57.18	160	3
	61.63	160	4
	55.93	160	5
Trial: 17	23.59	0	1
	26.42	0	2
Year: 88	33.97	0	3
	33.74	0	4
GridNo: 44	35.62	0	5
	38.54	40	1
Crop: SBAR	41.65	40	2
	43.57	40	3
Nmin: 0.00	43.81	40	4
	43.33	40	5
	42.72	80	1
	44.88	80	2
	47.52	80	3
	46.32	80	4
	45.60	80	5
	46.20	120	1
	46.68	120	2
	47.16	120	3
	43.31	120	4
	43.31	120	5
	44.82	160	1
	44.82	160	2
	44.09	160	3
	45.06	160	4
	42.89	160	5
Trial: 18	27.41	0	1
	23.82	0	2
Year: 88	24.14	0	3
	27.08	0	4
GridNo: 0	24.80	0	5
	33.27	40	1
Crop: SBAR	33.27	40	2
	28.33	40	3
Nmin: 0.00	33.60	40	4
	30.64	40	5
	36.39	80	1
	33.44	80	2
	33.11	80	3
	35.41	80	4
	33.44	80	5
	35.86	120	1
	34.22	120	2
	32.57	120	3
	38.17	120	4
	34.88	120	5
	38.08	160	1
	31.51	160	2
	34.46	160	3
	34.46	160	4
	34.79	160	5

## A.1. Danish Field Trials (continued)

	Yield	Nappl	RepliNo
Trial: 19	35.04	0	1
Year: 88	38.09	0	2
GridNo: 0	38.09	0	3
Crop: SBAR	41.14	0	4
Nmin: 0.00	38.09	0	5
	52.27	40	1
	49.19	40	2
	52.27	40	3
	47.65	40	4
	44.58	40	5
	52.07	80	1
	56.66	80	2
	56.66	80	3
	62.79	80	4
	53.60	80	5
	62.71	120	1
	64.24	120	2
	62.71	120	3
	62.71	120	4
	61.18	120	5
	65.76	160	1
	67.29	160	2
	65.76	160	3
	71.88	160	4
	62.71	160	5
Trial: 20	13.37	0	1
Year: 88	18.04	0	2
GridNo: 0	17.41	0	3
Crop: SBAR	19.90	0	4
Nmin: 0.00	31.96	40	1
	37.29	40	2
	32.90	40	3
	35.41	40	4
	42.30	80	1
	44.18	80	2
	38.54	80	3
	42.30	80	4
	46.88	120	1
	48.45	120	2
	48.76	120	3
	47.82	120	4
	50.94	160	1
	50.63	160	2
	50.94	160	3
	43.48	160	4
Trial: 21	8.73	0	1
Year: 88	18.72	0	2
GridNo: 0	19.96	0	3
Crop: SBAR	21.21	0	4
Nmin: 0.00	17.47	0	5
	24.95	40	1
	28.69	40	2
	27.44	40	3
	27.44	40	4
	28.69	40	5
	34.06	80	1
	35.32	80	2
	36.58	80	3
	36.58	80	4
	37.84	80	5
	43.11	120	1
	45.65	120	2
	43.11	120	3
	41.84	120	4
	43.11	120	5
	45.71	160	1
	50.79	160	2
	41.90	160	3
	43.17	160	4
	41.90	160	5

	Yield	Nappl	RepliNo
Trial: 22	26.99	0	1
Year: 88	29.75	0	2
GridNo: 0	33.12	0	3
Crop: SBAR	28.52	0	4
Nmin: 0.00	50.60	40	1
	46.31	40	2
	44.47	40	3
	46.61	40	4
	50.84	80	1
	52.07	80	2
	51.15	80	3
	49.62	80	4
	59.80	120	1
	56.73	120	2
	56.73	120	3
	54.89	120	4
	56.53	160	1
	57.75	160	2
	55.62	160	3
	55.62	160	4
Trial: 23	23.33	0	1
Year: 88	17.19	0	2
GridNo: 0	24.07	0	3
Crop: SBAR	27.01	0	4
Nmin: 0.00	24.80	0	5
	35.61	40	1
	30.45	40	2
	34.63	40	3
	37.82	40	4
	37.08	40	5
	42.73	80	1
	36.10	80	2
	42.49	80	3
	45.68	80	4
	39.54	80	5
	44.94	120	1
	43.71	120	2
	50.10	120	3
	52.31	120	4
	52.06	120	5
	49.36	160	1
	51.57	160	2
	55.01	160	3
	55.26	160	4
	53.54	160	5
Trial: 24	34.31	0	1
Year: 89	34.61	0	2
GridNo: 0	37.60	0	3
Crop: SBAR	37.30	0	4
Nmin: 0.00	33.72	0	5
	40.38	40	1
	46.36	40	2
	48.15	40	3
	42.47	40	4
	41.57	40	5
	43.47	80	1
	53.66	80	2
	52.16	80	3
	46.77	80	4
	51.26	80	5
	46.66	120	1
	55.03	120	2
	53.24	120	3
	50.25	120	4
	52.64	120	5
	50.07	160	1
	54.26	160	2
	53.36	160	3
	54.86	160	4
	53.96	160	5

	Yield	Nappl	RepliNo
Trial: 25	33.81	0	1
Year: 90	33.81	0	2
GridNo: 0	35.59	0	3
Crop: SBAR	39.15	0	4
Nmin: 0.00	33.81	0	5
	56.42	40	1
	55.42	40	2
	52.78	40	3
	52.78	40	4
	54.60	40	5
	64.30	80	1
	62.46	80	2
	62.46	80	3
	60.62	80	4
	60.62	80	5
	70.05	120	1
	68.21	120	2
	70.05	120	3
	68.21	120	4
	66.36	120	5
	75.14	160	1
	73.31	160	2
	78.81	160	3
	76.98	160	4
	73.31	160	5
Trial: 26	33.34	0	1
Year: 91	29.76	0	2
GridNo: 0	28.18	0	3
Crop: SBAR	23.41	0	4
Nmin: 0.00	26.59	0	5
	42.82	40	1
	45.20	40	2
	40.83	40	3
	38.46	40	4
	41.23	40	5
	54.10	80	1
	54.10	80	2
	50.92	80	3
	50.92	80	4
	53.30	80	5
	65.08	120	1
	61.12	120	2
	60.72	120	3
	61.91	120	4
	63.50	120	5
	68.89	160	1
	69.28	160	2
	65.72	160	3
	70.47	160	4
	68.49	160	5
Trial: 27	25.79	0	1
Year: 87	27.73	0	2
GridNo: 0	23.57	0	3
Crop: WBAR	26.34	0	4
Nmin: 0.00	24.96	0	5
	43.24	50	1
	41.85	50	2
	43.24	50	3
	44.64	50	4
	46.03	50	5
	55.80	100	1
	57.19	100	2
	51.61	100	3
	55.80	100	4
	55.80	100	5
	58.73	150	1
	60.13	150	2
	58.73	150	3
	55.93	150	4
	54.53	150	5
	69.18	200	1
	63.53	200	2
	59.29	200	3
	57.88	200	4
	56.47	200	5

## A.1. Danish Field Trials (continued)

	Yield	Nappl	RepliNo
Trial: 28	67.96	0	1
Year: 90	64.48	0	2
GrndNo: 0	53.42	0	3
Crop: WBAR	53.73	0	4
Nmin: 0.00	56.89	0	5
	79.39	50	1
	79.39	50	2
	72.51	50	3
	75.32	50	4
	70.64	50	5
	92.43	100	1
	90.55	100	2
	87.73	100	3
	89.30	100	4
	80.53	100	5
	93.69	150	1
	90.87	150	2
	90.55	150	3
	89.93	150	4
	88.36	150	5
	94.82	200	1
	93.24	200	2
	92.93	200	3
	95.46	200	4
	89.77	200	5
Trial: 29	46.56	0	1
Year: 88	53.21	0	2
GrndNo: 0	56.53	0	3
Crop: WWHE	48.22	0	4
Nmin: 0.00	53.21	0	5
	68.01	50	1
	69.67	50	2
	72.99	50	3
	66.35	50	4
	69.67	50	5
	78.06	100	1
	81.38	100	2
	84.70	100	3
	74.74	100	4
	78.06	100	5
	89.58	150	1
	89.58	150	2
	89.58	150	3
	86.26	150	4
	87.92	150	5
	95.53	200	1
	93.88	200	2
	90.59	200	3
	88.94	200	4
	92.24	200	5
	97.64	250	1
	92.67	250	2
	94.33	250	3
	92.67	250	4
	102.60	250	5
Trial: 30	36.62	0	1
Year: 88	34.99	0	2
GrndNo: 0	32.14	0	3
Crop: WWHE	36.21	0	4
Nmin: 0.00	34.18	0	5
	63.62	50	1
	51.80	50	2
	52.20	50	3
	55.47	50	4
	63.62	50	5
	71.07	100	1
	69.42	100	2
	67.78	100	3
	70.65	100	4
	68.60	100	5
	82.57	150	1
	79.28	150	2
	79.69	150	3
	81.34	150	4
	79.69	150	5
	87.09	200	1
	85.85	200	2
	85.44	200	3
	87.91	200	4
	86.68	200	5
	92.58	250	1
	87.28	250	2
	90.95	250	3
	92.17	250	4
	86.87	250	5

	Yield	Nappl	RepliNo
Trial: 31	48.59	0	1
Year: 89	35.83	0	2
GrndNo: 0	37.96	0	3
Crop: WWHE	42.51	0	4
Nmin: 0.00	44.64	0	5
	59.15	50	1
	57.63	50	2
	48.53	50	3
	57.02	50	4
	54.60	50	5
	68.58	100	1
	68.27	100	2
	62.48	100	3
	68.58	100	4
	66.14	100	5
	69.27	150	1
	74.46	150	2
	71.71	150	3
	73.85	150	4
	74.15	150	5
	70.30	200	1
	75.21	200	2
	70.30	200	3
	78.28	200	4
	71.83	200	5
	69.24	250	1
	72.62	250	2
	74.16	250	3
	69.54	250	4
	71.70	250	5
Trial: 32	44.65	0	1
Year: 89	43.88	0	2
GrndNo: 0	46.96	0	3
Crop: WWHE	46.58	0	4
Nmin: 0.00	48.12	0	5
	66.59	50	1
	71.60	50	2
	68.90	50	3
	75.06	50	4
	66.59	50	5
	81.32	100	1
	85.18	100	2
	84.41	100	3
	87.49	100	4
	82.48	100	5
	88.80	150	1
	91.12	150	2
	88.41	150	3
	86.86	150	4
	86.86	150	5
	89.79	200	1
	88.24	200	2
	88.24	200	3
	90.18	200	4
	87.46	200	5
	88.91	250	1
	92.01	250	2
	83.08	250	3
	90.07	250	4
	82.69	250	5
Trial: 33	60.98	0	1
Year: 89	53.85	0	2
GrndNo: 0	47.60	0	3
Crop: WWHE	50.87	0	4
Nmin: 0.00	74.89	50	1
	73.41	50	2
	72.22	50	3
	71.34	50	4
	93.89	100	1
	91.52	100	2
	89.14	100	3
	89.44	100	4
	100.60	150	1
	99.12	150	2
	101.79	150	3
	97.04	150	4
	104.79	200	1
	104.79	200	2
	105.39	200	3
	101.79	200	4
	104.98	250	1
	104.37	250	2
	111.01	250	3
	108.60	250	4

	Yield	Nappl	RepliNo
Trial: 34	73.22	0	1
Year: 90	57.95	0	2
GrndNo: 0	54.03	0	3
Crop: WWHE	59.90	0	4
Nmin: 0.00	50.51	0	5
	81.93	50	1
	77.22	50	2
	72.13	50	3
	75.26	50	4
	67.03	50	5
	96.94	100	1
	90.27	100	2
	91.84	100	3
	87.13	100	4
	87.13	100	5
	107.27	150	1
	99.81	150	2
	99.81	150	3
	94.70	150	4
	101.38	150	5
	108.32	200	1
	110.28	200	2
	109.50	200	3
	111.85	200	4
	111.07	200	5
	117.38	250	1
	116.98	250	2
	110.29	250	3
	114.23	250	4
	119.35	250	5
Trial: 35	37.00	0	1
Year: 91	37.98	0	2
GrndNo: 0	33.07	0	3
Crop: WWHE	32.42	0	4
Nmin: 0.00	28.82	0	5
	61.20	50	1
	63.17	50	2
	61.86	50	3
	58.57	50	4
	51.99	50	5
	81.50	100	1
	81.50	100	2
	82.16	100	3
	79.53	100	4
	75.58	100	5
	95.09	150	1
	98.05	150	2
	101.34	150	3
	95.42	150	4
	88.84	150	5
	103.23	200	1
	104.88	200	2
	107.19	200	3
	101.25	200	4
	99.60	200	5
	111.28	250	1
	108.63	250	2
	111.61	250	3
	102.69	250	4
	105.99	250	5
Trial: 36	32.01	0	1
Year: 87	34.42	0	2
GrndNo: 0	38.05	0	3
Crop: WWHE	30.50	0	4
Nmin: 0.00	31.40	0	5
	41.96	50	1
	47.47	50	2
	46.55	50	3
	46.86	50	4
	43.80	50	5
	52.40	100	1
	56.62	100	2
	57.22	100	3
	52.71	100	4
	55.42	100	5
	67.42	150	1
	66.52	150	2
	63.80	150	3
	67.42	150	4
	62.89	150	5
	70.12	200	1
	72.54	200	2
	77.70	200	3
	71.63	200	4
	68.29	200	5
	81.12	250	1
	82.33	250	2
	86.55	250	3
	92.88	250	4
	68.76	250	5

## A.1. Danish Field Trials (continued)

	Yield	Nappl	RepliNo
Trial: 37	54.83	0	1
	47.52	0	2
Year: 87	57.08	0	3
	53.99	0	4
GridNo: 0	64.09	50	1
	60.71	50	2
Crop: WWHE	66.07	50	3
	66.64	50	4
Nmin: 0.00	72.00	100	1
	69.74	100	2
	74.26	100	3
	72.56	100	4
	73.31	150	1
	72.18	150	2
	77.26	150	3
	69.36	150	4
	74.86	200	1
	73.18	200	2
	78.51	200	3
	81.87	200	4
	79.63	250	1
	71.78	250	2
	78.79	250	3
	68.42	250	4
Trial: 38	47.59	0	1
	51.08	0	2
Year: 89	46.00	0	3
	43.15	0	4
GridNo: 0	51.71	0	5
	64.24	50	1
Crop: WWHE	63.93	50	2
	64.88	50	3
Nmin: 0.00	60.76	50	4
	69.62	50	5
	70.05	100	1
	77.06	100	2
	65.60	100	3
	71.01	100	4
	69.42	100	5
	69.42	150	1
	72.92	150	2
	75.15	150	3
	74.51	150	4
	80.24	150	5
	70.55	200	1
	74.70	200	2
	72.14	200	3
	80.76	200	4
	74.70	200	5
	77.79	250	1
	72.37	250	2
	74.60	250	3
	88.00	250	4
	76.84	250	5
Trial: 39	31.35	0	1
	32.55	0	2
Year: 95	40.19	0	3
	34.16	0	4
GridNo: 30	31.35	0	5
	32.95	0	6
Crop: SBAR	40.33	40	1
	43.56	40	2
Nmin: 31.00	46.78	40	3
	40.73	40	4
	43.56	40	5
	40.33	40	6
	57.33	80	1
	53.30	80	2
	59.35	80	3
	53.30	80	4
	62.99	80	5
	49.26	80	6
	73.10	120	1
	64.98	120	2
	68.23	120	3
	69.04	120	4
	68.23	120	5
	69.85	120	6
	74.78	160	1
	80.85	160	2
	70.34	160	3
	72.76	160	4
	77.61	160	5
	71.95	160	6
	76.00	200	1
	85.29	200	2
	70.34	200	3
	81.66	200	4
	74.78	200	5
	74.78	200	6

	Yield	Nappl	RepliNo
Trial: 40	28.60	0	1
	27.15	0	2
Year: 97	24.97	0	3
	27.63	0	4
GridNo: 30	25.93	0	5
	42.37	40	1
Crop: SBAR	41.88	40	2
	39.46	40	3
Nmin: 14.04	39.70	40	4
	39.70	40	5
	53.64	80	1
	53.15	80	2
	49.98	80	3
	53.64	80	4
	51.69	80	5
	61.68	120	1
	61.68	120	2
	58.27	120	3
	59.97	120	4
	58.76	120	5
	63.97	160	1
	59.84	160	2
	60.56	160	3
	60.81	160	4
	62.27	160	5
	66.80	200	1
	66.56	200	2
	64.12	200	3
	64.85	200	4
	62.17	200	5
Trial: 41	27.14	0	1
	28.76	0	2
Year: 92	31.19	0	3
	33.21	0	4
GridNo: 34	29.16	0	5
	32.29	40	1
Crop: SBAR	33.10	40	2
	38.35	40	3
Nmin: 39.00	32.29	40	4
	42.39	40	5
	34.27	80	1
	36.69	80	2
	33.47	80	3
	37.10	80	4
	45.16	80	5
	34.16	120	1
	38.18	120	2
	38.18	120	3
	45.01	120	4
	47.42	120	5
	39.46	160	1
	41.45	160	2
	39.46	160	3
	44.64	160	4
	51.02	160	5
Trial: 42	35.16	0	1
	33.62	0	2
Year: 93	30.58	0	3
	32.53	0	4
GridNo: 34	34.34	0	5
	39.92	40	1
Crop: SBAR	39.25	40	2
	43.61	40	3
Nmin: 35.00	37.49	40	4
	39.51	40	5
	43.83	80	1
	45.08	80	2
	46.89	80	3
	39.93	80	4
	45.95	80	5
	49.03	120	1
	46.64	120	2
	49.35	120	3
	52.27	120	4
	47.41	120	5
	49.32	160	1
	49.59	160	2
	54.24	160	3
	49.37	160	4
	53.07	160	5

	Yield	Nappl	RepliNo
Trial: 43	31.85	0	1
	33.84	0	2
Year: 94	40.21	0	3
	41.01	0	4
GridNo: 30	34.24	0	5
	41.75	40	1
Crop: SBAR	44.93	40	2
	45.33	40	3
Nmin: 27.00	45.73	40	4
	39.37	40	5
	54.48	80	1
	50.50	80	2
	54.48	80	3
	54.48	80	4
	55.27	80	5
	63.55	120	1
	59.18	120	2
	65.14	120	3
	58.78	120	4
	57.99	120	5
	69.11	160	1
	62.36	160	2
	71.49	160	3
	64.34	160	4
	61.56	160	5
	64.34	200	1
	69.51	200	2
	71.89	200	3
	62.75	200	4
	52.82	200	5
Trial: 44	26.92	0	1
	29.69	0	2
Year: 94	24.94	0	3
	28.11	0	4
GridNo: 34	24.94	0	5
	41.13	40	1
Crop: SBAR	41.13	40	2
	37.18	40	3
Nmin: 31.00	39.95	40	4
	38.37	40	5
	56.30	80	1
	52.73	80	2
	49.16	80	3
	51.94	80	4
	53.12	80	5
	62.38	120	1
	55.23	120	2
	59.60	120	3
	60.39	120	4
	57.21	120	5
	67.64	160	1
	63.66	160	2
	69.63	160	3
	65.25	160	4
	63.66	160	5
	74.71	200	1
	68.75	200	2
	72.33	200	3
	73.12	200	4
	66.76	200	5
Trial: 45	32.21	0	1
	30.25	0	2
Year: 95	33.00	0	3
	32.21	0	4
GridNo: 34	30.25	0	5
	44.43	40	1
Crop: SBAR	44.43	40	2
	47.18	40	3
Nmin: 19.59	48.75	40	4
	46.00	40	5
	61.34	80	1
	60.95	80	2
	59.77	80	3
	60.56	80	4
	62.13	80	5
	64.17	120	1
	64.96	120	2
	66.92	120	3
	69.29	120	4
	67.32	120	5
	71.90	160	1
	71.51	160	2
	75.46	160	3
	75.46	160	4
	74.67	160	5
	71.03	200	1
	73.00	200	2
	72.21	200	3
	76.16	200	4
	70.63	200	5

A.1. Danish Field Trials (continued)

	Yield	Nappl	RepliNo
Trial: 46	69.49	0	1
	66.80	0	2
Year: 96	63.11	0	3
	62.10	0	4
GridNo: 33	58.07	0	5
	79.99	40	1
Crop: SBAR	77.97	40	2
	76.63	40	3
Nmin: 72.36	65.20	40	4
	70.58	40	5
	85.03	80	1
	85.36	80	2
	84.02	80	3
	78.64	80	4
	77.97	80	5
	86.94	120	1
	83.92	120	2
	83.59	120	3
	81.57	120	4
	82.24	120	5
	85.84	160	1
	84.49	160	2
	79.80	160	3
	77.79	160	4
	84.16	160	5
	83.59	200	1
	81.24	200	2
	77.21	200	3
	80.56	200	4
	80.90	200	5
Trial: 47	58.77	0	1
	53.53	0	2
Year: 96	56.89	0	3
	51.65	0	4
GridNo: 34	49.03	0	5
	70.79	40	1
Crop: SBAR	65.87	40	2
	71.54	40	3
Nmin: 56.16	69.27	40	4
	66.24	40	5
	75.42	80	1
	75.80	80	2
	75.80	80	3
	75.42	80	4
	72.77	80	5
	82.42	120	1
	82.42	120	2
	83.55	120	3
	83.17	120	4
	80.53	120	5
	79.69	160	1
	80.45	160	2
	81.21	160	3
	81.59	160	4
	77.79	160	5
	83.10	200	1
	83.48	200	2
	82.35	200	3
	82.35	200	4
	83.86	200	5
Trial: 48	54.44	0	1
	55.61	0	2
Year: 97	48.22	0	3
	52.11	0	4
GridNo: 34	51.72	0	5
	66.12	41	1
Crop: SBAR	64.15	41	2
	61.40	41	3
Nmin: 37.00	61.79	41	4
	64.15	41	5
	71.18	81	1
	70.39	81	2
	70.78	81	3
	71.18	81	4
	73.16	81	5
	74.61	122	1
	72.62	122	2
	73.81	122	3
	72.62	122	4
	75.40	122	5
	72.47	162	1
	74.45	162	2
	74.45	162	3
	72.47	162	4
	75.24	162	5
	73.77	203	1
	77.72	203	2
	77.32	203	3
	80.48	203	4
	73.77	203	5

	Yield	Nappl	RepliNo
Trial: 49	56.55	0	1
	63.01	0	2
Year: 93	64.63	0	3
	55.90	0	4
GridNo: 7	60.10	0	5
	70.31	50	1
Crop: WBAR	76.49	50	2
	74.86	50	3
Nmin: 49.00	71.28	50	4
	76.49	50	5
	87.99	100	1
	86.36	100	2
	88.97	100	3
	86.36	100	4
	87.99	100	5
	95.14	150	1
	103.91	150	2
	92.22	150	3
	95.14	150	4
	93.19	150	5
	99.68	200	1
	97.41	200	2
	100.01	200	3
	99.04	200	4
	92.22	200	5
Trial: 50	12.74	0	1
	11.47	0	2
Year: 95	9.77	0	3
	16.57	0	4
GridNo: 33	8.92	0	5
	28.36	50	1
Crop: WBAR	25.82	50	2
	28.36	50	3
Nmin: 9.87	29.63	50	4
	27.51	50	5
	45.40	100	1
	42.85	100	2
	45.82	100	3
	44.55	100	4
	44.55	100	5
	54.12	150	1
	52.85	150	2
	57.08	150	3
	54.96	150	4
	53.69	150	5
	60.10	200	1
	61.37	200	2
	60.10	200	3
	59.68	200	4
	60.10	200	5
Trial: 51	17.82	0	1
	26.89	0	2
Year: 95	18.14	0	3
	21.05	0	4
GridNo: 26	17.82	0	5
	31.26	50	1
Crop: WBAR	34.28	50	2
	34.28	50	3
Nmin: 22.38	36.30	50	4
	34.62	50	5
	47.03	100	1
	46.01	100	2
	48.39	100	3
	52.14	100	4
	52.82	100	5
	50.88	150	1
	57.67	150	2
	58.35	150	3
	62.08	150	4
	61.74	150	5
	54.44	200	1
	59.82	200	2
	59.15	200	3
	64.53	200	4
	63.52	200	5

	Yield	Nappl	RepliNo
Trial: 52	14.89	0	1
	23.17	0	2
Year: 90	21.51	0	3
	26.48	0	4
GridNo: 0	23.17	0	5
	34.84	50	1
Crop: WBAR	41.47	50	2
	34.84	50	3
Nmin: 17.00	49.76	50	4
	44.79	50	5
	51.61	100	1
	58.26	100	2
	53.27	100	3
	61.59	100	4
	59.93	100	5
	66.67	150	1
	66.67	150	2
	58.33	150	3
	61.67	150	4
	68.33	150	5
	59.93	200	1
	63.26	200	2
	68.25	200	3
	59.93	200	4
	69.92	200	5
Trial: 53	25.15	0	1
	21.18	0	2
Year: 96	25.15	0	3
	22.84	0	4
GridNo: 33	26.81	0	5
	29.47	50	1
Crop: WBAR	31.15	50	2
	35.50	50	3
Nmin: 44.28	33.49	50	4
	36.50	50	5
	37.93	100	1
	41.96	100	2
	48.34	100	3
	46.32	100	4
	47.00	100	5
	49.40	150	1
	49.74	150	2
	54.44	150	3
	55.79	150	4
	56.87	150	5
	56.87	200	1
	56.87	200	2
	53.52	200	3
	62.89	200	4
	64.56	200	5
	59.54	200	5
Trial: 54	36.50	0	1
	33.83	0	2
Year: 97	29.47	0	3
	29.47	0	4
GridNo: 33	51.24	50	1
	47.22	50	2
Crop: WBAR	46.89	50	3
	51.24	50	4
Nmin: 21.60	62.22	100	1
	58.87	100	2
	60.21	100	3
	62.89	100	4
	68.65	150	1
	61.62	150	2
	57.27	150	3
	69.32	150	4
	72.42	200	1
	61.02	200	2
	54.32	200	3
	72.42	200	4

## A.1. Danish Field Trials (continued)

	Yield	Nappl	RepliNo
Trial: 55	30.21	0	1
	40.28	0	2
Year: 90	52.03	0	3
	45.32	0	4
GridNo: 0	55.39	0	5
	55.39	50	1
Crop: WWHE	63.78	50	2
	72.17	50	3
Nmin: 33.00	70.49	50	4
	70.49	50	5
	80.28	100	1
	83.63	100	2
	85.30	100	3
	81.95	100	4
	85.30	100	5
	87.28	150	1
	88.96	150	2
	88.96	150	3
	90.64	150	4
	93.99	150	5
	93.99	200	1
	92.31	200	2
	97.35	200	3
	88.96	200	4
	104.06	200	5
	93.77	250	1
	100.47	250	2
	100.47	250	3
	97.12	250	4
	103.82	250	5
Trial: 56	57.27	0	1
	57.60	0	2
Year: 94	51.91	0	3
	51.91	0	4
GridNo: 43	66.65	50	1
	66.31	50	2
Crop: WWHE	65.31	50	3
	62.63	50	4
Nmin: 30.00	75.19	100	1
	70.16	100	2
	71.17	100	3
	71.84	100	4
	71.67	150	1
	70.66	150	2
	76.04	150	3
	73.35	150	4
	74.87	200	1
	66.10	200	2
	73.86	200	3
	74.87	200	4
	76.65	250	1
	69.22	250	2
	71.24	250	3
	76.31	250	4
Trial: 57	36.84	0	1
	34.83	0	2
Year: 95	41.86	0	3
	44.54	0	4
GridNo: 43	34.83	0	5
	60.02	50	1
Crop: WWHE	54.32	50	2
	64.71	50	3
Nmin: 44.00	62.36	50	4
	55.99	50	5
	72.43	100	1
	72.76	100	2
	78.15	100	3
	78.15	100	4
	70.74	100	5
	86.00	150	1
	81.62	150	2
	89.71	150	3
	82.29	150	4
	87.35	150	5
	88.80	200	1
	86.10	200	2
	88.46	200	3
	88.46	200	4
	88.13	200	5
	90.02	250	1
	88.33	250	2
	90.70	250	3
	86.98	250	4
	89.35	250	5

	Yield	Nappl	RepliNo
Trial: 58	34.90	0	1
	37.82	0	2
Year: 95	43.48	0	3
	43.28	0	4
GridNo: 42	57.64	50	1
	60.37	50	2
Crop: WWHE	65.04	50	3
	65.24	50	4
Nmin: 18.43	74.52	100	1
	77.43	100	2
	80.92	100	3
	81.31	100	4
	85.48	150	1
	88.58	150	2
	92.27	150	3
	92.27	150	4
	94.98	200	1
	97.11	200	2
	99.83	200	3
	98.66	200	4
	99.31	250	1
	99.90	250	2
	102.04	250	3
	104.77	250	4
Trial: 59	40.99	0	1
	44.40	0	2
Year: 95	34.84	0	3
	39.62	0	4
GridNo: 33	38.94	0	5
	65.58	50	1
Crop: WWHE	65.92	50	2
	57.04	50	3
Nmin: 20.49	65.92	50	4
	63.87	50	5
	83.25	100	1
	78.13	100	2
	80.52	100	3
	82.56	100	4
	83.59	100	5
	89.59	150	1
	83.10	150	2
	88.57	150	3
	92.33	150	4
	93.01	150	5
	92.93	200	1
	82.72	200	2
	96.67	200	3
	92.93	200	4
	96.67	200	5
	95.76	250	1
	93.72	250	2
	98.15	250	3
	95.08	250	4
	97.81	250	5
Trial: 60	31.72	0	1
	33.17	0	2
Year: 95	37.49	0	3
	34.61	0	4
GridNo: 26	31.72	0	5
	52.09	50	1
Crop: WWHE	53.54	50	2
	54.99	50	3
Nmin: 18.36	52.09	50	4
	52.09	50	5
	66.80	100	1
	68.25	100	2
	68.25	100	3
	69.70	100	4
	69.70	100	5
	77.05	150	1
	75.60	150	2
	81.41	150	3
	77.05	150	4
	77.05	150	5
	77.23	200	1
	83.06	200	2
	80.14	200	3
	83.06	200	4
	78.69	200	5
	81.79	250	1
	86.17	250	2
	89.09	250	3
	86.17	250	4
	81.79	250	5

	Yield	Nappl	RepliNo
Trial: 61	32.59	0	1
	36.54	0	2
Year: 96	35.80	0	3
	34.57	0	4
GridNo: 41	59.75	50	1
	63.21	50	2
Crop: WWHE	61.23	50	3
	62.22	50	4
Nmin: 28.08	81.48	100	1
	80.25	100	2
	82.22	100	3
	81.23	100	4
	95.04	150	1
	93.31	150	2
	92.32	150	3
	92.81	150	4
	100.48	200	1
	97.76	200	2
	100.24	200	3
	98.50	200	4
	102.96	250	1
	100.98	250	2
	101.72	250	3
	101.72	250	4
Trial: 62	58.92	0	1
	58.16	0	2
Year: 96	59.43	0	3
	54.86	0	4
GridNo: 42	81.69	50	1
	81.18	50	2
Crop: WWHE	82.96	50	3
	81.43	50	4
Nmin: 32.40	99.31	100	1
	95.24	100	2
	97.53	100	3
	97.53	100	4
	104.50	150	1
	103.49	150	2
	102.47	150	3
	106.54	150	4
	107.17	200	1
	102.33	200	2
	109.72	200	3
	107.42	200	4
	105.50	250	1
	105.50	250	2
	107.54	250	3
	107.54	250	4
Trial: 63	37.40	0	1
	40.46	0	2
Year: 97	31.96	0	3
	38.76	0	4
GridNo: 33	57.87	50	1
	59.56	50	2
Crop: WWHE	63.63	50	3
	61.93	50	4
Nmin: 27.00	69.88	100	1
	74.29	100	2
	67.50	100	3
	67.84	100	4
	80.39	150	1
	74.97	150	2
	83.11	150	3
	71.57	150	4
	85.48	200	1
	84.46	200	2
	83.45	200	3
	78.02	200	4
	89.55	250	1
	78.36	250	2
	91.25	250	3
	79.38	250	4

A.1. Danish Field Trials (continued)

	Yield	Nappl	RepliNo
Trial: 64	35.89	0	1
Year: 97	34.47	0	2
GndNo: 41	34.47	0	3
Crop: WWHE	33.77	0	4
Nmin: 17.28	58.66	50	1
	56.99	50	2
	56.75	50	3
	55.07	50	4
	76.98	100	1
	76.03	100	2
	75.32	100	3
	74.61	100	4
	88.94	150	1
	88.94	150	2
	88.46	150	3
	88.54	150	4
	97.55	200	1
	97.59	200	2
	97.07	200	3
	94.67	200	4
	100.91	250	1
	101.15	250	2
	98.98	250	3
	96.57	250	4
Trial: 65	44.08	0	1
Year: 98	40.51	0	2
GndNo: 42	42.93	0	3
Crop: WWHE	42.46	0	4
Nmin: 23.00	44.31	0	5
	57.63	50	1
	53.14	50	2
	56.52	50	3
	57.91	50	4
	60.09	50	5
	68.30	100	1
	67.54	100	2
	67.26	100	3
	67.14	100	4
	69.35	100	5
	81.69	150	1
	86.50	150	2
	81.93	150	3
	84.30	150	4
	84.08	150	5
	89.50	200	1
	87.85	200	2
	87.61	200	3
	89.70	200	4
	83.31	200	5
	98.10	250	1
	95.87	250	2
	103.06	250	3
	97.78	250	4
	94.21	250	5
Trial: 66	54.52	0	1
Year: 93	61.10	0	2
GndNo: 13	70.19	0	3
Crop: WWHE	56.40	0	4
Nmin: 24.00	53.89	0	5
	83.14	50	1
	88.14	50	2
	80.95	50	3
	77.82	50	4
	79.70	50	5
	95.49	100	1
	100.83	100	2
	98.32	100	3
	96.75	100	4
	99.89	100	5
	91.55	150	1
	103.86	150	2
	97.23	150	3
	97.86	150	4
	97.23	150	5
	104.38	200	1
	98.98	200	2
	93.91	200	3
	92.96	200	4
	92.00	200	5
	99.37	250	1
	103.17	250	2
	93.68	250	3
	96.52	250	4
	100.64	250	5

	Yield	Nappl	RepliNo
Trial: 67	37.21	0	1
Year: 94	38.18	0	2
GndNo: 20	41.38	0	3
Crop: WWHE	41.38	0	4
Nmin: 33.00	42.35	0	5
	64.26	52	1
	60.42	52	2
	64.90	52	3
	64.58	52	4
	58.51	52	5
	69.89	97	1
	74.04	97	2
	73.40	97	3
	71.80	97	4
	67.34	97	5
	79.64	149	1
	79.96	149	2
	79.00	149	3
	77.07	149	4
	72.90	149	5
	80.74	204	1
	80.41	204	2
	73.96	204	3
	74.60	204	4
	74.60	204	5
	75.73	256	1
	81.53	256	2
	78.63	256	3
	75.08	256	4
	74.11	256	5
Trial: 68	31.82	0	1
Year: 95	37.46	0	2
GndNo: 34	37.87	0	3
Crop: WWHE	40.28	0	4
Nmin: 16.10	28.20	0	5
	31.42	0	6
	58.69	50	1
	53.06	50	2
	60.70	50	3
	56.27	50	4
	55.87	50	5
	53.86	50	6
	72.36	100	1
	72.85	100	2
	71.16	100	3
	77.19	100	4
	72.36	100	5
	71.16	100	6
	76.13	150	1
	82.58	150	2
	75.73	150	3
	91.84	150	4
	79.36	150	5
	81.77	150	6
	89.23	200	1
	99.73	200	2
	93.67	200	3
	90.44	200	4
	89.23	200	5
	87.61	200	6
	98.93	250	1
	94.99	250	2
	104.99	250	3
	99.34	250	4
	89.24	250	5
	99.34	250	6
	88.03	250	7
Trial: 69	22.80	0	1
Year: 91	22.80	0	2
GndNo: 0	21.38	0	3
Crop: WWHE	34.21	0	4
Nmin: 26.00	29.45	0	5
	49.35	50	1
	60.74	50	2
	59.31	50	3
	62.64	50	4
	59.79	50	5
	69.26	100	1
	79.15	100	2
	76.80	100	3
	80.10	100	4
	76.33	100	5
	88.31	150	1
	89.73	150	2
	91.15	150	3
	90.20	150	4
	88.31	150	5
	92.42	200	1
	99.06	200	2
	97.16	200	3
	99.06	200	4
	98.11	200	5
	100.47	250	1
	100.95	250	2
	100.95	250	3
	100.47	250	4
	102.36	250	5

	Yield	Nappl	RepliNo
Trial: 70	60.28	0	1
Year: 93	61.18	0	2
GndNo: 36	55.29	0	3
Crop: WWHE	49.40	0	4
Nmin: 38.00	56.20	0	5
	82.28	50	1
	88.16	50	2
	82.73	50	3
	79.12	50	4
	77.76	50	5
	96.99	100	1
	93.36	100	2
	94.72	100	3
	95.18	100	4
	95.63	100	5
	100.53	150	1
	100.53	150	2
	101.44	150	3
	100.99	150	4
	101.90	150	5
	108.67	200	1
	99.99	200	2
	96.34	200	3
	108.21	200	4
	105.93	200	5
	107.30	250	1
	100.90	250	2
	99.99	250	3
	107.75	250	4
	110.95	250	5
Trial: 71	35.75	0	1
Year: 94	39.46	0	2
GndNo: 4	37.14	0	3
Crop: WWHE	39.46	0	4
Nmin: 45.00	61.99	50	1
	59.61	50	2
	60.56	50	3
	64.37	50	4
	75.61	100	1
	71.78	100	2
	76.57	100	3
	77.08	100	4
	82.85	150	1
	90.16	150	2
	80.41	150	3
	88.21	150	4
	83.13	200	1
	89.42	200	2
	96.67	200	3
	96.67	200	4
	99.29	250	1
	98.32	250	2
	94.45	250	3
	96.87	250	4
Trial: 72	32.27	0	1
Year: 94	33.25	0	2
GndNo: 4	22.64	0	3
Crop: WWHE	27.45	0	4
Nmin: 46.00	46.07	50	1
	49.37	50	2
	39.96	50	3
	52.66	50	4
	62.41	100	1
	61.43	100	2
	55.58	100	3
	68.26	100	4
	64.16	150	1
	60.75	150	2
	58.32	150	3
	71.45	150	4
	67.36	200	1
	63.43	200	2
	63.92	200	3
	76.21	200	4
	71.39	250	1
	61.68	250	2
	67.99	250	3
	75.28	250	4

## A.1. Danish Field Trials (continued)

	Yield	Nappl	RepliNo
Trial: 73	18.90	0	1
Year: 94	18.90	0	2
GridNo: 33	12.33	0	3
Crop: WWHE	15.20	0	4
Nmin: 23.00	15.62	0	5
	39.81	50	1
	43.92	50	2
	32.02	50	3
	34.07	50	4
	36.53	50	5
	51.00	100	1
	53.45	100	2
	46.52	100	3
	52.64	100	4
	48.96	100	5
	63.21	150	1
	66.08	150	2
	56.23	150	3
	56.23	150	4
	57.87	150	5
	64.18	200	1
	66.65	200	2
	61.71	200	3
	65.42	200	4
	62.54	200	5
	67.30	250	1
	72.25	250	2
	57.80	250	3
	66.06	250	4
	62.35	250	5
Trial: 74	30.46	0	1
Year: 94	23.40	0	2
GridNo: 18	23.72	0	3
Crop: WWHE	20.84	0	4
Nmin: 24.00	19.56	0	5
	39.55	50	1
	38.91	50	2
	38.91	50	3
	36.36	50	4
	37.00	50	5
	51.48	100	1
	44.45	100	2
	43.49	100	3
	44.45	100	4
	45.40	100	5
	54.12	150	1
	49.95	150	2
	45.79	150	3
	44.83	150	4
	50.59	150	5
	54.57	200	1
	49.43	200	2
	43.33	200	3
	43.98	200	4
	52.00	200	5
	52.57	250	1
	47.44	250	2
	45.20	250	3
	50.96	250	4
	50.32	250	5
Trial: 75	28.98	0	1
Year: 95	25.59	0	2
GridNo: 26	24.16	0	3
Crop: WWHE	20.64	0	4
Nmin: 15.71	17.39	0	5
	42.15	50	1
	35.00	50	2
	40.30	50	3
	39.76	50	4
	37.12	50	5
	55.58	100	1
	46.33	100	2
	54.58	100	3
	53.64	100	4
	54.31	100	5
	66.21	150	1
	60.28	150	2
	64.16	150	3
	65.56	150	4
	61.42	150	5
	66.11	200	1
	62.50	200	2
	63.90	200	3
	73.05	200	4
	68.49	200	5
	64.49	250	1
	69.99	250	2
	73.37	250	3
	73.94	250	4
	71.35	250	5

	Yield	Nappl	RepliNo
Trial: 76	48.34	0	1
Year: 95	41.58	0	2
GridNo: 26	37.86	0	3
Crop: WWHE	37.18	0	4
Nmin: 12.82	39.21	0	5
	60.85	50	1
	57.80	50	2
	58.48	50	3
	55.44	50	4
	56.79	50	5
	72.85	100	1
	73.52	100	2
	73.52	100	3
	69.47	100	4
	68.13	100	5
	80.45	150	1
	81.81	150	2
	80.12	150	3
	76.40	150	4
	73.02	150	5
	86.84	200	1
	89.55	200	2
	84.80	200	3
	84.80	200	4
	80.05	200	5
	86.54	250	1
	80.79	250	2
	87.21	250	3
	89.24	250	4
	81.81	250	5
Trial: 77	31.87	0	1
Year: 95	32.53	0	2
GridNo: 36	32.86	0	3
Crop: WWHE	33.86	0	4
Nmin: 42.12	46.63	50	1
	44.64	50	2
	49.97	50	3
	51.30	50	4
	62.29	100	1
	56.96	100	2
	62.62	100	3
	61.96	100	4
	65.44	150	1
	61.77	150	2
	68.78	150	3
	71.12	150	4
	69.45	200	1
	66.11	200	2
	73.45	200	3
	74.79	200	4
	72.28	250	1
	66.62	250	2
	74.28	250	3
	74.28	250	4
Trial: 78	38.11	0	1
Year: 96	37.11	0	2
GridNo: 25	30.49	0	3
Crop: WWHE	35.79	0	4
Nmin: 61.34	34.79	0	5
	55.21	50	1
	54.22	50	2
	51.90	50	3
	46.61	50	4
	49.59	50	5
	62.96	100	1
	61.30	100	2
	56.66	100	3
	59.65	100	4
	68.59	100	5
	78.70	150	1
	72.07	150	2
	66.12	150	3
	60.17	150	4
	71.08	150	5
	68.93	200	1
	67.60	200	2
	73.90	200	3
	72.90	200	4
	76.22	200	5
	77.15	250	1
	69.84	250	2
	68.17	250	3
	74.49	250	4
	73.16	250	5

	Yield	Nappl	RepliNo
Trial: 79	39.41	0	1
Year: 96	45.84	0	2
GridNo: 36	36.10	0	3
Crop: WWHE	48.02	0	4
Nmin: 47.52	44.36	0	5
	52.85	50	1
	51.55	50	2
	61.40	50	3
	51.56	50	4
	49.38	50	5
	70.16	100	1
	69.82	100	2
	80.24	100	3
	67.78	100	4
	67.65	100	5
	75.24	150	1
	78.47	150	2
	75.95	150	3
	76.43	150	4
	78.88	150	5
	80.20	200	1
	80.85	200	2
	75.22	200	3
	75.17	200	4
	84.82	200	5
	84.12	250	1
	84.75	250	2
	80.14	250	3
	75.96	250	4
	85.08	250	5
Trial: 80	63.00	0	1
Year: 97	65.47	0	2
GridNo: 18	51.88	0	3
Crop: WWHE	52.50	0	4
Nmin: 51.84	62.38	0	5
	80.95	50	1
	81.88	50	2
	75.72	50	3
	75.41	50	4
	80.34	50	5
	84.75	100	1
	85.97	100	2
	82.29	100	3
	85.36	100	4
	86.59	100	5
	86.81	150	1
	82.23	150	2
	83.14	150	3
	84.06	150	4
	85.28	150	5
	84.27	200	1
	82.74	200	2
	82.44	200	3
	83.05	200	4
	82.74	200	5
	86.06	250	1
	83.90	250	2
	84.83	250	3
	82.36	250	4
	80.51	250	5
Trial: 81	30.92	0	1
Year: 97	34.75	0	2
GridNo: 30	29.51	0	3
Crop: WWHE	26.70	0	4
Nmin: 17.28	30.92	0	5
	57.05	50	1
	58.46	50	2
	53.88	50	3
	51.41	50	4
	51.41	50	5
	74.39	100	1
	76.15	100	2
	73.33	100	3
	71.92	100	4
	72.62	100	5
	81.53	150	1
	83.29	150	2
	81.53	150	3
	80.47	150	4
	76.24	150	5
	88.08	200	1
	82.77	200	2
	81.36	200	3
	85.25	200	4
	79.94	200	5
	87.12	250	1
	85.35	250	2
	86.76	250	3
	83.58	250	4
	80.74	250	5



## A.1. Danish Field Trials (continued)

	Yield	Nappl	RepliNo
Trial:	56.37	0	1
82	57.42	0	2
Year:	54.60	0	3
97	54.25	0	4
GridNo:	53.90	0	5
36	77.68	50	1
Crop:	81.89	50	2
WWHE	77.33	50	3
Nmin:	75.22	50	4
21.60	76.97	50	5
	92.44	100	1
	91.38	100	2
	91.03	100	3
	88.92	100	4
	90.33	100	5
	102.28	150	1
	99.12	150	2
	97.71	150	3
	99.12	150	4
	97.71	150	5
	104.86	200	1
	103.81	200	2
	102.05	200	3
	99.23	200	4
	100.99	200	5
	108.61	250	1
	107.55	250	2
	104.04	250	3
	105.44	250	4
	103.33	250	5
Trial:	37.45	0	1
83	32.01	0	2
Year:	42.22	0	3
97	48.35	0	4
GridNo:	37.79	0	5
33	51.75	50	1
Crop:	48.35	50	2
WWHE	58.56	50	3
Nmin:	62.31	50	4
27.00	52.09	50	5
	67.08	100	1
	69.12	100	2
	74.91	100	3
	77.63	100	4
	66.39	100	5
	71.66	150	1
	74.74	150	2
	78.15	150	3
	81.90	150	4
	68.25	150	5
	79.26	200	1
	80.63	200	2
	84.73	200	3
	86.78	200	4
	75.50	200	5
	77.21	250	1
	80.97	250	2
	83.70	250	3
	86.44	250	4
	77.55	250	5
Trial:	49.06	0	1
84	54.26	0	2
Year:	50.80	0	3
97	46.76	0	4
GridNo:	73.14	50	1
0	69.68	50	2
Crop:	65.08	50	3
WWHE	69.11	50	4
Nmin:	84.28	100	1
27.00	87.74	100	2
	75.62	100	3
	81.97	100	4
	92.78	150	1
	95.10	150	2
	81.18	150	3
	81.76	150	4
	89.51	200	1
	101.13	200	2
	82.53	200	3
	88.93	200	4
	85.16	250	1
	94.43	250	2
	85.16	250	3
	82.26	250	4

## A.2 Climate Data

Source: Landskontoret for Planteavl  
Danish Farmers' Union and Danish Family Farmers' Association

### A.2.1 Average of Weekly Climate Observations for Relevant Years and Grids, 1993-1998

Year	Grid No.	Analysis No.	Whole growing season			First half of growing season			Second half of growing season		
			Temperature	Radiation	Water balance	Temperature	Radiation	Water balance	Temperature	Radiation	Water balance
1995	30	39	11.91	123.73	-7.89	8.03	107.65	-3.45	16.23	141.60	-12.83
1997	30	40	11.65	121.13	-4.61	7.59	102.42	-1.20	16.17	141.91	-8.40
1993	34	42	12.27	121.55	-10.36	10.19	123.92	-17.22	14.57	118.92	-2.74
1994	30	43	13.06	124.93	-10.95	9.37	101.72	-6.98	17.15	150.72	-15.36
1994	34	44	13.16	129.18	-14.09	9.28	106.23	-6.12	17.47	154.68	-22.94
1995	34	45	12.20	126.19	-12.28	8.13	108.37	-6.00	16.72	145.99	-19.27
1996	33	46	10.89	117.52	-10.87	7.52	101.52	-5.83	14.63	135.30	-16.48
1996	34	47	10.73	115.10	-9.89	7.36	99.59	-5.15	14.47	132.33	-15.16
1997	34	48	11.63	119.73	-7.28	7.42	101.94	-5.87	16.31	139.50	-8.84
1993	7	49	11.39	121.01	-8.44	9.59	118.49	-11.52	13.38	123.80	-5.02
1995	33	50	12.29	129.00	-12.07	8.30	112.60	-6.34	16.72	147.22	-18.44
1995	26	51	12.22	126.59	-13.08	8.16	108.99	-6.88	16.74	146.14	-19.97
1996	33	53	10.89	117.52	-10.87	7.52	101.52	-5.83	14.63	135.30	-16.48
1997	33	54	11.70	123.44	-8.92	7.50	105.92	-5.98	16.36	142.90	-12.18
1994	43	56	13.31	133.30	-14.25	9.51	111.42	-7.12	17.52	157.61	-22.17
1995	43	57	12.21	133.37	-12.76	8.32	118.17	-7.09	16.53	150.27	-19.07
1995	42	58	12.25	133.25	-13.55	8.35	118.07	-8.17	16.58	150.11	-19.52
1995	33	59	12.29	129.00	-12.07	8.30	112.60	-6.34	16.72	147.22	-18.44
1995	26	60	12.22	126.59	-13.08	8.16	108.99	-6.88	16.74	146.14	-19.97
1996	41	61	11.11	119.24	-11.02	7.73	103.13	-6.08	14.86	137.13	-16.50
1996	42	62	11.11	119.32	-9.96	7.74	103.26	-5.36	14.86	137.16	-15.08
1997	33	63	11.70	123.44	-8.92	7.50	105.92	-5.98	16.36	142.90	-12.18
1997	41	64	11.87	128.09	-7.43	7.74	108.84	-3.81	16.46	149.48	-11.44
1998	42	65	12.18	115.57	-5.11	9.60	104.73	-3.99	15.04	127.61	-6.34
1993	13	66	11.72	120.27	-9.86	9.76	120.14	-13.57	13.89	120.42	-5.73
1994	20	67	13.37	129.83	-13.48	9.46	107.77	-7.02	17.72	154.33	-20.67
1995	34	68	12.20	126.19	-12.28	8.13	108.37	-6.00	16.72	145.99	-19.27
1993	36	70	12.33	119.62	-8.33	10.39	116.48	-12.57	14.50	123.11	-3.62
1994	4	71	12.45	130.06	-12.03	8.90	110.76	-7.99	16.39	151.51	-16.51
1994	4	72	12.45	130.06	-12.03	8.90	110.76	-7.99	16.39	151.51	-16.51
1994	33	73	13.22	131.38	-13.27	9.37	108.80	-6.12	17.51	156.47	-21.21
1994	18	74	12.78	128.87	-14.66	9.00	105.43	-9.02	16.98	154.92	-20.92
1995	26	75	12.22	126.59	-13.08	8.16	108.99	-6.88	16.74	146.14	-19.97
1995	26	76	12.22	126.59	-13.08	8.16	108.99	-6.88	16.74	146.14	-19.97
1995	36	77	12.14	124.73	-8.21	8.28	109.26	-2.89	16.42	141.91	-14.11
1996	25	78	10.75	116.88	-11.66	7.35	98.95	-6.79	14.53	136.80	-17.07
1996	36	79	10.74	112.73	-8.80	7.55	101.70	-5.31	14.29	124.98	-12.68
1997	18	80	11.29	122.12	-4.14	7.11	100.85	-2.53	15.93	145.74	-5.92
1997	30	81	11.65	121.13	-4.61	7.59	102.42	-1.20	16.17	141.91	-8.40
1997	36	82	11.86	122.11	-3.65	7.82	103.70	-1.83	16.36	142.56	-5.68
1997	33	83	11.70	123.44	-8.92	7.50	105.92	-5.98	16.36	142.90	-12.18

## **Appendix B**

### **Comparison of Statistical Fit Between Functional Forms**

#### **B.1 Ranking of Functional Forms According to Statistical Fit (MSE)**

The tables in Section B.1 show a frequency count for all fitted functions based on the size of MSE. The figures illustrate how many times each functional form has the best fit, second best fit etc. Tables B.1.1 through B.1.4 deal with all 84 trial data sets with a break down by the three crops, spring barley, winter barley, and winter wheat. In Tables B.1.5 through B.1.8 the procedure is repeated for all 46 trial data sets, for which information about mineralized nitrogen (N1) at beginning of growing season is available. Table B.1.9 relates to 27 winter wheat trials with information about both N1 and climate factors. This sample is the basis for an extended model where all 27 trials are pooled in one analysis.

#### **B.2 Average MSE by Functional Forms and Trial Samples**

This section compares the calculated average MSE by functional form and by crops and trial samples as in Section B.1. A pairwise t-test is done for all functions. Where differences between averages are not significantly different the pertinent functions have been labeled with the same capital letter.

#### **B.3 Average MSE by Change of Trial Design**

The influence of trial design is illustrated in this section. Tables B.3.1 through B.3.3 compare sub samples of spring barley. Out of a total 36 trial data sets 24 trials were

treated with five levels of fertilizer nitrogen ranging from 0 to 160 kg per hectare with increments of 40 kg. In 12 trials there were six levels of nitrogen application with a maximum of 200 kg per hectare. A similar comparison is made in Table B.3.4 and B.3.5 for the 27 winter wheat trials. All 27 trials were treated with six levels of nitrogen ranging from 0 to 250 kg per hectare with increments of 50 kg. The effect of changed trial design is estimated by removing the highest N-application level and redoing the fitting of the functions to the truncated data sets.

#### **B.4 Residual Plots by Functional Forms and Field Trials**

The graphs show residual plots for all functions fitted to all trials. The horizontal axis has been normalized around the estimated optimal nitrogen application as calculated on the basis of output and input prices in Scenario 2. If optimal nitrogen application exceeds 120 per cent of highest nitrogen application observations are not included.

## B.1 Ranking of Functional Forms According to Statistical Fit (MSE)

### Frequency of Rank Scores by Functional Forms and Trial Samples

**Table B.1.1 Response Functions Fitted to 84 Data Sets, All Crops**

Rank	Frequency of Scores – Functional Forms Ranked by Mean Square Error									
	Quad-ratic	Cubic	Square root	Cobb-Dougl.	Mitsch-erlich	Quadr./Plateau 1	Quadr./Plateau 2	Cobb-D./Plateau	Mitsch./Plateau	Linear/Plateau
1	15	11	11	3	13	<b>16</b>	5	5		7
2	14	10	12	5	8	18	10	2	1	3
3	6	12	8	2	8	14	15	6	6	6
4	8	15	7	1	14	9	9	11	6	5
5	6	11	7	4	14	12	7	11	9	4
6	4	13	6	5	12	4	12	12	11	3
7	3	5	12	2	10	45	8	16	19	4
8	8	5	15	10	3	6	8	8	16	5
9	16	2	6	23	2		8	8	8	11
10	4			29			2	5	8	<b>36</b>
1+2	27	21	23	8	21	<b>34</b>	15	7	1	10
9+10	20	2	6	<b>52</b>	2		10	13	16	47

When including the linear model it scores 77 of 84 in rank group #11

**Table B.1.2 Response Functions Fitted to 36 Spring Barley Data Sets**

Rank	Frequency of Scores – Functional Forms Ranked by Mean Square Error									
	Quad-ratic	Cubic	Square root	Cobb-Dougl.	Mitsch-erlich	Quadr./Plateau 1	Quadr./Plateau 2	Cobb-D./Plateau	Mitsch./Plateau	Linear/Plateau
1	<b>8</b>	6	7	2	3	5	2	3		1
2	4	5	7	3	4	7	4			2
3	2	3	3	1	4	6	9	3	1	3
4	5	3	5	1	5	4	3	7	1	3
5	3	3	3	1	8	8	1	4	4	2
6	1	7	7	4	3	1	5	4	5	2
7		4	4		7	2	4	6	7	2
8	2	3	3	4		3	4	4	10	3
9	9	2	2	9	2		4	3	1	4
10	2			11				2	7	<b>14</b>
1+2	12	11	<b>14</b>	5	7	12	6	3		3
9+10	11	2	2	<b>20</b>	2		4	5	8	18

When including the linear model it scores 32 of 36 in rank group #11

**Table B.1.3 Response Functions Fitted to 8 Winter Barley Data Sets**

Rank	Frequency of Scores – Functional Forms Ranked by Mean Square Error									
	Quad-ratic	Cubic	Square root	Cobb-Dougl.	Mitsch-erlich	Quadr./Plateau 1	Quadr./Plateau 2	Cobb-D./Plateau	Mitsch./Plateau	Linear/Plateau
1		1			1	2				<b>4</b>
2	4	3					1			
3			1			3	2			2
4	1				3	2	1		1	
5	2	1	1	1	1	1	1			
6			1		2		2	2	1	
7		1	1	1	1			2	2	
8		2	4	1			1			
9	1			2				1	4	
10				<b>3</b>				3		2
1+2	4	<b>4</b>			1	2	1			<b>4</b>
9+10	1			<b>5</b>				4	4	2

When including the linear model it scores 7 of 8 in rank group #11

**Table B.1.4 Response Functions Fitted to 40 Winter Wheat Data Sets**

Rank	Frequency of Scores – Functional Forms Ranked by Mean Square Error									
	Quad-ratic	Cubic	Square root	Cobb-Dougl.	Mitsch-erlich	Quadr./Plateau 1	Quadr./Plateau 2	Cobb-D./Plateau	Mitsch./Plateau	Linear/Plateau
1	7	4	4	1	<b>9</b>	<b>9</b>	3	2		2
2	6	2	5	2	4	11	5	2	1	1
3	4	9	4	1	4	5	4	3	5	1
4	2	12	2		6	3	5	4	4	2
5	1	7	3	2	5	3	5	7	5	2
6	3	6	3	1	7	3	5	6	5	1
7	3		7	1	2	3	4	8	10	2
8	6		8	5	3	3	3	4	6	2
9	6		4	12			4	4	3	7
10	2			15			2		1	<b>20</b>
1+2	13	6	9	3	13	<b>20</b>	8	4	1	3
9+10	8		4	<b>27</b>			6	4	4	<b>27</b>

When including the linear model it scores 38 of 40 in rank group #11

**Table B.1.5 Response Functions Fitted to 46 Data Sets With N1 Data, All Crops**

Rank	Frequency of Scores – Functional Forms Ranked by Mean Square Error									
	Quad-ratic	Cubic	Square root	Cobb-Dougl.	Mitscherlich	Quadr./Plateau 1	Quadr./Plateau 2	Cobb-D./Plateau	Mitsch./Plateau	Linear/Plateau
1	6	8	3	1	8	9	2	3		7
2	10	5	5	1	4	11	7	1	1	1
3	3	7	3	1	2	9	10	2	5	3
4	4	11	2		8	5	5	7	3	2
5	5	5	3	2	8	5	6	6	4	2
6	2	6	5	2	8	3	4	8	7	
7	3	1	7	1	5	1	4	8	13	3
8	5	3	12	6	2	3	3	3	6	3
9	5		6	14	1		3	5	4	8
10	3			18			2	3	3	17
1+2	16	13	8	2	12	20	9	4	1	8
9+10	8		6	32	1		5	8	7	25

When including the linear model it scores 43 of 46 in rank group #11

**Table B.1.6 Response Functions Fitted to 10 Spring Barley Sets With N1 Data**

Rank	Frequency of Scores – Functional Forms Ranked by Mean Square Error									
	Quad-ratic	Cubic	Square root	Cobb-Dougl.	Mitscherlich	Quadr./Plateau 1	Quadr./Plateau 2	Cobb-D./Plateau	Mitsch./Plateau	Linear/Plateau
1	2	3	1	1	1			1		1
2	2	2	1		1	3	1			1
3					1	2	4		1	1
4	2	2	2		1	1		3		
5	2		1		3	3			1	
6		1	1	1		1	2	1	2	
7			1		2		1	2	2	2
8		2	1	2			1	1	2	1
9	1		2	1	1		1	2		2
10	1			5					2	2
1+2	4	5	2	1	2	3	1	1		2
9+10	2		2	6	1		1	2	2	4

When including the linear model it scores 9 of 10 in rank group #11

**Table B.1.7 Response Functions Fitted to 6 Winter Barley Sets With N1 Data**

Rank	Frequency of Scores – Functional Forms Ranked by Mean Square Error									
	Quad-ratic	Cubic	Square root	Cobb-Dougl.	Mitscherlich	Quadr./Plateau 1	Quadr./Plateau 2	Cobb-D./Plateau	Mitsch./Plateau	Linear/Plateau
1		1				1				<b>4</b>
2	3	2					1			
3						3	2			1
4	1				2	2	1			
5	2	1		1	1		1			
6			1		2			2	1	
7		1	1	1	1				2	
8		1	4				1			
9				2				1	3	
10				2				3		1
1+2 9+10	3	3		<b>4</b>		1	1	<b>4</b>	3	<b>4</b> 1

When including the linear model it scores 5 of 6 in rank group #11

**Table B.1.8 Response Functions Fitted to 30 Winter Wheat Sets With N1 Data**

Rank	Frequency of Scores – Functional Forms Ranked by Mean Square Error									
	Quad-ratic	Cubic	Square root	Cobb-Dougl.	Mitscherlich	Quadr./Plateau 1	Quadr./Plateau 2	Cobb-D./Plateau	Mitsch./Plateau	Linear/Plateau
1	4	4	2		7	<b>8</b>	2	2		2
2	5	1	4	1	3	8	5	1	1	
3	3	7	3	1	1	4	4	2	4	1
4	1	9			5	2	4	4	3	2
5	1	4	2	1	4	2	5	6	3	2
6	2	5	3	1	6	2	2	5	4	
7	3		5		2	1	3	6	9	1
8	5		7	4	2	3	1	2	4	2
9	4		4	11			2	2	1	6
10	2			11			2		1	<b>14</b>
1+2 9+10	9	5	6	1	10	<b>16</b>	7	3	1	2
	6		4	<b>22</b>			4	2	2	20

When including the linear model it scores 29 of 30 in rank group #11



**Table B.1.9 Functions Fitted to 27 Wheat Sets With N1 and Climate Data**

Rank	Frequency of Scores – Functional Forms Ranked by Mean Square Error									
	Quad-ratic	Cubic	Square root	Cobb-Dougl.	Mitsch-erlich	Quadr./Plateau 1	Quadr./Plateau 2	Cobb-D./Plateau	Mitsch./Plateau	Linear/Plateau
1	3	4	2		5	<b>8</b>	2	2		2
2	5	1	3	1	3	<b>8</b>	4	1		
3	3	5	3	1	1	3	4	2	4	1
4	1	8			5	2	4	3	2	2
5	1	4	1	1	4	2	4	6	3	1
6	2	5	2	1	6		2	5	4	
7	2		5		2	1	2	5	9	1
8	4		7	3	1	3	1	2	4	2
9	4		4	10			2	1		6
10	2			10			2		1	<b>12</b>
1+2	<b>8</b>	5	5	1	8	<b>16</b>	6	3		2
9+10	6		4	<b>20</b>			4	1	1	18

When including the linear model it scores 26 of 27 in rank group #11

## B.2 Comparison of Average MSE by Functional Forms and Trial Samples

### B.2.1 Average MSE, All 84 Trials, All Crops

Model	Avg. MSE	t-grouping *
Linear	35.53	A
LRP-Liebig	14.36	B
Cobb-Douglas	13.80	B C
Mitscherlich/plateau	12.70	B C
Square root polynomial	12.05	B C
Cobb-Douglas/plateau	11.96	B C
Quadratic polynomial	11.93	B C
Quadratic/plateau - top/left	11.72	B C
Mitscherlich	11.62	B C
Quadratic/plateau - top	11.35	C
Cubic polynomial	11.25	C

\* Means with same letter are not significantly different (95% level)

### B.2.2 Average MSE, 36 Spring Barley Trials

Model	Avg. MSE	t-grouping *
Linear	19.76	A
Mitscherlich/plateau	10.21	B
Cobb-Douglas	9.81	B
LRP-Liebig	9.58	B
Quadratic/plateau - top/left	8.53	B
Mitscherlich	8.49	B
Cobb-Douglas/plateau	8.48	B
Quadratic polynomial	8.47	B
Square root polynomial	8.47	B
Quadratic/plateau - top	8.31	B
Cubic polynomial	8.08	B

\* Means with same letter are not significantly different (95% level)

### B.2.3 Average MSE, 8 Winter Barley Trials

Model	Avg. MSE	t-grouping *
Linear	31.44	A
Cobb-Douglas	16.71	B
Cobb-Douglas/plateau	16.26	B
Square root polynomial	15.74	B
Mitscherlich/plateau	15.53	B
Mitscherlich	14.78	B
LRP-Liebig	14.42	B
Quadratic/plateau - top/left	14.21	B
Cubic polynomial	14.06	B
Quadratic polynomial	14.05	B
Quadratic/plateau - top	13.94	B

\* Means with same letter are not significantly different (95% level)

### B.2.4 Average MSE, 40 Winter Wheat Trials

Model	Avg. MSE	t-grouping *
Linear	50.54	A
LRP-Liebig	18.65	B
Cobb-Douglas	16.80	B C
Quadratic polynomial	14.63	B C
Square root polynomial	14.53	B C
Mitscherlich/plateau	14.39	B C
Cobb-Douglas/plateau	14.22	C
Quadratic/plateau - top/left	14.11	C
Mitscherlich	13.80	C
Quadratic/plateau - top	13.56	C
Cubic polynomial	13.53	C

\* Means with same letter are not significantly different (95% level)

### B.2.5 Average MSE, All 46 Trials with N1 Data

Model	Avg. MSE	t-grouping *
Linear	43.49	A
LRP-Liebig	16.55	B
Cobb-Douglas	15.96	B
Square root polynom.	14.10	B
Mitsch./plateau	13.87	B
Quadratic polynom.	13.80	B
Cobb-Douglas/plateau	13.65	B
Quad./plat. - top/left	13.39	B
Mitscherlich	13.33	B
Quad./plateau - top	12.94	B
Cubic polynomial	12.83	B

\* Means with same letter are not significantly different (95% level)

### B.2.6 Average MSE, 10 Spring Barley Trials

Model	Avg. MSE	t-grouping *
Linear	21.84	A
Cobb-Douglas	11.83	B
Square root polynom.	10.86	B
Mitsch./plateau	10.75	B
LRP-Liebig	10.64	B
Mitscherlich	10.53	B
Quad./plat. - top/left	10.25	B
Quadratic polynom.	10.16	B
Cobb-Douglas/plateau	10.01	B
Quad./plateau - top	9.97	B
Cubic polynomial	9.56	B

\* Means with same letter are not significantly different (95% level)

### B.2.7 Average MSE, 6 Winter Barley Trials

Model	Avg. MSE	t-grouping *
Linear	30.32	A
Cobb-Douglas	17.49	B
Cobb-Douglas/plateau	17.17	B
Square root polynom.	16.64	B
Mitsch./plateau	16.22	B
Mitscherlich	15.42	B
LRP-Liebig	14.56	B
Quad./plat. - top/left	14.51	B
Cubic polynomial	14.38	B
Quadratic polynom.	14.28	B
Quad./plateau - top	14.25	B

\* Means with same letter are not significantly different (95% level)

### B.2.8 Average MSE, 30 Winter Wheat Trials

Model	Avg. MSE	t-grouping *
Linear	53.34	A
LRP-Liebig	18.92	B
Cobb-Douglas	17.04	B C
Quadratic polynom.	14.92	B C
Square root polynom.	14.67	B C
Mitsch./plateau	14.45	B C
Quad./plat. - top/left	14.21	B C
Cobb-Douglas/plateau	14.17	B C
Mitscherlich	13.85	C
Quad./plateau - top	13.66	C
Cubic polynomial	13.61	C

\* Means with same letter are not significantly different (95% level)

### B.2.9 27 Winter Wheat Trials with Climate Information

Model	Avg. MSE	t-grouping *
Linear	50.53	A
LRP-Liebig	17.31	B
Cobb-Douglas	15.73	B
Quadratic polynom.	13.76	B
Square root polynom.	13.49	B
Mitsch./plateau	13.16	B
Quad./plat. - top/left	12.95	B
Cobb-Douglas/plateau	12.84	B
Mitscherlich	12.62	B
Cubic polynomial	12.36	B
Quad./plateau - top	12.34	B

\* Means with same letter are not significantly different (95% level)

### B.3 Average MSE by Change of Trial Design

#### B.3.1 Average MSE, All 36 Spring Barley Trials

Model	Avg. MSE	t-grouping *
Linear	19.76	A
Mitsch./plateau	10.21	B
Cobb-Douglas	9.81	B
LRP-Liebig	9.58	B
Quad./plat. - top/left	8.53	B
Mitscherlich	8.49	B
Cobb-Douglas/plateau	8.48	B
Quadratic polynom.	8.47	B
Square root polynom.	8.47	B
Quad./plateau - top	8.31	B
Cubic polynomial	8.08	B

\* Means with same letter are not significantly different (95% level)

#### B.3.2 24 Spring Barley Trials with 5 Levels of N-Application (Max 160 kg N/ha)

Model	Avg. MSE	t-grouping *
Linear	17.76	A
Mitsch./plateau	10.19	B
LRP-Liebig	9.40	B
Cobb-Douglas	8.99	B
Quadratic polynom.	8.07	B
Cobb-Douglas/plateau	8.06	B
Quad./plat. - top/left	8.02	B
Quad./plateau - top.	7.83	B
Mitscherlich	7.78	B
Cubic polynomial	7.76	B
Square root polynom.	7.74	B

\* Means with same letter are not significantly different (95% level)

#### B.3.3 12 Spring Barley Trials with 6 Levels of N-Application (Max 200 kg N/ha)

Model	Avg. MSE	t-grouping *
Linear	23.76	A
Cobb-Douglas	11.46	B
Mitsch./plateau	10.24	B
LRP-Liebig	9.96	B
Square root polynom.	9.92	B
Mitscherlich	9.92	B
Quad./plat. - top/left	9.53	B
Cobb-Douglas/plateau	9.32	B
Quad./plateau - top.	9.29	B
Quadratic polynom.	9.28	B
Cubic polynomial	8.73	B

\* Means with same letter are not significantly different (95% level)

**B.3.4 27 Winter Wheat Trials with Climate Information**  
6 Levels of N-Application (Max=250 Kg/Ha)

Model	Avg. MSE	t-grouping *
Linear	50.53	A
LRP-Liebig	17.31	B
Cobb-Douglas	15.73	B
Quadratic polynom.	13.76	B
Square root polynom.	13.49	B
Mitsch./plateau	13.16	B
Quad./plat.- top/left	12.95	B
Cobb-Douglas/plateau	12.84	B
Mitscherlich	12.62	B
Cubic polynomial	12.36	B
Quad./plateau - top	12.34	B

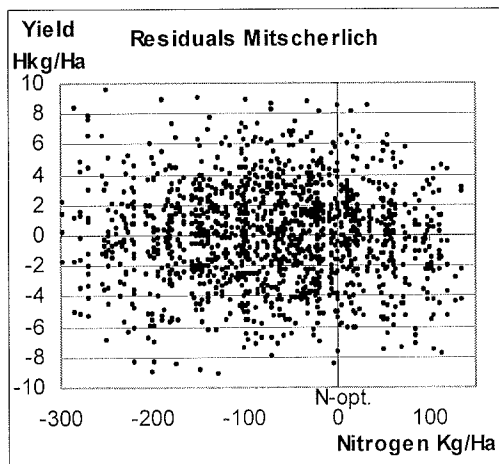
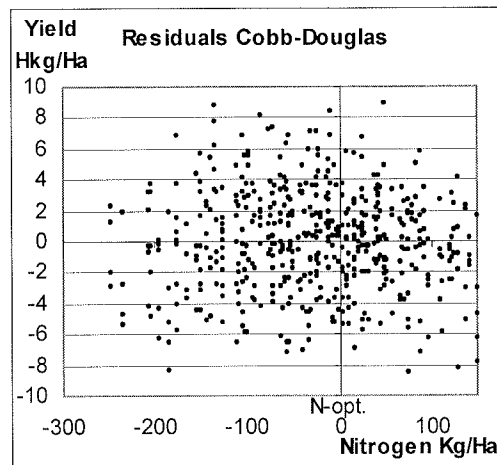
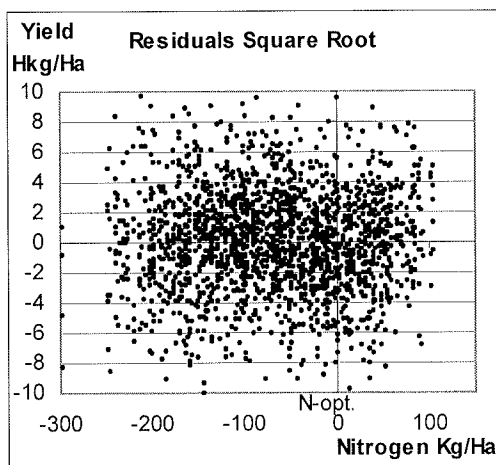
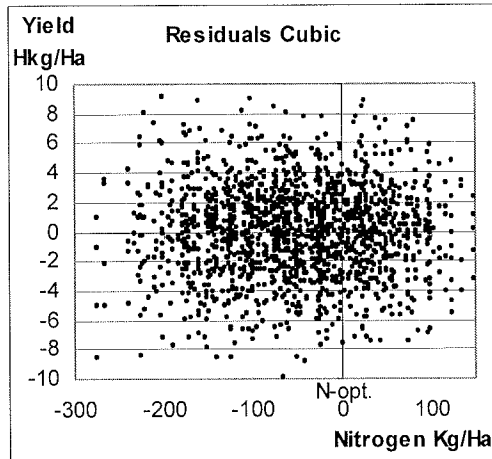
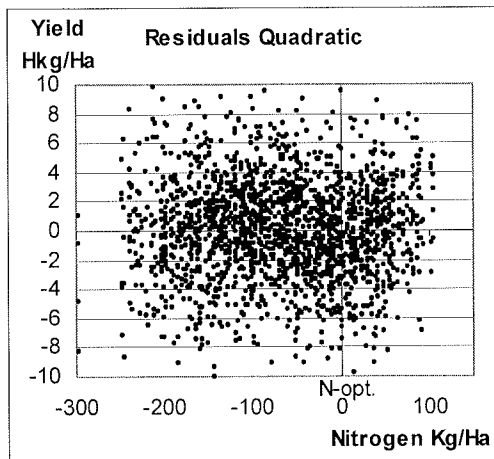
\* Means with same letter are not significantly different (95% level)

**B.3.5 27 Winter Wheat Trials With Climate Information**  
5 Levels of N-Application (250 Kg/Ha Cancelled)

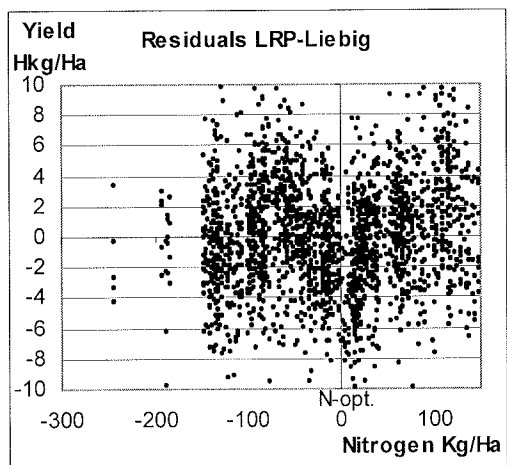
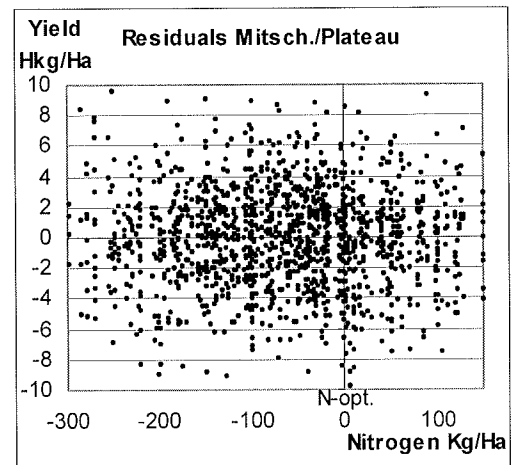
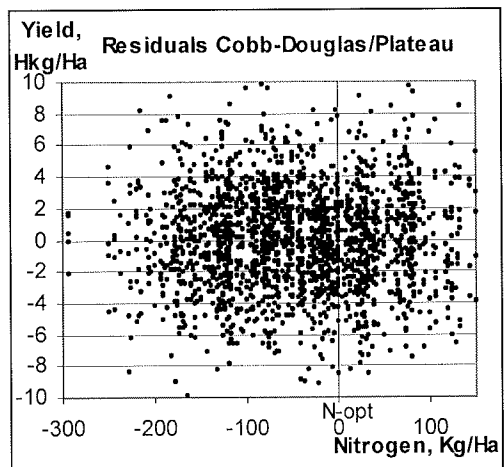
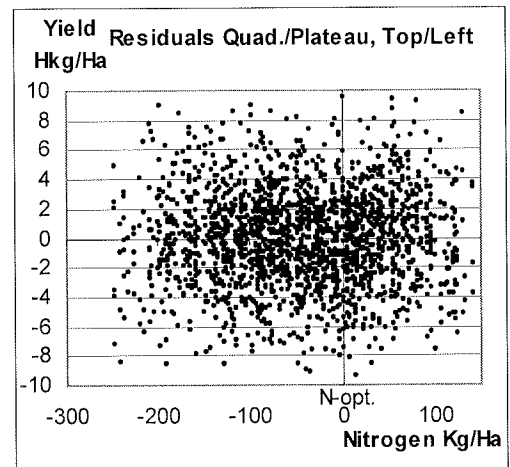
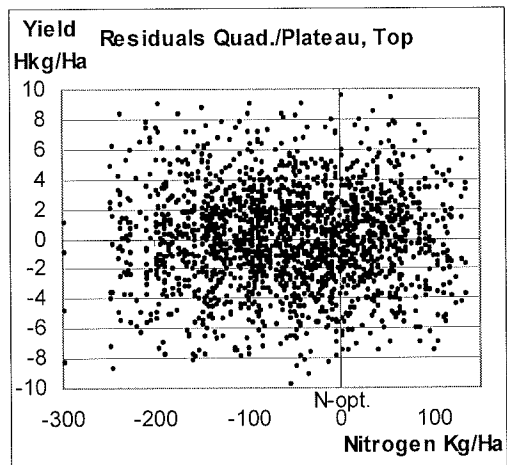
Model	Avg. MSE	t-grouping *
Linear	37.64	A
LRP-Liebig	15.32	B
Cobb-Douglas	14.41	B
Cobb-Douglas/plateau	14.35	B
Mitsch./plateau	13.29	B
Square root Polynom.	13.14	B
Mitscherlich	12.67	B
Quadratic polynom.	12.51	B
Quad./plat.- top/left	12.35	B
Cubic polynomial	12.34	B
Quad./plateau - top	12.27	B

\* Means with same letter are not significantly different (95% level)

#### B.4 Residual Plots by Functional Forms and Field Trials



(B.4 continued)





## **Appendix C**

### **Estimated Averages of Optimal Nitrogen Application**

#### **C.1 Optimal Nitrogen Application at 4 Nitrogen/Cereals Price Ratios**

Estimated optimal nitrogen applications are compared between functional forms and four price scenarios as outlined in Chapter 4, Section 4.2.2. Calculations are made for spring barley, winter barley, and winter wheat in all 84 field trials and in the sample of 46 trials with available information about mineralized nitrogen at the beginning of the growing season. Averages have been corrected for outliers, i.e. trial estimates of optimal nitrogen application that exceed 120 per cent of the highest nitrogen application level. Cobb-Douglas, square root, and Mitscherlich with plateau are not included in the table.

#### **C.2 Optimal Nitrogen Application, Price Scenario 2, 84 Trials**

Calculated optimal nitrogen averages are compared between all the examined functional forms applied to all 84 trials. Averages are corrected by omission of outliers as explained in Section C.1. Data are shown for spring barley and winter wheat only, while the small sample of winter barley trials has been omitted. The tables contain information about the actual number of trials in each average calculation

#### **C.3 Optimal Nitrogen Application, Price Scenario 2, 46 Trials**

Calculated optimal nitrogen averages are compared between all the examined functional forms. Averages are corrected by omission of outliers as explained in Section C.1. Calculations refer to the sample of trials, in which each trial data set contains information about N1 (plant available mineralized soil nitrogen at beginning of growing season). Data are shown for spring barley and winter wheat only, while the small sample of winter barley trials has been omitted. The tables contain information about the actual number of trials in each average calculation.

### C.1 Optimal Nitrogen Application at 4 Nitrogen/Cereals Price Ratios, Kg/Ha

Averages corrected for outliers

		QUA	CUB	MIT	QP1	QP2	CDP	LRP
<b>All 84 trials:</b>								
36 SBAR	N-opt. 1	140.3	129.2	120.5	129.8	119.1	126.0	103.0
	N-opt. 2	138.7	121.3	121.7	129.1	117.0	125.8	03.0
	N-opt. 3	135.9	119.6	125.0	127.1	117.0	124.2	103.0
	N-opt. 4	97.2	90.6	86.9	93.8	93.2	98.1	96.1
(Range of Std.dev.)		(34 - 41)	(34 - 41)	(46 - 53)	(42 - 46)	(36 - 39)	(34 - 51)	(32 - 39)
8 WBAR	N-opt. 1	183.3	181.1	197.8	178.3	150.6	154.4	130.4
	N-opt. 2	179.4	177.8	193.3	174.5	150.2	154.4	130.4
	N-opt. 3	175.6	174.7	180.7	170.9	149.9	154.4	130.4
	N-opt. 4	140.3	140.8	140.9	137.4	146.6	156.8	130.4
(Range of Std.dev.)		(22 - 30)	(26 - 29)	(8 - 40)	(25 - 34)	(26 - 29)	(40 - 43)	(24)
40 WWHE	N-opt. 1	202.5	192.4	218.8	188.6	179.8	173.5	130.8
	N-opt. 2	200.7	186.4	213.6	187.8	177.7	173.5	130.8
	N-opt. 3	196.5	183.4	210.9	184.2	175.5	173.5	130.8
	N-opt. 4	160.0	149.7	145.4	153.3	152.5	168.6	130.8
(Range of Std.dev.)		(27 - 36)	(42 - 46)	(47 - 58)	(41 - 43)	(36 - 44)	(46 - 49)	(28)
<b>All 46 trials with N2</b>								
10 SBAR	N-opt. 1	171.9	156.6	164.0	164.4	145.4	141.3	123.7
	N-opt. 2	172.7	151.2	169.3	166.3	142.2	141.3	123.7
	N-opt. 3	166.6	146.5	169.4	160.6	139.2	141.3	123.7
	N-opt. 4	110.8	109.8	109.1	108.0	107.5	114.9	109.0
(Range of Std.dev.)		(32 - 53)	(27 - 51)	(49 - 64)	(41 - 54)	(36 - 50)	(29 - 51)	(26 - 46)
6 WBAR	N-opt. 1	191.6	183.2	189.2	189.2	156.4	171.7	136.9
	N-opt. 2	187.5	180.4	200.1	185.2	156.4	171.7	136.9
	N-opt. 3	183.6	177.8	187.3	181.4	156.4	171.7	136.9
	N-opt. 4	147.1	151.6	154.0	145.7	156.4	171.7	136.9
(Range of Std.dev.)		(23 - 32)	(25 - 28)	(36 - 41)	(26 - 35)	(27)	(37 - 43)	(26)
30 WWHE	N-opt. 1	202.9	191.0	227.3	188.7	179.5	171.9	126.1
	N-opt. 2	198.6	184.8	217.0	185.1	177.2	171.9	126.1
	N-opt. 3	194.5	182.5	213.5	181.6	174.9	171.9	126.1
	N-opt. 4	160.6	150.2	143.8	153.4	151.6	169.1	126.1
(Range of Std.dev.)		(26 - 35)	(41 - 42)	(42 - 58)	(39 - 41)	(34 - 44)	(46)	(21)

Note: The actual number of trials represented in each average may vary due to correction for outliers.

## C.2 Optimal Nitrogen Application, Price Scenario 2, 84 Trials

Total Sample of Field Trials

Spring Barley, 36 Trials			Winter Wheat, 40 Trials		
Function	Kg N/Ha	t-Comp.	Function	Kg N/Ha	t-Comp.
Quadratic (34)	138.7	A	Mitscherlich (29)	213.6	A
Quad./Plat. 1 (34)	129.1	AB	Square Root (25)	212.8	A
Cobb-D./Plat. (35)	125.8	AB	Mitsch./Plat. (29)	209.8	AB
Mitscherlich (24)	121.7	ABC	Quadratic (38)	200.7	ABC
Cubic (29)	121.4	ABC	Quad./Plat. 1 (38)	187.8	ABC
Quad./Plat. 2 (35)	117.0	ABC	Cubic (28)	186.4	ABC
Square Root (21)	115.9	ABC	Quad./Plat. 2 (39)	177.7	BC
Mitsch./Plat. (24)	107.3	BC	Cobb-D./Plat. (39)	173.5	C
LRP-Liebig (36)	103.0	BC	Cobb-Douglas (7)	172.9	C
Cobb-Douglas (14)	95.4	C	LRP-Liebig (40)	130.8	D

Note: Numbers in brackets are lower than total number of trials due to omission of outliers or no solution. Function averages with same letter are not significantly different.

## C.3 Optimal Nitrogen Application, Price Scenario 2, 46 Trials

Field Trials with N1 Information

Spring Barley, 10 Trials			Winter Wheat, 30 Trials		
Function	Kg N/Ha	t-Comp.	Function	Kg N/Ha	t-Comp.
Quadratic (10)	172.7	A	Square Root (20)	219.0	A
Mitscherlich (6)	169.3	A	Mitscherlich (23)	217.0	AB
Mitsch./Plat. (6)	166.7	A	Mitsch./Plat. (23)	212.2	ABC
Quad./Plat. 1 (10)	166.3	A	Quadratic (29)	198.6	ABCD
Cobb-Douglas (2)	155.1	A	Quad./Plat. 1 (29)	185.1	ABCD
Cubic (8)	151.2	A	Cubic (21)	184.8	ABCD
Square Root (4)	145.3	A	Quad./Plat. 2 (30)	177.2	BCD
Quad./Plat. 2 (10)	142.2	A	Cobb-D./Plat. (30)	171.9	CD
Cobb-D./Plat. (10)	141.3	A	Cobb-Douglas (5)	165.7	DE
LRP-Liebig (10)	123.7	A	LRP-Liebig (30)	126.2	E

Note: Numbers in brackets are lower than total number of trials due to omission of outliers or no solution. Function averages with same letter are not significantly different.

## **Appendix D**

### **Estimates of Nitrogen Mineralization**

#### **D.1 Nitrogen Mineralization By Crops and Functions**

Total availability of mineralized nitrogen (NS) is the sum of available nitrogen at beginning of growing season (N1) and mineralization during the growing season (N2). NS is estimated on the basis of fitted functions. In 46 trials N1 is measured in soil samples and for these trials N2 can be derived as the difference,  $NS - N1$ . Averages of calculated nitrogen mineralization are shown for all 84 trials and for the mentioned 46 trials by functional forms and crops. In all sub-samples a t-test has been applied to compare all averages two by two.

#### **D.2 Estimated NS and N2 in Individual Field Trials**

In 41 out of the 46 trials mentioned in D.1, climate information is available. Estimates of N2 in these trials are regressed on climate variables. To avoid impact of differences between crops only data from the largest sub-sample, 27 individual winter wheat trials, are used in the analysis. The table shows N2-estimates for each of the all 41 trials as calculated by fitting quadratic, quadratic with plateau from top of parabola, and Mitscherlich.

## D.1 Nitrogen Mineralization By Crops and Functions

### D.1.1 Total Availability of Soil Nitrogen (N1 + N2) Kg/Ha, All 84 Trials

36 Spring Barley Trials			8 Winter Barley Trials			40 Winter Wheat Trials		
Funct.	NS	t-com.	Funct.	NS	t-com.	Funct.	NS	t-com.
LRP	163.8	A	LRP	116.0	A	LRP	133.6	A
QUA	92.5	B	QP2	80.9	AB	QUA	90.9	B
QP2	90.6	B	QUA	73.0	B	QP2	86.6	BC
QP1	83.7	B	QP1	71.0	BC	QP1	82.5	BC
CUB	72.9	BC	CUB	56.2	BCD	CUB	72.3	CD
MIP	54.8	CD	MIP	55.8	BCD	MIP	57.4	D
MIT	54.6	CD	MIT	55.8	BCD	MIT	57.1	D
SQU	33.1	DE	CDP	38.5	BCD	CDP	30.5	E
CDP	29.5	E	SQU	29.4	CD	SQU	29.2	EF
C-D	12.3	E	C-D	20.6	D	C-D	13.2	F

### D.1.2 Total Availability of Soil Nitrogen (N1 + N2) Kg/Ha, 46 Trials

10 Spring Barley Trials			6 Winter Barley Trials			30 Winter Wheat Trials		
Funct.	NS	t-com.	Funct.	NS	t-com.	Funct.	NS	t-com.
LRP	191.3	A	LRP	105.0	A	LRP	121.4	A
QP2	121.1	B	QP2	77.1	AB	QUA	84.5	B
QUA	109.6	B	QUA	68.0	BC	QP2	79.9	BC
QP1	103.2	BC	QP1	67.3	BC	QP1	75.7	BC
CUB	85.4	BCD	MIP	56.0	BCD	CUB	66.4	CD
MIP	83.1	BCD	MIT	56.0	BCD	MIP	52.2	D
MIT	74.1	BCD	CUB	48.2	BCD	MIT	51.7	D
CDP	54.9	CDE	CDP	36.4	CD	CDP	26.6	E
SQU	43.2	DE	SQU	29.5	D	SQU	25.6	E
C-D	21.9	E	C-D	24.7	D	C-D	10.4	E

### D.1.3 Mineralization During Growing Season (N2) Kg/Ha, 46 Trials

10 Spring Barley Trials			6 Winter Barley Trials			30 Winter Wheat Trials		
Funct.	N1	t-com.	Funct.	N1	t-com.	Funct.	N1	t-com.
LRP	155.0	A	LRP	77.6	A	LRP	91.6	A
QP2	84.9	B	QP2	49.8	AB	QUA	54.8	B
QUA	73.4	B	QUA	40.6	BC	QP2	50.2	B
QP1	66.9	BC	QP1	39.9	BC	QP1	46.0	B
CUB	47.3	BCD	CUB	32.1	BCD	CUB	36.7	BC
MIP	46.9	BCD	MIP	28.7	BCDE	MIP	22.4	C
MIT	37.9	BCD	MIT	28.7	BCDE	MIT	22.0	C
CDP	18.7	CDE	CDP	9.1	CDE	CDP	-3.1	D
SQU	7.0	DE	SQU	2.1	DE	SQU	-4.1	D
C-D	-14.3	E	C-D	-2.7	E	C-D	-19.3	D

Note: Averages with same capital letter are not significantly different (95%).

## D.2 Estimated Mineralization

### D.2.1 Mineralization, Kg N/Ha, Quadratic, Quadratic with Plateau, and Mitscherlich, 41 Trials with Climate Data

Trial No.	Crop	Quadratic		Quadr./Plateau		Mitscherlich	
		NS	N2	NS	N2	NS	N2
39	SBAR	75.1	44.1	75.1	44.1	70.36	39.36
40	SBAR	59.0	45.0	57.8	43.8	42.45	28.41
42	SBAR	143.7	108.7	143.7	108.7	114.81	79.81
43	SBAR	94.4	67.4	94.4	67.4	78.51	51.51
44	SBAR	66.9	35.9	66.9	35.9	56.39	25.39
45	SBAR	59.3	39.7	59.6	40.0	44.28	24.70
46	SBAR	142.8	70.4	118.5	46.2	57.06	-15.30
47	SBAR	118.0	61.8	102.5	46.4	60.29	4.13
48	SBAR	140.4	103.4	116.4	79.4	69.63	32.63
49	WBAR	126.5	77.5	126.5	77.5	98.58	49.58
50	WBAR	26.6	16.7	26.6	16.7	24.37	14.50
51	WBAR	46.7	24.3	46.7	24.3	41.30	18.93
53	WBAR	100.0	55.7	100.0	55.7	97.63	53.35
54	WBAR	68.5	46.9	64.2	42.6	43.89	22.29
56	WWHE	181.8	151.8	135.1	105.1	70.88	40.88
57	WWHE	71.6	27.6	69.2	25.2	49.05	5.05
58	WWHE	75.3	56.9	75.3	56.9	57.53	39.09
59	WWHE	70.1	49.6	63.4	42.9	44.48	23.99
60	WWHE	70.6	52.2	69.0	50.6	49.84	31.48
61	WWHE	54.8	26.7	53.6	25.5	38.90	10.82
62	WWHE	96.0	63.6	85.6	53.2	53.63	21.23
63	WWHE	82.7	55.7	78.7	51.7	49.84	22.84
64	WWHE	60.6	43.4	60.6	43.4	47.90	30.62
65	WWHE	123.4	100.4	123.4	100.4	112.87	89.87
66	WWHE	106.5	82.5	74.0	50.0	43.57	19.57
67	WWHE	85.2	52.2	67.8	34.8	39.98	6.98
68	WWHE	70.8	54.7	70.8	54.7	54.75	38.66
70	WWHE	100.0	62.0	84.6	46.6	52.08	14.08
71	WWHE	77.7	32.7	77.7	32.7	55.32	10.32
72	WWHE	67.9	21.9	58.5	12.5	39.59	-6.41
73	WWHE	35.8	12.8	34.6	11.6	24.82	1.82
74	WWHE	74.9	50.9	58.6	34.6	36.39	12.39
75	WWHE	56.0	40.3	56.0	40.3	44.62	28.91
76	WWHE	90.7	77.8	90.5	77.7	64.60	51.78
77	WWHE	81.0	38.9	79.1	37.0	55.55	13.43
78	WWHE	87.6	26.3	84.4	23.0	57.20	-4.15
79	WWHE	100.5	53.0	99.2	51.7	74.75	27.23
80	WWHE	143.4	91.6	76.3	24.5	35.50	-16.34
81	WWHE	52.5	35.2	47.2	29.9	32.96	15.68
82	WWHE	106.3	84.7	98.9	77.3	64.87	43.27
83	WWHE	87.9	60.9	86.8	59.8	61.13	34.13

## References

- Ackello-Ogut, C., Q. Paris, and W.A. Williams. "Testing a von Liebig Crop Response Function against Polynomial Specifications." *American Journal of Agricultural Economics* 67(November 1985):873-80.
- Anderson, R.L., and L.A. Nelson. "A Family of Models Involving Intersecting Straight Lines and Concomitant Experimental Designs Useful in Evaluating Response to Fertilizer Nutrients." *Biometrics* 31(June 1975):308-18.
- Beattie, B.R. "Diminishing Marginal Productivity and the Liebig Production Function." Unpublished, Department of Agricultural and Resource Economics, College of Agriculture and Life Sciences, University of Arizona, 1998.
- Beattie, B.R., and C.R. Taylor. *The Economics of Production*. Malabar, FL: Krieger Publishing Company, 1993.
- Berck, P., and G. Helfand. "Reconciling the von Liebig and Differentiable Crop Production Functions." *American Journal of Agricultural Economics* 72(November 1990):985-96.
- Black, C.A. *Soil-Plant Relationships*. New York: John Wiley and Sons, Inc., 1968.
- Black, J.P. "Book Review of Spillman, W.J and E. Lang: The Law of Diminishing Returns." *Journal of Farm Economics* 7(April 1925):274-80.
- Bock, B.R., and F.J. Sikora. "Modified-Quadratic/Plateau Model for Describing Plant Response to Fertilizer." *Soil Science Society American Journal* 54(November-December 1990):1784-89.
- Boyd, D.A., L.T.K. Yuen, and P. Needham. "Nitrogen Requirement of Cereals. Response Curves." *Journal of Agricultural Science* 87(August 1976):149-62.
- Boyd, D.A., P.B.H. Tinker, A.P. Draycott, and P.J. Last. "Nitrogen Requirement of Sugar Beet Grown on Mineral Soils." *Journal of Agricultural Science* 74(February 1970):37-46.
- Bullock, D.G., and D.S. Bullock. "Quadratic and Quadratic-Plus-Plateau Models for Predicting Optimal Nitrogen Rate of Corn: A Comparison." *Agronomy Journal* 86(January-February 1994):191-96.

- Bulman, P., and L.A. Hunt. "Relationship among Tillering, Spike Number and Grain Yield in Winter Wheat (*Triticum aestiva* L.) in Ontario." *Canadian Journal of Plant Science* 68(July 1988):583-96.
- Burt, O.R. "Pooling Experiments over Time to Quantify the Dynamics of Dryland Winter Wheat Yield Response." *Agronomy Journal* 87(March-April 1995):156-64.
- Cerrato, M.E., and A.M. Blackmer. "Comparison of Models for Describing Corn Yield Response to Nitrogen Fertilizer." *Agronomy Journal* 82(January-February 1990):138-43.
- Danish Farmers' Union. *Landboforeningerne om kvaelstof, fosfor og vandmiljoe*. Copenhagen: De Danske Landboforeninger, 1997.
- Danish Farmers' Union. *Landoekonomisk Oversigt 1999*. Copenhagen: De Danske Landboforeninger, 1999a.
- Danish Farmers' Union. *Beretning 1999*. Copenhagen: De Danske Landboforeninger, 1999b.
- Dillon, J.L. *The Analysis of Response in Crop and Livestock Production*. Oxford: Pergami Press, 1977.
- Environmental Group, The Danish Agricultural Advisory Center. *Environment '99. Rules Applying to Danish Agriculture*. Aarhus: Danish Farmers' Union and Danish Family Farmers' Association, 1999.
- European Commision, Directorate General for Economic and Financial Affairs. *European Economy, EC Agricultural Policy for the 21th Century*. Bruxelles: European Commission, 1994.
- Frank, M.D., B.R. Beattie, and M.E. Embleton. "A Comparison of Alternative Crop Response Models." *American Journal of Agricultural Economics* 72(August 1990):597-603.
- Frederick, J.R. "Crop Ecology, Production and Management." *Crop Science* 37(November-December 1997):1816-26.
- Frederick, J.R., and J.J. Camberato. "Water and Nitrogen Effects on Winter Wheat in the Southeastern Coastal Plains: II. Physiological Responses." *Agronomy Journal* 87(May-June 1995):527-33.



- Grimm, S.S., Q. Paris, and W.A. Williams. "A von Liebig Model for Water and Nitrogen Crop Response." *Western Journal of Agricultural Economics*. 12(December 1987):182-92.
- Heady, E.O., and J. Pesek. "A Fertilizer Production Surface with Specification of Economic Optima for Corn Grown on Calcareous Ida Silt Loam." *Journal of Farm Economics* 36(August 1954):466-82.
- Jarvis, S.C., E.A. Stockdale, M.A. Shepherd, and D.S. Powlson. "Nitrogen Mineralization in Temperate Agricultural Soil: Process and Measurement." *Advances in Agronomy* 57(1996):188-237.
- Jonsson, L. "On the Choice of a Production Function Model for Fertilization and on Small Grains in Sweden." *Swedish Journal of Agricultural Research* 4(April 1974):87-97.
- Kennedy, P.E. *A Guide to Econometrics*. Cambridge: MIT Press, 1998.
- Landskontoret for Planteavl. *Oversigt over landsforsogene 1997*. Aarhus: Danish Agricultural Advisory Center, Danish Farmers' Union and Danish Family Farmers' Association, 1997.
- Langer, R.H.M., and F.K.Y. Liew. "Effects of Varying Nitrogen Supply at Different Stages of the Production Phase on Spikelet and Grain Production and on Grain Nitrogen in Wheat." *Australian Journal of Agricultural Research* 24(September 1973):647-56.
- Lanzer, E.A., and Q. Paris. "A New Analytical Framework for the Fertilization Problem." *American Journal of Agricultural Economics* 63(February 1981):93-103.
- McTaggart, I.P., and K.A. Smith. "Estimation of Potentially Mineralisable Nitrogen in Soil by KCI Extraction II. Comparison with Plant Uptake in the Field." *Plant and Soil* 157(December 1993):175-84.
- Myers, R.J.K., C.A. Campbell, and K.L. Weier. "Quantitative Relationship between Net Nitrogen Mineralization and Moisture Content of Soil." *Canadian Journal of Soil Science* 62(February 1982):111-24.
- Olesen, E.E. "Vinterhvedes (*Triticum aestivum* L) udbytte-respons ved tilførsel af stigende mængder kvælstof." MS thesis, Royal Veterinary and Agricultural University of Copenhagen, 1999.
- Paris, Q. "The von Liebig Hypothesis." *American Journal of Agricultural Economics* 74(November 1992a):1019-28.

- Paris, Q. "The Return of von Liebig's Law of the Minimum." *Agronomy Journal* 84(November-December 1992b):1040-46.
- Paris, Q., and K. Knapp. "Estimation of von Liebig Response Function." *American Journal of Agricultural Economics* 71(February 1989):178-86.
- Perrin, R.K. "The Value of Information and the Value of Theoretical Models in Crop Response Research." *American Journal of Agricultural Economics* 58(February 1976):54-61.
- Plantedirektoratet. *Vejledning og skemaer, mark- og goedningsplan, goedningsregnskab, plantedaekke, harmoniregler 1999/2000*. Copenhagen: Plantedirektoratet, 1999.
- Raun, W.R., and G.V. Johnson. "Soil-Plant Buffering of Inorganic Nitrogen in Continuous Winter Wheat." *Agronomy Journal* 87(September-October 1995):827-34.
- Redman, J.C., and S.Q. Allen. "Some Interrelationships of Economic and Agronomic Concepts." *Journal of Farm Economics* 36(August 1954):453-65.
- Rude, S., and B.S. Frederiksen. *National and EC Nitrate Policies*. Copenhagen: Statens Jordbrugsokonomiske Institut, 1994.
- SAS Institute Inc. *SAS User's Guide: Statistics, Version 5 Edition*. Cary, N.C.: SAS Institute Inc., 1993.
- Sparrow, P.E. "Nitrogen response curves of spring barley." *Journal of Agricultural Science* 92(April 1979):307-17.
- Spiertz, J.H.J., and J. Ellen. "Effects of Nitrogen on Crop Development and Grain Growth of Winter Wheat in Relation to Assimilation and Utilization of Assimilates and Nutrients." *Netherlands Journal of Agricultural Science* 26(April 1978):210-31.
- Stanford, G., M.H. Frere, and D.H. Schwaninger. "Temperature Coefficient of Soil Nitrogen Mineralization." *Soil Science* 115(4)(April 1973):321-23.
- Thompson, T. Personal communication. Department of Soil, Water and Environmental Science, College of Agriculture and Life Sciences, University of Arizona, July 2000.
- Tronstad, R., and C.R. Taylor. *An Economic and Statistical Evaluation of Functional Forms and Estimation Procedures for Crop Yield Response to Primary Plant Nutrients*. Staff Paper. Auburn: College of Agriculture and Alabama Agricultural Experiment Station, Auburn University, 1989.

- Vigil, M.F., and D.E. Kissel. "Rate of Nitrogen Mineralized from Incorporated Crop Residues as Influenced by Temperature." *Soil Science Society American Journal* 59(November-December 1995):1636-44.
- von Liebig, J. *Die Chemie in ihrer Anwendung auf Agricultur und Physiologie*. Braunschweig: Verlag von Friedrich Bieweg und Sohn, 1846.
- Whingwiri, E.E., and D.R. Kemp. "Spikelet Development and Grain Yield of the Wheat Ear in Response to Applied Nitrogen." *Australian Journal of Agricultural Research* 31(July 1980):637-47.
- Wild, A. "Historical." *Russell's Soil Conditions and Plant Growth*, 11th ed. Alan Wild ed., pp. 1-30. New York: John Wiley and Sons, 1988.
- Wild, A., and J.H.P. Jones. "Mineral Nutrition of Crop Plants." *Russell's Soil Conditions and Plant Growth*, 11th ed. Alan Wild ed., pp. 69-112. New York: John Wiley and Sons, 1988.