CEREAL RESPONSE TO NITROGEN

by

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STATEMENT BY AUTHOR

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Abstract

This study examines various mathematical formulations of cereal response to nitrogen. Questions about the curvature of the response function and its shape after yield maximum have occupied scientists for decades. Available agronomic knowledge combined with statistical fit to field trial data were criteria for comparing and ranking ten different functional forms. A quadratic response function with a maximum yield plateau was judged to be the best performing model overall, although several other models seem worthy of further considerations.

Another goal of the study was to estimate the extent of nitrogen mineralization from soil resources. Estimation was based on the functions' nitrogen axis intercept assuming a zero yield without nitrogen and that the fitted functions were representative outside the observable fertilizer domain. Available climate data could not satisfactorily explain mineralization variation among trial sites. Economic evaluation showed that incorrect choice of response model entails significant costs, especially if the choice is between linear and non-linear specifications. Insurance against unpredictable mineralization variation in the form of a permanent over-fertilization seems profitable only if a linear response specification were correct.

Chapter 1

Introduction

For many years, it has been a goal for scientists to establish the functional relationship of crop response to fertilizer application. Computational costs no longer prohibit extensive examinations of how functional forms of varying complexity comply with empirical observation. As a result, over the last four or five decades, knowledge in this field has accumulated at an increasing pace. However, this does not mean that today one specific mathematical model can be nominated as best describing the relationship between crop yield and input of plant nutrients in arable production. The long-standing dispute over Justus von Liebig's more than 150-year old linear/plateau model versus E.A. Mitscherlich's 50-year younger theory of diminishing marginal productivity is still very much alive (Frank, Beattie, and Embleton 1990; Cerrato and Blackmer 1990; Paris 1992a).

Plant scientists know a great deal about how plant nutrient availability influences photosynthetic activity, number of straws per plant, and number and weight of kernels. Laboratory research in these and other fields is invaluable as a basis for hypothesizing the nature of input-output relationships in farming. However, unlike laboratory conditions, climate, soil quality, occurrence of pests, and availability of soil resources of nutrients for example, vary significantly in space and time in commercial production environments. Often, such variation is unknown or can not be appropriately quantified. Econometric estimation of yield response to the application of nutrients based on empirical data from practical farming and even from organized field trials may be subject to its own "law of the minimum." That is, improvements in model prediction are difficult beyond a certain plateau where availability of quality data becomes a limiting factor.

Endeavors to discover the truth about the fundamental nature of crop response functions have a direct bearing on real life economic problems. Economically optimal fertilizer application requires knowledge of the physical productivity function in addition to information about expected output and input prices. Optimal (efficient) public policy to deal with environmental damage associated with crop fertilization also demands reliable knowledge about the nature of crop response functions.

Throughout this thesis monoperiodic production (a single timeless growing season) under certainty is assumed. Matters involving multiperiod production and timing of fertilizer application within a single production period are beyond the scope of this study. The same is true regarding the consideration of risk and uncertainty as they relate to the fertilizer-yield relationship and product and factor prices, notwithstanding their importance in commercial farming. The statistical analysis is based on several years of extensive dryland field trial activity undertaken by Danish farm organizations.

Again, focus of this study is on the nature of the crop response relationship. It seems obvious that the exact nature of this relationship would vary considerably among crops, rotation systems, etc. The estimation of functional relationships is for cereal crops grown on soil with high water retaining capacity. The field trial results only allow for study of the impact of nitrogen application on yield. The question of possible interaction or substitution between nitrogen and other macro plant nutrients is not considered. The crop response to nitrogen in the field trials is predicated on the assumption that other nutrients are available in non-limiting amounts.

1.1 Objective of Study

This thesis has two main objectives. The first objective is to examine which functional forms best describe cereal yield in response to application of nitrogen. By examining the statistical fit of functions with different mathematical properties it may be possible to assess, for example, whether a linear or non-linear form is more appropriate. This issue remains in dispute in the literature (Paris 1992a; Paris 1992b). Also, whether yield tends towards a maximum plateau, or if there is an abrupt inversion at higher levels of nitrogen is investigated. A comparison of the empirical viability of several commonly used functional forms fitted on a rather extensive Danish field trial data set is a main objective of the statistical analysis.

A second general aim is to assess the importance of plant available nitrogen from natural nitrogen soil resources, which often amounts to several tons per hectare. The major part of soil nitrogen is tied up in organic compounds and it is not immediately available as nutrient to growing plants. A certain amount becomes plant available each year through so-called mineralization. Plant available soil nitrogen at the beginning of growing season can be estimated on the basis of soil samples. Mineralization during the growing season can not be directly observed. Indirectly, the functional forms, which will be fitted to field trial data, can yield an estimate of the amount of mineralized nitrogen. Assuming that plant growth would be zero if no nitrogen were available, nitrogen provided from soil resources might be estimated on the basis of functions' negative nitrogen-axis intercept values for otherwise well-behaved and precisely-estimated functional forms. Variations in the amount of mineralized nitrogen diminish the inferences that can be made when functional relationships are based only on applied amounts of fertilizer and plant-available soil nitrogen at planting. For example, with above normal mineralization and other things equal a given level of output can be achieved with a lower application of fertilizer. Therefore, to get a more complete picture of output response to fertilizer application a part of the empirical analysis was to estimate the extent of the mineralization process and how it is influenced by climatic variation among field trial locations and years.

1.2 Organization of Thesis

Chapter 2 reviews the agronomic theory of nitrogen's influence on cereal yield. This physiological information provides guidance for hypothesis formulation regarding the fundamental nature of the crop response models. Properties of various functional forms are reviewed in Chapter 3. Important economic considerations and differences among competing model specifications are discussed assuming various linear and non-linear functional forms with plateaus, upper asymptotes, or decreasing yield at high rates of nutrient application. Chapter 4 discusses the specific functional forms advanced for empirical estimation and testing using the Danish field trial data.

Chapter 5 presents results of the empirical analysis for each of the functional forms, including the estimated mineralization of organic soil nitrogen. The influence of climatic variation on mineralizing soil nitrogen is examined in chapter 6. Chapter 7 is devoted to farm economic and environmental policy implications. Chapter 8 provides a summary with concluding remarks and considerations about need for further research.

Chapter 2

The Influence of Nitrogen on Cereal Yield

This chapter explores how certain plant physiological mechanisms determine cereal yield. Agronomic theory and knowledge provides guidance for modeling proper functional relationships between yield and applied nitrogen. Further, the question of ascertaining total plant-available nitrogen – applied fertilizer nitrogen plus nitrogen from "natural" sources – is addressed.

2.1 Nitrogen and Production of Organic Matter in Cereal Plants

Grain yield from a given area of land depends on the number of head-bearing straws, kernels per head, and kernel weight (Bulman and Hunt 1988). This specification provides a useful framework for evaluating yield response by focusing attention on how nitrogen effects these three yield components. This section also draws from an extensive literature summarized by Olesen (1999).

2.1.1 Number of Head-Bearing Straws

The potential number of straws per plant (main shoot plus tillers) is large but only a fraction of initiated straws make it to the head-bearing state. In field experiments with winter wheat, the number of heads (spikes) was found to increase linearly at approximately one for every three tillers (Bulman and Hunt 1988). Another field trial study showed that about 40 percent of all tillers result in fertile spikes. Nitrogen was found to influence the tillering capacity and thereby also the number of heads per plant and per area unit. As described in Table 2.1, heads per m^2 increased from about 500 when applied nitrogen was 50 to 100 kg per ha, to about 540 for the highest levels of nitrogen (Spiertz and Ellen 1978).

Table 2.1 Nitrogen Effect on Yield and Yield Components

	Nitrogen dressing, kg/ha			
	50	100	150	200
Heads per m^2 *	509	498	540	538
Kernels per head *	32.8	35.9	37.8	38.2
Avg. kernel weight, mg *	40.0	41.3	42.1	42.1
Grain total, g per m^2 **	640	732	799	821

Note: * Based on observations on 0,3 m^2. ** By harvest of 60 m^2.
Source: Spiertz and Ellen, 1978.

In a wheat field trial on low fertility soil, Whingwiri and Kemp (1980) found the following on how heads per plant increase with the amount of applied nitrogen:

Nitrogen, kg per ha per week for 10 weeks	Heads per plant at maturity
0	1.0
3	1.1
10	2.8
30	3.3

The Spiertz and Ellen (1978), and Whingwiri and Kemp (1980) results suggest that the number of heads on a given area increase at a diminishing rate with nitrogen application.

The number of heads also depend on the timing of nitrogen application(s). Fertilizing in the early phases of the growing season has a positive effect on the number of heads. Concentrations of plant hormones, plant nutrients, assimilated carbon compounds, and water in the plants have significant impact on the number of initiated straws. It has been shown that the amount of certain essential hormones for the straw initiating process is positively influenced by high concentrations of nitrogen in the plant (Olesen 1999).

It is important for straw initiation that plants develop leaves with high photosynthetic activity as early as possible. Carbon compounds assimilated in the leaves are directed to the part of the plant where branching takes place. The level of carbon concentration positively influences the number of initiated straws.

Photosynthetic assimilation depends on leaf area, which is positively related to the amount of nitrogen (Langer and Liew 1973; Frederick and Camberato 1995). However, plants will utilize most of the available light in the photosynthetic process when leaf area reaches an optimum of 6-9 times the planted area. When leaf area exceeds the optimum level, upper leaves will shadow for lower leaves where formation of carbon assimilates will cease (Olesen 1999).

Increasing the application of nitrogen causes an increase in the concentration of nitrogen in the plants (Frederick 1997). Photosynthesis is positively correlated with nitrogen concentration but at a decreasing rate. As concentration goes up an increasing amount of nitrogen will be inactivated and stored in the plant. Cereal plants are generally very tolerant to high nitrogen concentrations. Therefore, the generation of organic matter in the plants can stabilize over a rather extended plateau. It is only when nitrogen reaches an extraordinary high level of concentration that poisonous effects may appear and cause a yield decrease.

2.1.2 Kernels per Head

Spiertz and Ellen (1978) found that the number of kernels per head is positively related with applied nitrogen. The figures in Table 2.1 suggest a concave relationship within the domain of nitrogen application. Whingwiri and Kemp (1980) found the same functional relationship, see Table 2.2. They report figures for both main shoot and tillers. No tillers were observed for the treatment levels 0 and 3 kg nitrogen per week. Kernels per head is the main contributor to yield increases when nitrogen dressing goes up from N-0 to N-3 and from N-3 to N-10. Between N-10 and N-30 there is a slight decline in the number of grains per main shoot, whereas the decline is more evident per tiller.

Table 2.2. Nitrogen Effect on Yield Components

	Nitrogen dressing							
	N-0	N-3	N-10			N-30		
	MS*	MS*	MS*	T_1*	T_2*	MS*	T_1*	T_2*
Spikelets per head	8.6	14.2	17.8	16.6	6.6	18.8	16.2	15.0
Kernels per spikelet	1.74	2.07	2.60	2.64	2.60	2.40	2.10	1.97
Kernels per head	15.0	20.4	46.2	43.8	43.2	45.2	34.0	29.6
Grain weight, mg	26.0	29.8	32.8	30.2	30.1	32.1	30.8	26.2

Note: * MS denotes main shoot and T_1 and T_2 are first and second tillers.
Source: Whingwiri and Kemp 1980.

2.1.3 Kernel Weight

Whingwiri and Kemp (1980) report an increase in average kernel weight from increasing amounts of nitrogen up to N-10. A relatively stable kernel weight was found between N-10 and N-30, see Table 2.2. On average, kernel weight is lower for tillers than for the main shoot. Spiertz and Ellen (1978) also found increasing kernel weight for the lower nitrogen applications and stability at higher fertilization rates, see Table 2.1. The timing of nitrogen application may influence the grain filling period and thereby also the final kernel weight (Langer and Liew 1973).

2.1.4 Total Yield

Spiertz and Ellen (1978) show that the combined effect of the different yield components is an increasing grain yield on a given area of land when nitrogen application is increased. Their figures in Table 2.1 show decreasing yield growth for uniform increments of nitrogen. Whingwiri and Kemp (1980) report the same pattern, however with a yield decrease between N-10 and N-30. (N-30 equals 300 kg nitrogen per ha.) Spiertz and Ellen (1978) use 200 kg for their maximum application. The actual shape of the curve between the nitrogen applications can not be assessed on the basis of the data. Both studies indicate decreasing marginal response and plateau and/or decrease in total yield at high nitrogen levels. Increasing kernel numbers per area unit is the main contribution to increased yield whereas average kernel weight is less affected by increasing amounts of nitrogen.

Bulman and Hunt (1988) found that yield is linearly related to the number of spikes, which increases with higher rates of applied nitrogen. They report that in other studies the relationship between spikes and yield is described as a flat-topped parabolic curve because younger tillers yield less than the main shoot and older tillers. Bulman and Hunt underline the importance of high spike numbers per area unit because more kernels per spike and higher kernel weight cannot compensate sufficiently for low spike numbers to make up yield.

2.1.5 Negative Indirect Effects of High Nitrogen Application

Besides these direct physiological relationships between plant growth and nutrients, certain indirect effects play an important role. Increasing amounts of nitrogen lead to more lush crops and a moister microclimate, which promotes attacks from various fungicidal pests. Further, it is found that fast growing plants are more easily infested. A high concentration of fungi on leaves is found to reduce both the number of kernels per head and the kernel weight, primarily because of diminishing photosynthetic activity in the leaves. Also, lush crops may attract more harmful insects. Insects cause reduced yield through their direct feeding on the plants, secretions that reduce photosynthetic activity, and an increased risk of fungicidal infections (Olesen 1999).

Another indirect effect of high nitrogen levels can be lodging (Landskontoret for Planteavl 1997). Lodging negatively influences the transport of water and nutrients to the upper parts of the plant. Plants try to compensate for the lack of water by narrowing stomatas to reduce evaporation. This also impedes the intake of carbon dioxide from the air with the result that carbon assimilation in the plants is reduced.

2.1.6 Summary

Based on the foregoing information it can be concluded that within certain limits, increasing nitrogen results in increased yields. Nitrogen influences the number of head-bearing straws per plant, kernels per head, and kernel weight. Some of these factors may manifest as a linear yield response, but in combination they tend to suggest a curvilinear or concave relationship between yield and nitrogen. A maximum grain yield will be reached, as the crop becomes increasingly lush and the leaf area approaches 6-9 times the planted area. Beyond that level, additional leaves will shadow existing leaves, causing photosynthetic activity of the shadowed leaves to cease. Given the ability of plants to store available nitrogen, which can not be utilized in the photosynthetic assimilation process, a yield plateau level of a certain extent can be anticipated after maximum yield has been reached. Poisonous effects may be found at higher levels of applied nitrogen so that a yield decline may follow the plateau. Yield loss at high nitrogen levels is more likely to be the indirect result of pests caused by fungi and insects in lush crops. Also lodging of lush crops may reduce yields because the bent of straws restricts water supply to upper parts of the plants.

2.2 Importance of Nitrogen in Soil

Nitrogen is essential for plant growth. Amino acids, proteins, and certain enzymes require nitrogen as an important building block. Natural sources of nitrogen are plentiful. Atmospheric gasses are 78 percent nitrogen, and one hectare of arable land may contain several tons of nitrogen.

2.2.1 Plant Available Nitrogen

Legumes and certain other plant species can assimilate atmospheric nitrogen in a symbiotic cohabitation with bacteria. Cereal crops do not possess this ability so that they depend on available nitrogen in soil. Their annual need is but a fraction of the total stock of nitrogen in most soils. However, almost all soil resources of nitrogen are part of inaccessible organic compounds (e.g. plant residues, bacteria, and complex connections with soil particles). Through mineralization, organic nitrogen may be transformed whereby ammonium ions and nitrate ions are set free. Only such inorganic compounds when dissolved in soil water can be absorbed by the root system of cereal plants. Part of the mineralized nitrogen leaches or evaporates from the root zone before it can be utilized by growing plants. High yielding crops require nitrogen beyond the amount made available from soil stocks of nitrogen. The deficit is normally provided for in the form of commercial fertilizers or livestock manure.

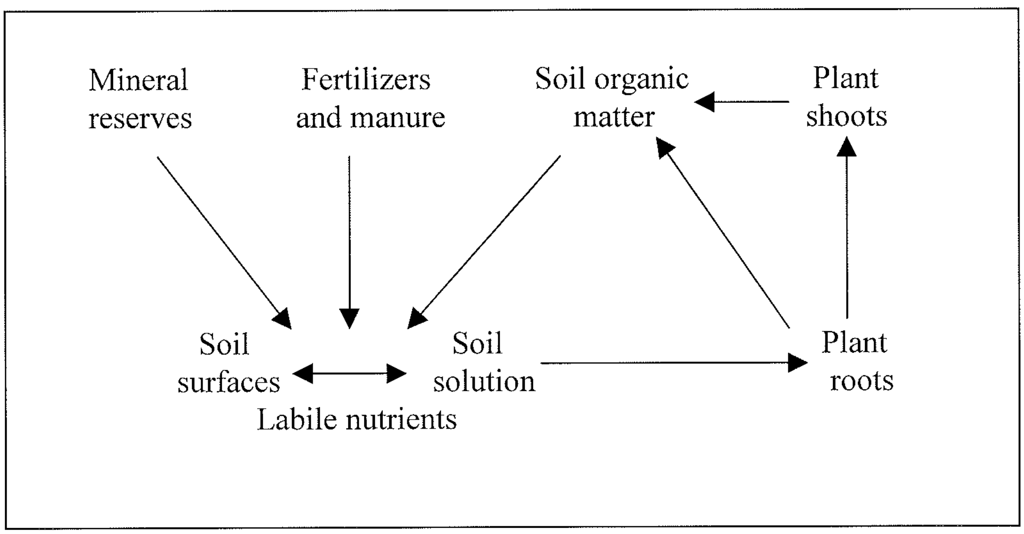
The extent of mineralization varies in space and time depending largely upon the organic matter in the soil and climate related factors. This is an important issue for determining the proper application of nitrogen fertilizer to secure an adequate total nitrogen supply for crops.

2.2.2 Factors Influencing Nitrogen Availability

Growth in the stock of nitrogen in organic material takes place when the root system and other plant residues are left on a field after harvest. Application of livestock manure has the same effect. When the total stock of organic nitrogen increases, potential mineralization increases as well. Figure 2.1 provides a schematic representation of available plant nutrients in soil, their replenishment, and their transfer to plants (Wild and Jones 1988).

The composition of organic material, notably the relation between carbon and nitrogen content, is important for the mineralization process. If the carbon/nitrogen ratio exceeds 20-25, the active microbial biomass will suffer a nitrogen deficit (Black 1968). In this situation ammonium and nitrate ions in soil water will be absorbed and net mineralization will become negative. If for instance straw from the crop is

Figure 2.1. Availability of Nitrogen in Soil



Source: Adapted from Wild and Jones (1988).

ploughed into the soil, soil bacteria will require nitrogen to digest cellulose in the straw. In this process plant available nitrogen may be immobilized. At a later stage, when microbial biomass and other organic nitrogen compounds erode, plant available nitrogen is set free.

Soil texture has an impact on speed of mineralization. Fine clay and silt particles can bind and encapsulate organic material, thereby protecting it from mineralization. The average topsoil organic nitrogen content in English arable fields has been found to range between three tons per hectare for coarse sand and thirteen tons per hectare for clay (Jarvis et al. 1996). Danish arable land contains five to eight tons of nitrogen per hectare (Danish Farmers' Union 1997).

Temperature plays an important role for the speed of microbial activity and mineralization, which almost ceases at around the freezing point and is at optimum between 25 and 35 °C. However, periods with alternating frost and thaw cause a degradation of microbial biomass, whereby organic nitrogen becomes accessible. Disruption of soil aggregates can have the same effect (Jarvis et al. 1996). The Q-10 value, which is the mineralization growth rate for a temperature increase of 10 °C, was found to be around two and relatively constant over the temperature interval 5-35 °C (Stanford, Frere, and Schwaninger 1973). Vigil and Kissel (1995) found similar results although Q-10 values tended to increase with increasing temperature. Their results also showed some variation for different types of organic material. High lignin content slows down the mineralization process.

Adequate amounts of soil water in combination with high temperatures enhance mineralization activity. Mineralization increases almost linearly with the relative moisture content in the soil. Extreme water deficit limits the biological activity and hence mineralization. Excess amounts of water reduce the oxygen content in soil. Most bacteria in the biomass are aerobe, i.e. they need oxygen for their activity. Therefore, a high water saturation level can slow down the bacterial activity and thereby also impede the mineralization process (Myers, Campbell, and Weier 1982; Jarvis et al. 1996).

There is little doubt that the supply of nitrogen from soil resources is substantial but measurements of nitrogen mineralization are difficult and little exact information is available. McTaggart and Smith (1993) conducted several experiments using ^{15}N-labelling to determine the ratio between applied fertilizer nitrogen and soil nitrogen in plant material. ^{15}N is a stable, non-radioactive nitrogen isotope that occurs naturally. It can be isolated from the normal ^{14}N by cold fractional distillation and compounded in fertilizers. ^{15}N is absorbed and utilized by plants in the same way as ^{14}N. By the use of light gas spectrometry ^{15}N can be traced and quantified when analyzing plant material samples (Thompson 2000). The results of some of the McTaggart and Smith experiments from Southern Scotland with spring barley grown after cereals crops were reported by Jarvis et al. (1996). It was found that the uptake of soil nitrogen averaged 57 kg per ha, which was close to 50 percent of total uptake. Variation was substantial between sites, types of soil and tended to increase with increasing content of organic matter in soil. According to Olesen (1999) a maximum of about two percent of organic nitrogen compounds in the soil are mineralized per year.

Mineralization of organic material is a slow process, which primarily adds to the amount of inorganic nitrogen in the form of ammonium ions. The same happens when manure and certain types of fertilizers are applied. Positively charged ammonium ions may be bound by negatively charged clay particles. Ammonium ions serve as nutrients in further microbial activity, and plants can absorb them directly. Most important for plant growth is the transformation by oxidation of ammonium ions to nitrate ions. Nitrate is the principal form of inorganic nitrogen, which is absorbed by plants (Black 1968).

2.2.3 Leaching and Evaporation of Nitrogen

Nitrate ions are easily soluble in soil water, and therefore, they are also prone to leach in periods when water moves downward in the soil. Leaching occurs when the soil is already sated with water and precipitation exceeds evaporation. In four-season regions, downward water movement normally occurs between harvest and the following spring, i.e. in a period when plant growth and plant absorption of nitrogen is minimal. The amount of nitrogen leaching to (and contaminating) groundwater depends on the amount of soil water leaving the root zone and the nitrate concentration in that water (Danish Farmers' Union 1997). Nitrogen concentration may vary with the amount and composition of organic material in the soil, type and acidity of soil, plant growth activity, and temperature. Model studies indicate that in a temperate-climate area, like Denmark, annual leaching may amount to over 90 kilo of nitrogen per hectare (Olesen 1999). The risk of leaching ceases by onset of spring when water movement in the soil is reversed and becomes upward. Remaining nitrate in soil water after the end of winter season is generally retained in the root zone. Analysis of soil samples can determine the amount of plant available nitrogen at the beginning of growing season (Jarvis et al. 1996).

Nitrogen may also be lost from soil through evaporation of ammonium, especially in the application of manure and certain types of fertilizers. Also, when the oxygen concentration in soil is low, nitrate ions may be reduced to a gaseous state (nitrification) and released to the atmosphere. On the positive side, soil receives small additional amounts of nitrate, which is formed during lightning storms, and delivered through precipitation.

Adequate farm management measures can reduce the loss of nitrogen. Whether such measures are profitable depends on their cost in relation to possible reduction of fertilizer expenditures. Leaching and evaporation of nitrogen are important environmental issues, which give rise to various social costs. Restrictions on the use of fertilizers and manure have been introduced in many countries to reduce pollution. Concerns primarily focus on nitrate levels in ground water (drinking water). In addition, eutrophication of streams, lakes, and coastal waters is considered a problem. High concentrations of nutrients cause algae growth and a reduction of oxygen content, which subsequently results in fish mortalities. Air pollution through ammonium evaporation from fields and manure storage facilities is also cause for concern. All of these undesired effects imply social costs (Rude and Frederiksen 1994).

2.2.4 Summary

Soil resources provide a share of total annual nitrogen needed in modern agriculture. The contribution of soil sources of nitrogen varies in time and space, among other things as a result of climatic differences. In this study, crop yield response to varying levels of applied nitrogen is estimated on the basis of Danish field trial data. The plant growth theory and knowledge discussed in Section 2.1 provided guidance as to which functional forms were considered for empirical investigation. There is also a need to include information about the relative importance of nitrogen made available from soil resources to get an accurate picture of the "actual" nitrogen available under differing experimental trials. This will enable a more accurate assessment of the true functional relationship between yield and nitrogen. Data on mineralized nitrogen in the soil at the beginning of the growing season are available dating back to the early 1990s in the Danish experimental data. However, estimates of mineralization during the growing season are not available. Indirectly, this amount may be estimated on the basis of fitted response curves, assuming that no yield would result if the nitrogen level were zero. The climatic influence on mineralization was examined by combining field trial data with local climate data, which were available, by years and geographical regions for post 1992 field trials.

Chapter 3

Production Economics Theory and Alternative Production Function Specifications

This chapter surveys various mathematical functional forms that have been used to describe (model) crop response to fertilizers. Three considerations are important in crop response modeling. First, crop response models should comply, insofar as possible, with established plant physiology and agronomic theory. Chapter 2 discussed some key relationships of the law-of-nature type. A second consideration that is particularly germane for economic modeling is to have functional forms mirror as accurately as possible observed empirical data. Lastly, hypothesized functional forms must be amenable to appropriate statistical procedures to enable estimation of model parameters and evaluation of the quality of fit.

Drawing upon production economics theory, Section 3.1 outlines important properties to be considered in modeling crop response to fertilizer and other inputs. Emphasis in the present study is on models relating arable output to nitrogen fertilization. Section 3.2 surveys principal functional forms that have been tested against crop response data.

3.1 Some Theoretical Aspects of Functional Forms

Like other production processes, crop production is a complex process of combining and coordinating a great number of input materials and productive forces in the creation of an output. The fact that crop production involves biological processes adds to the complexity of modeling the input-output relationship. Specification of the quantitative (mathematical) relationship between yield and factors of production is a compact means of describing technical production opportunities. This abstract representation of the relationship between inputs and output is referred to as a production function (Beattie and Taylor 1993). The term response function is also used (Dillon 1977). [1]

In generalized notation, a single-product production function (model) is given by

(3.1) $Q = f(Z_1, Z_2, Z_3, \cdots, Z_n)$

where output (Q) is a function of various inputs or factors ($Z_1, Z_2, Z_3, \cdots, Z_n$). Normally, knowledge about productive forces is far from complete. Most often, production functions must be hypothesized as simplified models of the real world. Usually, only the most significant factors of production are identified.

Land is an important input in the classical list of production forces. Land's special significance in crop production is obvious. However, in this study, land is treated as a fixed factor; specifically, land is fixed at the level of a hectare and all other inputs and output are measured per unit of land. That is, if Z_n in equation (3.1) is land, then the following transformed model is obtained:

(3.2) $Y = f(X_1, X_2, X_3, \cdots, X_{n-1})$

where Y is output per hectare (Q/Z_n) or simply yield and $X_i = Z_i/Z_n$ for $i = 1, 2, \cdots, n-1$ are the various non-land input levels per hectare.

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[1] Throughout it will be assumed that best available technology is embedded in the production function. Land tillage, pest management, timeliness of planting and fertilization, and harvesting are conducted in such a way that maximum utilization of applied nitrogen is achieved.

3.1.1 Single Factor Variation

To simplify further, the present application represents an extreme short-run scenario with applied nitrogen (X_1) as the single choice/variable input. Other (non-land) inputs are considered fixed ($X_2, X_3, \cdots, X_{n-1}$), although not in the traditional sense. Rather, the field trials, which are the source of the empirical data, were so designed that other inputs (except, of course, land) may be presumed available in sufficient amounts so as to not limit yield for any level of nitrogen input.

Strictly speaking, since yield is specified here as output per hectare, then, by definition, land is not assumed available in a non-limiting amount. Thus, if no factors other than land and nitrogen are in limiting supply, yield can be expected to take on a maximum value as determined by the crop's genetic potential (Dillon 1977). An absolute upper yield limit given by genetic capacity seems plausible. However, the possibility of a limiting effect of land may complicate matters. Defining yield as output per hectare might inhibit attainment of the maximum genetic capability of the plants. There is a physical limit to the number of plants and straws that can be squeezed into a given area. Also, there is the indirect "crowding" effect on attainable yield manifested in relation to photosynthetic utilization of light, see Chapter 2. Whether influence of the spatial barrier sets on gradually as input of nitrogen increases, or more abruptly when input intensity reaches a certain level, or at all is an open question.

However, for purposes of this study we are not concerned whether the limiting factor is genetic potential or the one-hectare land constraint or some combination of the two. That is, in this study it is assumed that genetic potential and land set an upper barrier to yield beyond which no further increase is possible even if the amounts of other inputs increase. The limiting influence of genetic potential and/or land provides appealing motivation for the existence of a yield maximum or a yield plateau, which is a central question relative to the discussion about possible functional forms. Traditionally, plateau yield is described as a situation where availability of other variable inputs limits yield. If input of a limiting factor is increased a new and higher plateau will occur at a point where the limiting factor is once again in deficit. However, in our case everything is assumed to be non-limiting except for genetic potential and one hectare of land.

Yield decline may occur if at high intensity levels certain nutrients exert a poisonous effect. Excess amounts of water may have the same result, e.g. by reducing the oxygen content in the soil or by lodging of the crop. The point at which yield decline occurs determines the extent (width) of a possible yield plateau.

The foregoing discussion highlights the hypotheses that are explored in this thesis: (1) The nature of the crop response relationship from a zero level of applied nitrogen (X_1) up to the point of yield maximum. Is the yield-nitrogen relationship linear, strictly concave (e.g. quadratic) or quasi-concave (e.g. cubic)? (2) Is there a yield plateau of some discernable width? (3) Is there an eventual poisoning effect giving rise to a yield decline?

3.1.2 The Two Variable Factor Model

Although the focus is on a one-product one-factor relationship between yield of grain and applied nitrogen, this study also takes steps to include climate as an explanatory factor. Analyses were limited to an evaluation of climatic impact on mineralization of organic soil nitrogen. Examination of direct climatic influence on the response function would be a natural continuation of these analyses. In anticipation of such expansion at a later stage, this chapter also discusses some of the salient features of a two-factor model. In simplified,[2] general notation, let

(3.3) $Y = f(X_1, X_2)$

where X_1 denotes quantity of nitrogen per hectare and X_2 could denote a continuous climate index. If X_2 is unchanged for all levels of X_1 the functional relationship only relates yield to varying amounts of X_1. However, the response curve is determined by the interplay between X_1 and X_2. Even with only two factors the response surface can take many different forms depending on the actual relationships between yield and those factors. Two different situations are illustrated in the examples below.

When no substitution is assumed between nitrogen and climate the relationship takes the form of a Leontief production function with a plateau:

(3.4) $Y = \min(f(X_1), f(X_2), Y_{max})$

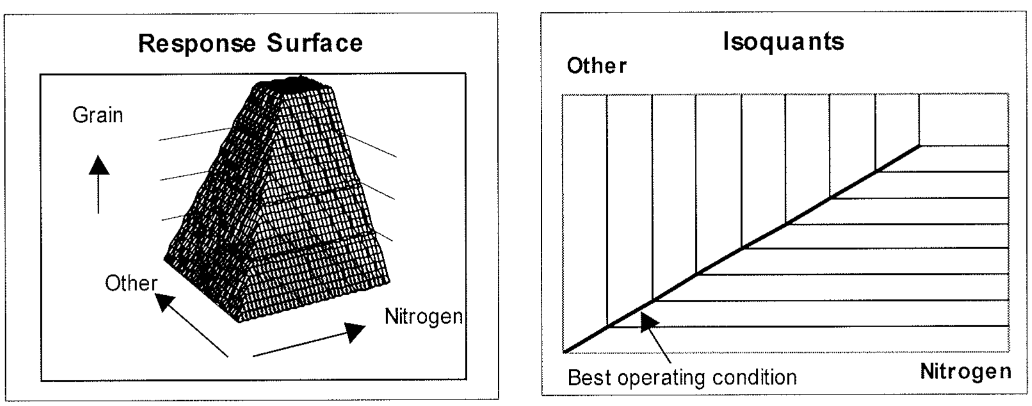
Figure 3.1 illustrates a case where the functional relationships, $f(X_1)$ and $f(X_2)$, are assumed to be linear so that the response surface is formed by two intersecting planes. Isoquants are L-shaped signifying that the factors combine in a constant proportion. The rate of technical substitution is not everywhere defined. The best and only operating condition is found on the factor ray that connects the knots of the isoquants. In this case,

[2] The other factors ($X_3, \cdots X_{n-1}$) in equation (3.2) are held constant in the two variable factor model and are not explicitly shown in (3.3) for expositional simplicity.

with equidistant isoquants,[3] the relationship between nitrogen and yield will be traced out as a straight line like that in Figure 3.1, Panel B. This description parallels the first scientific formulation of a plant growth model made by Justus von Liebig more than 150 years ago (von Liebig 1846). von Liebig's two-factor model generally assumes two different macronutrients as the relevant choice variables. In another situation it may be assumed, for example, that the pre-plateau response surface follows a quadratic functional relationship such as

$$\text{(3.5)} \quad Y= \min (b_0 + b_1X_1 + b_2X_1^2 + b_3X_2 + b_4X_2^2 + b_5X_1X_2, Y_{max}).$$

Figure 3.1 Production Function – No Substitution between Factors

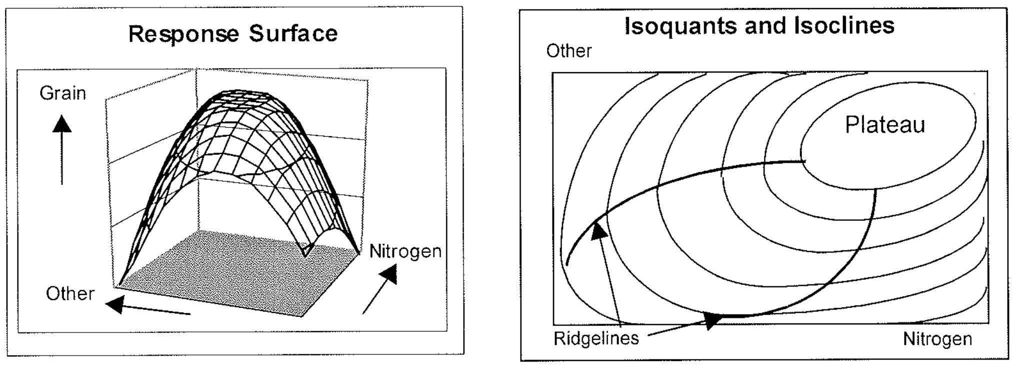


Panel A **Panel B**

A yield plateau is reached at Y_{max}; X_1 and X_2 are complementary factors if $b_5 > 0$; b_1 and b_3 are positive; and b_2 and b_4 are negative. A graphic representation is given in Figure3.2. The response surface is concave and isoquants are everywhere differentiable.

[3] The case depicted is that of constant returns to quasi-scale (Beattie and Taylor, pp. 58-59).

Figure 3.2 Quadratic Function – Imperfect Factor Substitution



Panel A **Panel B**

When the other factor (e.g. climate) is available in non-limiting amount, the best operating condition, or maximum attainable output, coincides with the upper ridgeline, that is the loci for an undefined isoquant slope. The combination of nitrogen and other factors along this ridgeline represents the maximum attainable output for varying amounts of nitrogen. The functional relationship between yield and nitrogen, which is traced out along the ridgeline, is strictly concave (marginal physical productivity decreasing in both factors) until it reaches the plateau. Other functional forms that do not assume a yield plateau are discussed in Section 3.2.

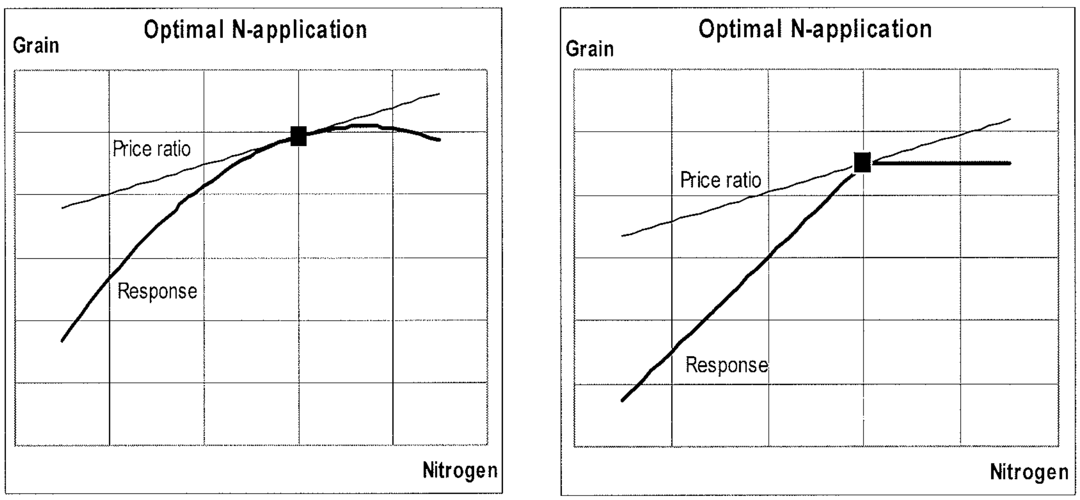
3.1.3 Economic Optimization

Under conditions of monoperiodic production with certainty, and single factor variation, the profit maximizing factor level is easily determined.[4] Much of the interest in

[4] It is assumed that yield is independent of fertilization and production in previous years. Carry-over of nutrients is captured by measured nitrogen supplied from soil resources at the beginning of the growing season. Expected output and all prices are known with certainty. Quality differences, such as varying protein percent in grains, are disregarded, as is the amount of straw.

the functional forms in Section 3.2 relates principally to different specifications, e.g. polynomial or linear with plateau as illustrated in Figure 3.3, and is concerned with implications for the optimal choice (profit-maximizing level) of applied nitrogen. With

Figure 3.3 Economic Optimum - Different Functional Forms



Panel A **Panel B**

total physical product of grain measured along the vertical axis and application of nitrogen along the horizontal, the function, TPP = f (N), is obviously continuous in both Panel A and B. However, in Panel B the marginal physical productivity, MPP = $\partial f(N)/\partial N$, is not defined at the knot point, where the response curve is not differentiable.

For given unit prices of output, p, and nitrogen, r, economic optimal application of nitrogen can be derived in Panel A. Optimum is defined as maximum monetary profit. For this single-factor case, profit is given by

$$(3.6) \quad \pi = pf(N) - rN - C$$

where p and r are unit prices for grain and fertilizer nitrogen and C denotes fixed costs associated with production factors other than nitrogen. For simplicity it is assumed that nitrogen application costs and output handling costs are zero. Differentiating the profit equation with respect to N, $\partial\pi/\partial N = p\partial Y/\partial N - r$, the critical N-value can be found where the slope of the response function equals the inverse price ratio, $\partial Y/\partial N = r/p$, i.e. at the point of tangency. It is assumed that concavity of the profit function is ensured in the form of a negative second derivative, $\partial^2\pi/\partial N^2 < 0$, so that the point of tangency is a maximum. This is clearly the case in Panel A where the function exhibits decreasing marginal productivity.

In the example given in Panel B the same procedure can not be applied because the response function is not everywhere differentiable. Economic optimum for N occurs at the knot on the response curve, that is at maximum TPP, for positive r/p values less than the slope of the pre-plateau segment of the response curve. If the r/p ratio exceeds the slope, optimum applied N is zero. Had the increasing segment of the response curve been curvilinear, an economic optimum would still be at the knot point if the marginal productivity everywhere exceeds the inverse price ratio. Statistical techniques allow for fitting splined curves even when they, as in B, are only almost everywhere differentiable. Care must be exercised to ensure that the knot point is indeed the economic optimum.

It is evident that the calculated economic optimum in Panel A is sensitive to changes in the r/p price ratio. In Panel B the optimum is stable at the kink point for all positive price ratios less than the slope of the increasing branch of the curve. It should be obvious from a cursory examination of Figure 3.3 that knowing the nature of the response function is crucial in determining the optimal application of nitrogen.

3.2 Review of Some Principal Functional Forms

Since Justus von Liebig more than 150 years ago formulated his "law of the minimum," numerous alternative theories and crop response models have been tested against experimental data and field trial results. This research area is important since fertilizer recommendations are based on what is considered the most appropriate model (Paris 1992b). Decisions on adequate rates of fertilization have attracted increasing attention over the past 50 years with a significant expansion in the use of fertilizer (Cerrato and Blackmer 1990; Bock and Sikora 1990). Optimal rates are closely linked with the question of farm profit and increasing environmental concern associated with agriculture's use of nutrients, notably nitrogen.

Most response models are based on one of two distinctly different "laws" of plant growth. One is the previously mentioned von Liebig's "law of the minimum." The other is Mitscherlich's "law of decreasing response," which was first published at the beginning of the 19th century. The two scientists should perhaps share the paternity of the two competing functional forms with some of their contemporaries (Paris 1992b). Nevertheless, their names are used in the following sections when grouping functional forms, some of which combine properties from the von Liebig and Mitscherlich laws, while others have unique characteristics.

3.2.1 von Liebig's Law of the Minimum

Justus von Liebig's law states that the yield of a crop is governed by the change in the quantity of the most scarce factor called the minimum or limiting factor (von Liebig 1846). As the minimum factor is increased the yield will increase in proportion to the supply of that factor until another factor becomes limiting. If a non-minimum factor is increased or decreased, the yield would not be affected (Redman and Allen 1954). There is no possibility of substitution of one factor for another (the isoquants are L-shaped).

Other translated quotes from the more elaborate later editions of von Liebig's "Die Chemie in ihrer Anwendung auf Agricultur und Physiologie" say that the crops on a field decrease or increase in exact proportion to the diminution or increase of the mineral substances conveyed to it in manure (Wild 1988). The productivity of a field is in direct relation to the necessary constituents contained in the soil in smallest quantity (Black 1925).

von Liebig did not present his theory in mathematical terms. Most modern day scientists have applied his model assuming that it should be interpreted as follows:

$$(3.7)\ Y = \min(Y^*, \beta_1 + \beta_2 N, \beta_3 + \beta_4 P)$$

where in this example N and P are nitrogen and phosphorus, respectively, and Y^* is maximum yield. The formula can easily be extended to accommodate several nutrients. The von Liebig function is consistent with the idea that macronutrients perform different biochemical functions in plant growth. Further the function implies that plants respond (in a linear fashion) only to the most limiting nutrient. After some level of application (say N^* and P^*) the plant will no longer respond to the applied nutrients. At this point, the plant reaches maximum growth at Y* (Frank, Beattie, and Embleton 1990). This linear version of the von Liebig function is called the LRP-Liebig (linear response with plateau). These three central features - no substitution between nutrients, linear response, and a plateau maximum yield - were outlined in Figure 3.1 of Section 3.1.

The LRP model was discarded for a long period when curvilinear functional forms attracted the attention of agronomists and economists as being in better compliance with the law of diminishing marginal return. The LRP model was revived when Boyd et al. (1970) suggested that two intersecting straight lines gave a better approximation to crop response than the traditional curvilinear models. In their analysis of UK field trials with nitrogen on sugar beets Boyd et al. found that such a model closely fits the individual replications of a trial. Anderson and Nelson (1975) used the von Liebig theory as the basis for a proposed family of models involving intersecting straight lines. Some of these models were of the LRP type. Anderson and Nelson found that for agronomic purposes and for certain crops the response relationship between yield and major nutrients is linear until yield reaches a maximum plateau. Their examinations showed that some curvilinear models tended to suggest optimal fertilization rates that were too high. Perrin (1976) utilized the LRP relationship in his economic analyses of crop response. He found that it provided recommendations as valuable as those derived from the then widely used polynomial forms. He believed that his results should cast some doubt on the widespread notion among economists that fertilizer response should always be analyzed by fitting a smooth response surface.

In their study Boyd et al. showed that the calculated optimal nitrogen application rate (knot of the two intersecting lines) might differ between replications in an experiment. Also the height of the plateau could be different from one replication to another. Therefore, it is possible that a smooth, curvilinear function fitted to the average of many replications of an experiment can yield a better fit than two intersecting straight lines. However, this process (averaging across replications) does not reflect the true underlying physiological relationship (Boyd et al. 1970).

Berck and Helfand (1990) argued a reconciliation of the LRP function with smooth, curvilinear curves. They found that even if at plant level the LRP function is correct, the estimation of a production function at the level of a whole field, a farm, or a county is a different matter. Variation in distribution of inputs (soil quality, amount of nitrogen etc.) across any large area causes plants in different locations of a field to be limited by different limiting values of inputs. Such heterogeneity of one or more inputs can result in a smooth production function when estimated as an aggregate for a whole field.

This point of view is not supported by Olesen's analysis of Danish field trials with winter wheat. By fitting curves to individual replications in a large number of field trials he found no evidence that the LRP function performed better on individual replications than on aggregate data (Olesen 1999). In experiments with barley Boyd, Yuen, and Needham (1976) showed that results were well represented by two intersecting straight lines, and on average the von Liebig LRP model had the least residual variation compared with alternative functions.

A series of studies carried out during the 1980s by Paris and co-authors gave strong support to the von Liebig model. By assuming weak separability between nutrients and other factors like soil quality and climate the functional relationship between yield and application of nutrients can be estimated (Lanzer and Paris 1981). Further, non-substitution between nutrients makes it possible to examine yield response for one-product, one-variable factor relationships, when other factors are assumed to be available in non-limiting amounts. The validity of the LRP form was supported by an examination of data from extensive experiments with application of potassium and phosphorus to several crops (Ackello-Ogutu, Paris, and Williams 1985). The LRP also performed well in competition with other model specifications in a study of field trials with application of water and nitrogen to different crops (Grimm, Paris, and Williams 1987). The von Liebig response function's simplicity, its good fit to experimental data, and its explanation of non-substitution between nutrients were underlined as positive features (Paris and Knapp 1989).

In Paris's studies of the 1980s, the tested von Liebig theory was taken to be synonymous with plateau yield, non-substitution and linear pre-plateau response. In his 1992-articles, Paris argues that von Liebig's thesis about "direct relation" between yield and nutrient need not imply linear response (Paris 1992a; Paris 1992b). In an unpublished paper, Beattie (1998) advocates the linear interpretation for the pre-plateau curve segment with reference to the term "exact proportion" in Wild's (1988) translation. The question is important not only for historical correctness but also relative to unambiguous specifications of von Liebig models. The term LRP leaves no doubt as to the question of linearity. Paris's interpretation of von Liebig opens the door for useful model constructions in which plateau yield can be combined with non-linear pre-plateau functional forms. A principal thrust of this thesis is to explore the empirical viability of both versions of the von Liebig model.

3.2.2 Mitscherlich's Law of Diminishing Response

Like von Liebig, Mitscherlich was a German natural scientist. His examination of large numbers of field trial results led to his plant growth theory. Mitscherlich proposed a smooth, curvilinear, and concave response function that approaches a maximum attainable yield. Under the Mitscherlich law, yield increases continuously at a decreasing rate with the amount of applied nutrient and approaches a maximum yield asymptotically. The nearer the yield to the maximum, the less the increase with each increment of fertilizer (Black 1925). Key features of Mitscherlich's basic hypothesis are that there is a maximum attainable yield and that yield increases diminish for additional increments of nutrients the closer actual yield is to maximum (Redman and Allen1954; Jonsson 1974; Olesen 1999).

In his review of Spillman's treatise on diminishing returns, Black (1925) welcomed Mitscherlich's approach in contrast with von Liebig's theory. Black believed von Liebig's theory was inconsistent with the principle of diminishing returns and marginal productivity theory, which are generally taken as starting point in value theory. (Recall that prior to Paris' interpretation, the Liebig model was assumed to be LRP). Since 1839, when von Liebig for the first time formulated the law of the minimum, soil and plant chemists had struggled trying to reconcile "the law" with practical field experience. Mitscherlich in a series of experiments, the first results of which were published in 1909, claimed to show that von Liebig's law of the minimum was wrong, and that the principle of diminishing return was the true principle involved. Mitscherlich's work was well received as providing a formula better aligned with the general conception of physical and chemical processes (Black 1925).

Mitscherlich specified his law as an exponential function:

$$(3.8) \quad Y = m\,(1 - ke^{-\beta N})$$

where k and β are response parameters and m is the asymptotic yield plateau for a single-factor response function (Paris 1992b). The German mathematician, Baule, generalized the formula to multiple factors. The Mitscherlich-Baule function is given as

$$(3.9) \quad Y = m\Pi\,(1 - k_j e^{-\beta_j X_j})$$

where the subscript j denotes the individual nutrients. This formula allows substitution between nutrients in contrast to a von Liebig multifactor function (Frank, Beattie, and Embleton 1990). It also differs from the LRP-Liebig by being curvilinear, and its "plateau" is an upper asymptote, which is only gradually approached (and never reached). Like von Liebig's function, equation (3.9) never displays decreasing yield.

With product and nitrogen prices, p and r, profit in the monofactor case is given by

$$(3.10) \quad \pi = pm\,(1 - ke^{-\beta N}) - rN - C$$

When the first-order derivative, $\delta\pi/\delta N = pmk\beta e^{-\beta N} - r$, is set equal to zero and solved for N, the critical point is found at $N^* = \frac{-1}{\beta}\ln\left[\frac{r}{pmk\beta}\right]$. The second-order derivative

$$(3.11)\quad \delta^2\pi/\delta N^2 = -\, pmk\beta^2\, e^{-\beta N}$$

is negative for all N > 0 since all parameter values are positive. Thus, the stationary value is guaranteed to be a maximum.

Until computing facilities improved in the last half of the 20th century, practical application of the Mitscherlich-Baule function was rather limited because parameter estimation requires non-linear estimation methodology.

3.2.3 Polynomials

First to research the adequacy of polynomials were Heady and Pesek (1954). They fitted quadratic and square root polynomials to data from field trials with nitrogen and phosphorus on corn (Heady and Pesek). Polynomial models assume curvilinearity for all input levels as in the Mitscherlich model. The notion of a plateau yield is not maintained. Polynomial specifications also introduce the possibility of substitution between nutrients in opposition to von Liebig models. Single nutrient response curves can be estimated for different fixed levels of the other nutrient(s).

Following Heady and Pesek the use of polynomial models was definitely in vogue in research, extension, and farm management (Jonsson 1974; Boyd, Yuen, and Needman 1976; Sparrow 1979; Cerrato and Blackmer 1990; Bullock and Bullock 1994). Polynomial formulas were readily estimated without incurring substantial computational costs. Of particular interest in this study, Danish farm organizations have, for many years, based normative nitrogen use recommendations mainly on results using cubic polynomials. Since 1998, Danish environmental legislation has restricted farmers' use of nitrogen based on the same procedure of calculating economic optimum with a deduction of ten percent from the result (Plantedirektoratet 1999).

The properties of polynomial functions are presented in Table 3.1 for quadratic, cubic and square root response curves for single nutrient models. Other types of polynomials, like the so-called three halves, have been applied in some of the cited studies. Yield maximizing nitrogen application and maximum yield can be established by means of the usual first and second derivative procedures. The yield maximizing input of nitrogen is determined as a function of estimated parameter values. As discussed

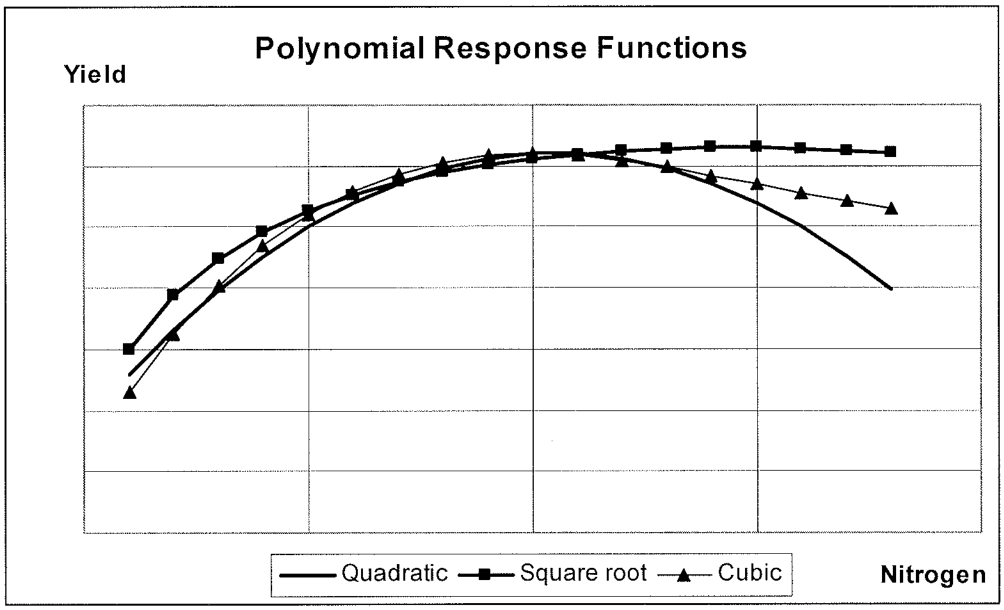
Table 3.1 Properties of Single-Factor Polynomial Functions

	Yield function, Y	N at max. yield	N at economic optimum
Quadratic	$a + b_1N + b_2N^2$	$-b_1/2b_2$	$(r/p - b_1)/\ 2b_2$
Cubic	$a + b_1N + b_2N^2 + b_3N^3$	$(-b_2 \pm (b_2^2 - 3b_1b_3)^{0.5})/3b_3$	$(-b_2 \pm (b_2^2 - 3b_3(b_1 - r/p))^{0.5})/3b_3$
Square root	$a + b_1N + b_2N^{0..5}$	$b_2^2/(-2b_1)^2$	$b_2^2/(2(r/p - b_1))^2$

previously, economic optimum values reported in Table 3.1 were found by differentiation of the profit function (see equation 3.6).

The examples given in Figure 3.4 illustrate the symmetry of the quadratic polynomial with respect to its maximum. As such it can not approximate a plateau yield. The asymmetric cubic and especially the square functions have the ability to display an approximate plateau over the post-maximum curve segment. The cubic function deserves special attention. Its flexible form – a convex segment and a concave segment – ensures a good fit to data.[5]

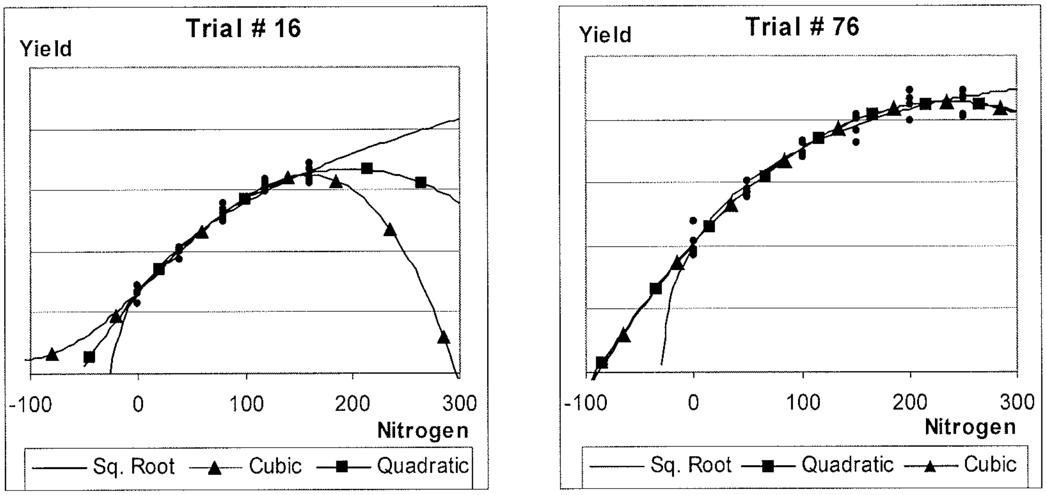
Figure 3.4 Comparison of Polynomial Response Curves



The cubic curve in Panel A of Figure 3.5 exhibits all three stages of production (increasing average productivity followed by decreasing average productivity and finally negative marginal productivity) whereas the cubic in Panel B, like the quadratic, exhibits only Stages II and III over the domain represented in the graphs. The curves have been fitted to two randomly selected data sets from the many included the present study. Observations in the data sets cover a limited segment of N-axis values. It is not always possible to predict in advance which segment of the cubic function will be revealed over

[5] Although it is possible depending on relative parameter signs and magnitudes for the concave segment to precede the convex segment, most investigators would reject such a model as being inconsistent with production theory. Also when the curve displays an overall decrease from left to right (which is possible) that too is inconsistent with production theory.

Figure 3.5 Different Behaviors of Cubic Function



Panel A **Panel B**

the domain of observed data. In Panel A, the cubic curve has no N-axis intercept to the left of the observed data. Over nitrogen levels, for which predictions may be desirable, the cubic displays all three stages of an S-shaped production function. Yet the "interesting" Stage I and III segments are revealed (for the most part) outside the domain of data. In Panel B, the cubic curve intersects the X-axis not far from the smallest N-observation and it displays no Stage I segment for positive nitrogen values. Further, in Panel A the cubic curve bends down sooner and more steeply than the quadratic curve. In this case the cubic is even less capable than the quadratic of indicating a somewhat flat (plateau) segment. In Panel B the two functions are almost identical over the observed domain and its immediate neighborhood. With other data sets, new and different variations are likely to appear. The examples given in Figure 3.5 emphasize the need for caution in curve fitting especially when using cubic models. Further, if functions are used, as they sometimes are, for inferences and prediction of values outside the domain of the observed data particular care must be exercised.

3.2.4 Other Functional Forms

The literature is abundant with other mathematical models and modifications of the ones mentioned above, which have been explored and compared. Inverse polynomials and hyperbolic curves (Jonsson 1974), Cobb-Douglas functions (Tronstad and Taylor 1989), modifications of exponential models of the Mitscherlich type (Burt 1995) are just a few examples.

A quadratic polynomial with plateau is a modification of the LRP model that attracted much interest in the 1990s. As the name indicates a quadratic pre-plateau segment is combined with plateau yield and no factor substitution. Other non-linear curves can be substituted for the linear response, consistent with Paris's findings (Paris 1992a; Paris 1992b). As early as 1975, a quadratic plateau model was considered a possibility (Anderson and Nelson 1975). Its general formulation

$$(3.12)\quad Y = \min(Y^*, a + b_1N + b_2N^2)$$

does not indicate how the pre-plateau segment and the plateau should be joined. To avoid the question of non-differentiability, which was discussed relative to the LRP model as shown in Figure 3.3, some authors (Bock and Sikora 1990; Bullock and Bullock 1994; Olesen 1999) favor a model where the plateau is joined with the quadratic polynomial at its maximum point. The mathematical formulation of this version of the model is

(3.13) $Y = a + b_1N + b_2N^2$ for $N < -b_1/2b_2$, and
$Y = a + b_1(-b_1/2b_2) + b_2(-b_1/2b_2)^2$, i.e. constant for $N \geq -b_1/2b_2$

By constraining the model in this way the first derivative of the function equals zero for both left-hand and right-hand segments at the joint point. In other words, the function is everywhere differentiable and economic optimum can be found by applying the usual optimization procedure.

A less restrictive version is given by

(3.14) $Y = a + b_1N + b_2N^2$ for $N < J$, and
$Y = a + b_1J + b_2 J^2$, i.e. constant for $N \geq J$
subject to $J \leq -b_1/2b_2$

where J denotes the nitrogen rate, which occurs at the intersection of the quadratic response and the plateau line. It is only required that the joining point not be to the right of the maximum of the parabola. While (3.14) has greater fitting flexibility than (3.13), the "drawback", of course, is the slight complication of a stable economic optimum point for certain r/p ratios.[6] Both versions of the quadratic with plateau model are examined in this study. It is not immediately clear which of the two versions Cerrato and Blackmer (1990) applied. They do not explicitly tie the joining point to the maximum of the quadratic curve. On the other hand, they state that economic optimum is found by equating the first derivative of the response function to the fertilizer-output price ratio and solving for the independent variable without making reservation with regard to differentiability everywhere on the curve. This suggests the same approach as used by Bullock and Bullock (1994) and by others.

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[6] Whether or not this feature is a drawback depends on one's view regarding the possibility of discontinuities in the marginal productivity curve. The author does not find this feature disconcerting.

Figure 3.6 Quadratic and Quadratic with Plateau Functions

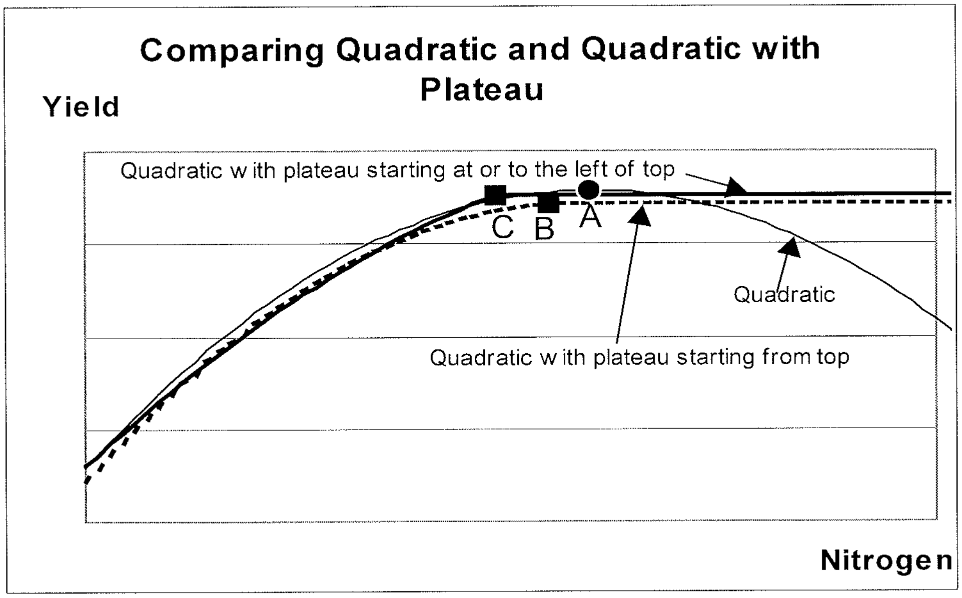


Figure 3.6 compares fits of a quadratic curve with the two versions of the quadratic/plateau model to one of the data sets. In this example, both location and curvature of the increasing curve segment display certain deviations between the three functional forms. The yield maximizing N-application is highest for the quadratic function (represented by A). It is lower (B) when a plateau starting at the top of the parabola is introduced and even lower yet (C) when the plateau is allowed to start before the parabola's maximum has been reached. The estimated maximum yield also differs between the curves. Other data sets produce different patterns. Further details are provided in Chapter 5.

3.3 Summary

Functions used to represent yield in response to application of nutrients should comply with agronomic theory, mirror accurately observed empirical data, and be amenable to appropriate statistical procedures so that parameter values can be estimated and empirically evaluated for statistical fit. It is generally assumed that there is no substitutability between different plant nutrients. The relationship between yield and a single nutrient can therefore be examined when ascertaining that other factors of production are available in non-limiting quantities. Because yield and variable inputs are measured per unit of land, land can be treated as a fixed factor. Agronomic theory suggests that yield response is bounded by a maximum and, further, that yield may be constant over a certain range for additional nutrient application. There is a long-standing discussion about the shape of the response curve up to the maximum. Is it linear or is it strictly concave or quasiconcave?

Various functional forms can accommodate an evaluation of the appropriateness of different response theories. Continuous curves, which combine a growth segment with a maximum/plateau segment at a kink, do not always satisfy the generally held requirement that they be twice differentiable everywhere. However, this does not preclude establishing an economically optimal application of a nutrient, and with basis in agronomic theory growth/plateau curves are indeed possible candidates for well-behaved functional forms.

Historically, von Liebig's more than 150 years old LRP-curve – non-substitution, linear response and a yield plateau – represents the first scientific formulation of a yield response theory. Mitscherlich's plant growth theory from the beginning of the 20th century found widespread support as being in better compliance with the economic law of diminishing marginal return. For a number of years, the more manageable polynomial models as introduced by Heady around the middle of the 20th century, were also considered superior to the LRP-model. Over the last three decades, a number of scientists have underlined the virtues of plateau models and nutrient non-substitution, which led to a revival of von Liebig's theory. Today, many economists consider a combination of plateau yield with curvilinear pre-plateau response as an appropriate functional form. This recent development is not least the result of Paris's research activity. The next chapter examines how well a number of different response functions such as those reported in the present chapter match empirical field trial data on crop response to application of nitrogen.

Chapter 4

Models and Data

This thesis examines how well different functions fit observed data on crop response to applied nitrogen. Estimation of the amount of nitrogen supplied to the plants from soil resources is a second topic. Models, statistical procedures, and the data base for the analyses are explained in the following sections.

4.1 Models and Statistical Procedures

4.1.1 Comparison of Different Functional Forms

Model comparison involved ten different functional forms, which were fitted to the empirical base of trial data on yield response to nitrogen application. The functions shown in Table 4.1 do not cover all relevant possibilities but were selected to encompass a range of plausible growth patterns and combinations of growth curves with plateau segments. It was also an aim to accommodate comparison with models that have been featured/advocated in the literature. Observed data can only represent yield over the domain where available and applied nitrogen can be measured directly. A substantial share of total nitrogen supply comes from soil resources of organic nitrogen compounds. Observed data on cereal grain yield include yield owing to the total nitrogen supply, including unmeasured soil nitrogen.

In the functions in Table 4.1, Y denotes yield, N is the level of applied fertilizer, and j is the replication number in a given trial. The residuals between observed and estimated yield, ε, are assumed normally distributed with a mean of zero. The estimated yield-axis intercept is generally denoted α. However, in the Mitscherlich function, which is a slight modification of equation (3.8), α is the asymptotic maximum yield and the Y-intercept is α - β, which results from setting N to zero. In the square root and the Cobb-Douglas models the Y-intercept is a complex term involving two or more of the other parameters and the term, α, does not appear in those two functions.

Table 4.1 Functions Fitted to Trial Data

Model	Functional Form
1. Quadratic	$Y_j = \alpha + \beta_1 N + \beta_2 N^2 + \varepsilon_j$
2. Cubic	$Y_j = \alpha + \beta_1 N + \beta_2 N^2 + \beta_3 N^3 + \varepsilon_j$
3. Square root	$Y_j = \beta_1(\beta_2 + N) + \beta_3(\beta_2 + N)^{0.5} + \varepsilon_j$
4. Cobb-Douglas	$Y_j = \beta_1(\beta_2 + N)^{\beta_3} + \varepsilon_j$
5. Mitscherlich	$Y_j = \alpha - \beta_1 e^{(-\beta_2 N)} + \varepsilon_j$
6. Linear with plateau (von Liebig)	$Y_j = \alpha + \beta_1 N + \varepsilon_j$ for $N < N^*$ and $= \alpha + \beta_1 N^* + \varepsilon_j$ for $N \geq N^*$
7. Quadratic with plateau	$Y_j = \alpha + \beta_1 N + \beta_2 N^2 + \varepsilon_j$ for $N < N^*$ and $= \alpha + \beta_1 N^* + \beta_2 (N^*)^2 + \varepsilon_j$ for $N \geq N^*$ s.t. $N^* \leq -\beta_1/2\beta_2$ which is N-coordinate for top of parabola
8. Cobb-Douglas w/ plateau	$Y_j = \beta_1(\beta_2 + N)^{\beta_3} + \varepsilon_j$ for $N < N^*$ and $= \beta_1(\beta_2 + N^*)^{\beta_3} + \varepsilon_j$ for $N \geq N^*$
9. Mitcherlich with plateau	$Y_j = \alpha - \beta_1 e^{(-\beta_2 N)} + \varepsilon_j$ for $N < N^*$ and $= \alpha - \beta_1 e^{(-\beta_2 N^*)} + \varepsilon_j$ for $N \geq N^*$

Note: Two versions of the quadratic with plateau were included: one where the onset of the plateau is tied to the top of the parabola, and the other where the onset can be to the left of the parabola top. Thus, 10 functions were fitted to trial data in Chapter 5.

The first three functions – quadratic, cubic and square root – all have the ability to reveal a concave response curve with a unique maximum. The properties of the cubic function call for special attention when inferences are made outside the domain of observed data, as was explained in Section 3.2.3. Models 4 and 5 – Cobb-Douglas and Mitscherlich – have no maximum. The Cobb-Douglas curve increases indefinitely contrary to agronomic theory. Nevertheless, this popular constant-elasticity function was retained in the calculations to check its ability to represent the response curvature for lower levels of nitrogen. A plateau version of the Cobb-Douglas function was included to remedy the indefinite growth feature of this historically popular and robust model. The Mitscherlich function approaches a yield maximum asymptotically. It can be regarded as a step in the direction of models 6, 7, 8, and 9 where the plateau represents maximum yield.

In order not to force the Cobb-Douglas curve through the origin when applied nitrogen is zero a shifter, β_2, was included in the formula for both the non-plateau and plateau version. In the square root function a shifter makes it possible to define values of the dependent variable for negative values of the independent variable. These modifications allow for estimation of negative nitrogen axis intercepts, which serve as an estimate of the amount of mineralized nitrogen made available to plants during the growing season, see Section 4.1.2.

Models 6, 7, 8 and 9 combine a growth segment with a constant-value plateau segment. The location of the intersection between the two curve segments (knot point) is determined in the fitting procedure, which minimizes the sum of squared deviations over the two segments. Two versions of the quadratic with plateau were examined, one where the knot point coincides with the parabola top, and one where the onset of the plateau can be to the left of the parabola top. The effect of including a plateau can be directly assessed for the quadratic with plateau, the Cobb-Douglas with plateau, and the Mitscherlich with plateau, which all have counterparts in non-plateau versions – models 1, 4 and 5, respectively. A similar comparison is not provided for linear with plateau. A simple linear function as counterpart of the LRP-Liebig was excluded because of its poor fit. Overall linearity is also inconsistent with agronomic theory. Visual inspection of trial data plots clearly reveals concavity and the linear relationship ranks last, far below the other functions in terms of goodness-of-fit. However, by comparing the LRP-Liebig with the other plateau models the validity of pre-plateau linearity can be assessed.

All functions were fitted to data using the SAS software, Model Procedure. Minimization of the sum of squared errors was applied as the common criterion for all of the functions – linear in parameters, non-linear in parameters, and compound plateau functions. In all cases the residuals between observed and estimated yield values were assumed normally distributed with zero mean. By logarithmic transformation certain functions, e.g. quadratic and cubic, could have been rendered linear in both variables and parameters, so that the usual OLS technique could have been applied. However, had this been done logarithmic values of residuals instead of absolute values would be minimized. Other functions, like e.g. Mitscherlich and models with shifters, can not be transformed to yield a linear in parameter specification. In these cases a search procedure was used for the estimation. Likewise, in the plateau models the estimated intersection coordinates between the growth segment and the plateau were determined using an iterative search procedure. Thus, all functions were estimated in non-transformed state. The default convergence criteria setting of .001 was used in all cases. Adequate starting values for parameters were supplied to ensure convergence when fitting the different functions using the SAS Model Procedure.

Various methods can be used to evaluate how well different functions describe observed data. The coefficient of determination, R^2, is a commonly used expression for how well a curve fits observed data. Total yield variation (SST), i.e. sum of squared deviations between observed dependent variable data and their global mean, can be decomposed as

$$(4.1)\quad \sum(Y_i - \overline{Y})^2 = \sum(\hat{Y}_i - \overline{Y})^2 + \sum(Y_i - \hat{Y}_i)^2 \text{ or}$$

$$\text{SST} \quad = \quad \text{SSR} \quad + \quad \text{SSE}$$

where SSR is variation accounted for by the fitted function and SSE is residual variation. R^2 is defined as SSR/SST. The applied fitting procedures are designed to minimize residual variation (SSE), which is synonymous with maximizing R^2. Ranking of alternative models fitted to a given set of data according to R^2 is a logical procedure for finding the function that best fits observed data. R^2 was used as the evaluation criterion in some of the literature cited in Section 3.2 (Heady and Pesek 1954; Jonsson 1974; Anderson and Nelson 1975; Bock and Sikora 1990).

However, searching for a high R^2 is not unconditionally an adequate procedure. It is possible to improve R^2 simply by including additional explanatory variables. Even if such an extended model fits the data well it may be incorrect if it captures accidental variation in the particular data set rather than the true underlying relationship (Kennedy 1998). An alternative for overcoming this problem is to use adjusted R^2:

$$(4.2) \quad \overline{R}^2 = 1 - \frac{K-1}{T-K}(1-R^2)$$

where K is the number of independent variables and T is the number of observations. Models with many explanatory variables are thereby "punished" according to their extra use of degrees of freedom.

A similar adjustment is to use calculated mean square error (MSE) as a selection criterion. MSE is the residual variation, SSE, divided by its degrees of freedom. This procedure was used in several previous studies (Boyd, Yuen, and Needham 1976; Sparrow 1979; Cerrato and Blackmer 1990; Olesen 1999). MSE was adopted as the preferred criterion in this study because it – like adjusted R^2 – better than R^2 compares parsimonious models with functions based on more variables. Moreover, the absolute numerical MSE value, more directly than the relative R^2, indicates the magnitude of unexplained variation in comparisons across models.

The comparison of statistical fit between different functional forms was based on a ranking of functions according to numeric MSE values. Frequency counts revealed how often the individual functions ranked as number 1 through 9. For all ten functions this procedure was executed for all 84 trials and for various sub-samples of trials, e.g. by crops, using the Danish data set.

Further, the MSE-performance was evaluated by testing differences of MSE averages for the different functional forms where the tested H_0-hypothesis was that average MSE values are not significantly different between functional forms as explained in Chapter 5. Results were obtained in a two-way analysis of variance where

$$(4.1) \quad MSE = f(\text{Functional Form, Trial})$$

Unfortunately, the MSE goodness-of-fit criterion only relates to the domain of the production function where data observations are available. It does not necessarily reflect how well the function complies with agronomic theory, and whether it is representative outside the data domain. These questions concern the basic model structure and call for supplementary evaluation criteria. Therefore, the MSE criterion can only be one element when assessing the adequacy of different functional forms. Other properties, which must be considered, are the possibilities of deriving meaningful maximum and optimal values for nitrogen application and plausible negative nitrogen-axis intercepts as an estimate of nitrogen supplied by soil resources.

Formulas for deriving maximum and optimum values were generally discussed in Chapter 3. Following the modification of certain functions as shown in Table 4.1 the amount of applied N, which corresponds with maximum yield, was calculated on the basis of estimated parameter values as given in Table 4.2. Depending on the parameter values the N_{max} expression may not be defined in all cases. Given information about unit prices of cereals and nitrogen, p and r, respectively, economic optima were established via usual derivation procedures based on the formulas shown in Table 4.2.

Table 4.2 N-Application at Estimated Maximum Yield and Economic Optimum

	Yield function, Y =	N at max. yield	N at economic optimum
Quadratic	$\alpha + \beta_1 N + \beta_2 N^2$	$-\beta_1/2\beta_2$	$(r/p - \beta_1)/ 2\beta_2$
Cubic	$\alpha + \beta_1 N + \beta_2 N^2 + \beta_3 N^3$	$(-\beta_2 \pm (\beta_2^2 - 3\beta_1\beta_3)^{0.5})/3\beta_3$	$(-\beta_2 \pm (\beta_2^2 - 3\beta_3(\beta_1 - r/p))^{0.5})/3\beta_3$
Square root	$\beta_1 (N+\beta_2) + \beta_3 (N+\beta_2)^{0..5}$	$\beta_3^2/(-2\beta_1)^2 - \beta_2$	$\beta_3^2/(2(r/p - \beta_1))^2 - \beta_2$
C-Douglas	$\beta_1 (\beta_2+N)^{\beta}{}_3$	no maximum	$(r/(p\beta_1\beta_3))^{(1/(\beta 3-1))} - \beta_2$
Mitscherl.	$\alpha - \beta_1 e^{(-\beta_2 N)})$	α is asymptotic maximum	$-\ln(r/(p\beta_1\beta_2))/ \beta_2$

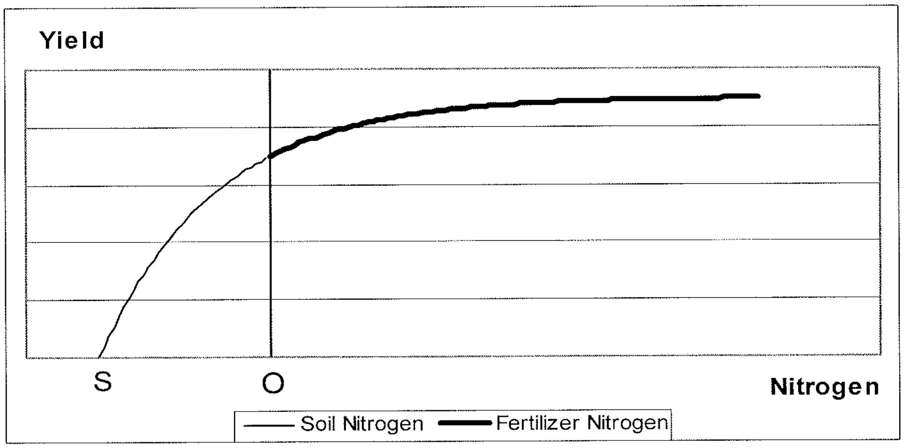
For the plateau models yield-maximizing nitrogen application corresponds with the N-coordinate of the intersection between the functions' growth segment and plateau. N-application at economic optimum is also found here if at this point the slope of the growth segment of the response curve is greater than the r/p ratio. Otherwise, the optimum is found somewhere in the growth segment of the function. If, for the whole domain of observed data, r/p is greater than the slope of the growth segment the optimum nitrogen application is set at zero. Further details are provided in Chapter 5, which reports analytical results.

4.1.2 Estimation of Nitrogen Supply from Mineralization of Soil-N

Total nitrogen uptake in cereal crops can be divided into that derived from applied fertilizer and that from the soil. It was assumed that fertilizer nitrogen and mineralized soil nitrogen are equally accessible to the growing plants and that each has the same nutritional value. Further, the estimation was based on the condition that yield would be zero if no nitrogen were available. Provided that the functional form is representative for the yield/nitrogen relationship over the total domain of nitrogen supply, in particular over the low N-level unobserved portion of the domain, the amount of plant available soil nitrogen can be estimated on the basis of the intersection between the response curve and the nitrogen axis as indicated in Figure 4.1.

The validity of this assumption is central for the relevancy of the procedure that is applied to estimate mineralization. If a specific function both displays a superior statistical fit compared with other functions and generally behaves in accordance with

Figure 4.1 Estimation of Nitrogen Supplied from Soil Resources



agronomic theory outside the observed data domain it could be considered a natural candidate for the estimation process. If several functions display adequate statistical fits the choice becomes more difficult. By definition, exact knowledge about the "true" yield/nitrogen relationship outside the data domain is not available. However, the assumed zero-yield in the absence of nitrogen and various indications about the order of magnitude for mineralization may provide some guidance for the evaluation of the examined functional forms and serve as a basis for narrowing down the range of mineralization. Moreover, even if one had to accept that the intercept procedure can not provide the exact magnitude of mineralization it is possible that the variation in mineralization among trial sites in space and time would be well represented. In that case – and given the lack of actually observed mineralization data – the estimates resulting from the procedure applied in this study could still be a valuable data input in a model evaluation of how different factors – e.g. temperature and moisture – influence mineralization. However, it is important to note/remember that extrapolation outside the domain of the data is nevertheless speculative.

The distance SO on the horizontal axis in Figure 4.1 represents the amount of mineralized soil nitrogen used by the crop. Algebraically it corresponds to setting Y equal to zero and solving for N in the pertinent response function

(4.2) $Y = f(N)$

where N denotes applied fertilizer. For most response functions this procedure is straightforward. Certain functions like the quadratic and square root have two N-axis intercepts and care must be taken to chose the lower intercept value. The cubic function has 1 or 3 N-intercept solutions, which must be found by iteration, e.g. via SAS Solver Procedure. Depending on the data set the cubic function is not always well behaved when it comes to inferences outside the data domain, and it may have no stationary point, see Section 3.2.3 and the reported results in Chapter 5.

For simplicity an abbreviated notation will be used in the following when referring to different sources of nitrogen supply. The total amount of nitrogen made available to plants by mineralization of soil resources (NS) consists of the plant-available nitrogen that is in the soil at the beginning of growing season, i.e. available N from pre-season mineralization and/or carry-over fertilization from the previous season (N1), and an amount provided by mineralization during the growing season (N2), so that

(4.3) $NS = N1 + N2$

For 46 of 84 trials, information about N1 is available so that for those trials an estimate of N2 is possible. Applied fertilizer nitrogen will be denoted NF, so that total nitrogen supply (NT) is

$$(4.4)\quad NT = NS + NF = N1 + N2 + NF$$

Chapter 6 presents modeling and analysis of climatic impact on nitrogen mineralization.

4.2 Data

4.2.1. Field Trials – Cereal Response to Nitrogen

The empirical analyses in this study were based on data from Danish field experiments. The National Department of Plant Production, The Danish Agricultural Advisory Center, which is owned by The Danish Farmers' Union, and The Family Farmers' Association, provided the data.

Each year the Danish farm organizations carry out more than 2,000 field experiments. Advisors in local farm organizations conduct the experiments in fields belonging to members of the organizations. Experiments enable evaluation of yield potentials for both common crops and potential new crops. For each crop, different varieties are compared. Pest management programs, yield response to varying fertilizing intensity, etc. are evaluated. Results of all experiments are recorded, examined and interpreted by National Department of Plant Production. An extensive annual report on this activity is made available to local advisors and other interested parties. Results are accessible on line in the integrated computer system, which links the national center with all local advisory service units.

Results of ongoing field experiments are an important platform for advisory services directed toward individual farmers. In recent years, field experiment results have also played an important role in environmental legislation, e.g. provisions for limited of nitrogen use in field crops.

The present examination focuses on cereal yield response to increasing amounts of nitrogen. Data from about 1,200 dry land field experiments were available from the period 1987-1998. In order to rule out various uncertainties and disturbing factors, which could not be handled in the study, a number of restrictions were applied for the final selected data sets. Experiments were excluded when manure and slurry had been applied to the field trial site in previous years. In addition, only experiments with cereal as the preceding crop soil types JB#6 or JB#7 with high clay content (and high water retaining capacity) were included in the final sample. The selection procedure resulted in a sample of 84 data sets, which are presented in Appendix A, Table A.1.

Experiments normally have five or six levels of nitrogen application with five replications. Each plot is a minimum 30 square meters (about 330 square feet). The content of plant available N in the soil at the beginning of the growing season is established for each experiment on the basis of lab tests of 16 soil samples from the trial site. Measurement of N was phased in after 1990. Samples are taken from the upper 75 cm of the soil. An average result of the 16 samples represents the pre-season plant available N for each plot in an experiment. The plots in an experiment are arranged systematically in blocks, each block representing one replication, and all levels of nitrogen application in each block.

Only yield of grain is measured; straw yield and protein content are not recorded. Standardization is made for moisture content of the grain. Possible quality differences, e.g. due to varying protein content in grain, are not taken into account.

Experiment data have been coded with their geographic location (denoted grid number) since the beginning of the 1990s. In the analysis this information is combined with climate data differentiated by years and grid numbers.

The numbers of retained data sets by year and crop are shown in Table 4.3.

Table 4.3 Number of Field Trials by Year and Crop

Year:	Spring Barley	Winter Barley	Winter Wheat	Total
1987	4	1	2	7
1988	13	0	2	15
1989	4	0	4	8
1990	3	2	2	7
1991	2	0	2	4
1992	1	0	0	1
1993	1	1	2	4
1994	2	0	6	8
1995	2	2	8	12
1996	2	1	4	7
1997	2	1	7	10
1998	0	0	1	1
Total	36	8	40	84

The field trials are ordered by number of nitrogen applications and replications in Table 4.4. The 27 data sets with wheat, which are pooled in a single calculation (see Chapter 7), all have six levels of nitrogen applications with 250 kg nitrogen per ha as the highest level of nitrogen application. In 17 trials the number of replications is five, in nine trials it is four, and in one trial the number of replications is six.

Table 4.4 Field Trials by Number of Nitrogen Applications and Replications

No. of Nitrogen Applications:	Number of Replications			Total
	4	5	6	
5	7	25	0	32
6	13	37	2	52
Total	20	62	2	84

Table 4.5 shows that in 46 of the 84 trials measurements were made of N1 or mineralized nitrogen available at the beginning of the growing season.

Table 4.5 Field Trials by Crops and Information about N1

	Number of Trials			Total
	Spring Barley	Winter Barley	Winter Wheat	
N1 available	10	6	30	46
N1 not available	26	2	10	38
Total	36	8	40	84

Climate data are available for 41 (9 trials with spring barley, 5 with winter barley, and 27 with winter wheat) of the 46 trials with N1 information.

4.2.2 Unit Prices of Cereals and Nitrogen

To establish economic optimum, price data on cereals and nitrogen were utilized, see Section 3.1.3. Different functional forms display varying degrees of sensitivity to changes in the input/output price ratio. A range of price ratios was established based on actual conditions over various time periods, as shown in Table 4.6. Unit prices are expressed in terms of the appropriate quantity units of the productivity functions, i.e. kroner per hkg of grain and per kg of nitrogen.

Table 4.6 Cereals and Nitrogen Prices

	Unit Prices, Danish Kroner	
	Per Hkg of Grain	Per Kg of Nitrogen
Scenario 1: Before 1993-Reform	120	4.0
Scenario 2: After 1993-Reform	85	3.5
Scenario 3: After Agenda 2000	72	3.5
Scenario 4: Sc. 3 plus nitrogen levy	72	8.5

The prices in scenario 1 illustrate the situation as it was before the 1993-reform of the Common Agricultural Policy (CAP) of the European Union. Cereal prices were lowered by about 30 percent as a result of the 1993-reform, and fixed hectare subsidies were introduced as compensation (European Commission 1994). The second scenario represents the after-reform situation at the turn of the century. The third scenario is based on anticipated cereals price reductions, which will occur in 2001, when the so-called Agenda 2000 Reform is fully in force (Danish Farmers' Union 1999b). The fourth scenario adds the effect of a nitrogen levy to scenario 3. The levy of 5 DKr per kg nitrogen was enacted in Denmark in 1998 (Environmental Group, The Danish Agricultural Advisory Center 1999). The levy is waved if a farmer annually submits plans and documentation showing that his/her use of fertilizers and manure does not exceed prescribed norms by crops, type of soil, etc. (Plantedirektoratet 1999). Estimated cereal and nitrogen prices for the four scenarios are based on Danish Farmers' Union (1999a) and author calculations.

4.2.3 Climate Data

Climate data on a weekly basis are available for the period 1993-1998 in 44 areas (grid numbers). The data include information on precipitation, potential evaporation, air temperature, and global short-wave radiation. Averages for the growing season (calendar weeks 13 through 31), the first half (weeks 13 through 22), and the second half (weeks 23 through 31) were calculated by the author for each trial in the analyses, i.e. by pertinent years and grid numbers. Calculations are presented in Appendix A, Table A.2.

Climatic conditions during the fall and winter seasons also influence mineralization as well as leaching of nitrogen to ground water. Therefore, the amount of plant available nitrogen at the beginning of the growing season (N1) may vary depending on temperature and precipitation during especially the winter period. In the calculations, where climatic factors are taken into account, only field trials with N1 information are considered. Effects of fall and winter climate are captured by the measurement of N1, so that only climate information for the growing season was needed for this study.

Chapter 5

Results of Statistical Analyses

This chapter compares the behavior of different functional forms that were fitted to data on crop response to nitrogen in 84 cereal field trials. The statistical fit to observed data is compared among functions and each functions' ability to provide "reasonable" estimates of economic optimum and of mineralized soil nitrogen is evaluated. The influence of climatic variations on mineralization of soil nitrogen is appraised in Chapter 6.

Analytical results are based on data for cereal crops under specific field conditions. Therefore, results are not generally applicable to other crops or other circumstances. A main purpose in this study is to compare different functional forms, not different crops, different soil types, etc.

5.1 Comparison of Results from Fitting Different Functional Forms

5.1.1 Goodness of Statistical Fit to Trial Data

Each of the ten functional forms are ranked in Table 5.1 according to how well they fit observed data in 84 field trials. The ranking criterion is mean squared error, MSE, so that rank number one denotes the lowest MSE. The numbers in the table are a frequency count of how many times each function ranks as number 1 through 10. Columns therefore sum to 84, i.e. the number of data sets. Rows also sum to 84 except for a few cases where ties occur between functions.

Table 5.1 Comparison of 10 Response Functions Fitted to All 84 Data Sets

	Frequency of Scores – Functional Forms Ranked by Mean Square Error									
Rank	QUA	CUB	SQR	CD	MI	QP1	QP2	CDP	MIP	LRP
1	15	11	11	3	13	**16**	5	5		7
2	14	10	12	5	8	18	10	2	1	3
3	6	12	8	2	8	14	15	6	6	6
4	8	15	7	1	14	9	9	11	6	5
5	6	11	7	4	14	12	7	11	9	4
6	4	13	6	5	12	4	12	12	11	3
7	3	5	12	2	10	5	8	16	19	4
8	8	5	15	10	3	6	8	8	16	5
9	16	2	6	23	2		8	8	8	11
10	4			29			2	5	8	**36**
1+2	29	21	23	8	21	**34**	15	7	1	10
9+10	20	2	6	**52**	2		10	13	16	47

When including the linear model it scores 77 of 84 in rank group #11

On this basis the quadratic function with plateau starting at the parabola top comes out as number one in the comparison with 16 placements in rank group 1, and the LRP model is in last place with 36 placements in rank group 10. Considering scores in the top two and the bottom two rank groups, respectively, the quadratic with plateau is still number one with the quadratic as a close runner-up. The Cobb-Douglas function is now in the last place. The Cobb-Douglas also ranks lowest (group 9 + 10) for spring barley and winter barley, and it shares this placement with the LRP-Liebig in the case of winter wheat, see Table 5.2. The quadratic scores most frequently in group one for spring barley, but the square root is number one when group 1 and 2 are merged. Quadratic with plateau from parabola top ranks highest for winter wheat. The LRP-Liebig displays a high ranking in the winter barley sample, which counts only eight data sets. More details are provided in Appendix B, where frequency tables on ranking are also shown for other sub samples of the 84 trials.

Table 5.2 Comparison of Fit for 10 Response Functions, by Crops

Rank	Frequency of Scores – Functional Forms Ranked by Mean Square Error									
	QUA	CUB	SQR	CD	MIT	QP1	QP2	CDP	MIP	LRP
Spring Barley, 36 Data Sets:										
1+2	12	11	**14**	5	7	12	6	3		3
9+10	11	12	2	**20**	2		4	5	8	18
Winter Barley, 8 Data Sets:										
1+2	4	4			1	2	1			**4**
9+10	1			**5**				4	4	2
Winter Wheat, 40 Data Sets:										
1+2	13	6	9	3	13	**20**	8	4	1	3
9+10	8		4	27			6	4	4	**27**

Note: Linear would appear 32, 7, and 38 times in rank group # 11 for s.barley, w.barley, and w.wheat respectively

Although a certain pattern emerges from the tables the ranking figures do not clearly identify one functional form that is superior to all others from a goodness–of–fit point of view. Comparison of the actual MSE values that result from fitting each of the functional forms can broaden the basis for evaluation.

Table 5.3 shows the average MSE values for each of the different functional forms that were fitted to all 84 field trials. For comparison, a linear specification is included in the table together with the ten functions that were seriously considered. As one would expect, the fit of the simple linear equation is inferior to the other functions in the table. A t-test can reveal whether the difference between the average MSE values for any two functions is statistically significant at the five percent error level.

However, with eleven functions, there will be $11x(11 - 1)/2 = 55$ pairs of MSE averages to compare and in each comparison there is a five percent probability of falsely rejecting the null hypothesis (type I error). The probability of making at least one type I error increases with the number of comparisons. With eleven different averages and 55 pairs of averages to compare an upper bound estimate of the probability of a type I error is $1-(1-0.05)^{55}$ or 94 percent. A simultaneous test of all averages where the probability of making at least one type I error is controlled was therefore preferred over the repeated t-test.

Accordingly a Tukey-Kramer test was applied in a simultaneous comparison of MSE averages and averages of other key factors like optimal N-application and estimated N-mineralization. A SAS procedure is available for execution of the Tukey-Kramer test (SAS Institute Inc. 1993). The means of two functions, i and j, are considered statistically different when the following criterion is satisfied:

$$(5.1) \qquad \frac{\left|\overline{\text{MSEi}}-\overline{\text{MSEj}}\right|}{s\sqrt{\dfrac{1}{\text{Ni}}+\dfrac{1}{\text{Nj}}}} \geq q_{.05,k,v}$$

N denotes the number of observations, s is the estimated root mean square deviation, σ_{i+j}, and q is a studentized range distribution of k (11) variables with v (830) degrees of freedom. In Table 5.3 the same letter was assigned to the functions for which the multiple test shows that averages are not significantly different at the 5 percent error level. After the linear specification, the LRP-Liebig model has the highest average MSE of all other models, which suggests that linearity should be ruled out as the growth pattern for the pre-plateau curve segment of plateau models. However, besides the linear function, the LRP-Liebig can only be discerned at the five percent error level from the quadratic with plateau from top of parabola and the cubic function. For the other functions, the average MSE values are not significantly different.

Table 5.3 Comparison of Average MSE by Models, 84 Trials

Function	Avg. MSE	t-grouping *		
Linear	35.53	A		
LRP-Liebig	14.36		B	
Cobb-Douglas	13.80		B	C
Mitsch./plateau	12.70		B	C
Square root polynom.	12.05		B	C
Cobb-Douglas/plateau	11.96		B	C
Quadratic polynom.	11.93		B	C
Quad./plat.- top/left	11.72		B	C
Mitscherlich	11.62		B	C
Quad./plateau - top	11.35			C
Cubic polynomial	11.25			C

* Means with same letter are not significantly different (95% level)

Surprisingly, the quadratic with plateau tied to the top of the parabola has a lower MSE value than the version where the plateau can start from any point of the growth segment. For the latter, the increased fitting flexibility was expected to result in a lower MSE value. However, when the knot point is not pre-specified, an extra degree of freedom is used, which increases the calculated MSE. The plateau version of the Cobb-Douglas function is placed in the group of functions with low average MSE. On average, the flexible cubic function displays the best fit to trial data despite the fact that the cubic uses one more degree of freedom than the quadratic.

The overall impression of the ranking of models is similar when comparing data in Table 5.1 and Table 5.3 even if certain differences occur. The quadratic with plateau is less convincing in Table 5.3 where all placement information, and not just top or bottom placements, is taken into account. For wheat trials alone, the conclusion is similar to that of all 84 trials. As expected the LRP-Liebig average MSE is relatively low in barley trials, especially in the small sample of winter barley trials, where the LRP-Liebig scored well in the ranking calculations. The general pattern is the same in the reduced sample of 46 trials with information about N1. This is true also in the restricted sample of 27 wheat trials that will be used in the examination of climate impact on mineralization.

As was the case in relation to the ranking/frequency approach in Table 5.1 and 5.2, it is not possible to nominate one single, ideal function based on the MSE averages in Table 5.3. However, the numbers in both tables indicate that linear growth should be ruled out, at least in wheat trials. The LRP-Liebig with a linear growth segment scored consistently low in wheat trials. It ranked better in barley trials, especially in the small sample of winter barley trials. The Cobb-Douglas without plateau performed relatively poorly in almost all the samples. The variation in MSE averages among different functional forms is lower for barley samples than for wheat. More details are available in Appendix B.

The number of fertilizer application levels used in field trials and the range of nutrient application spanned may influence the ranking of models. The impacts of different trial designs were evaluated for all models using different samples of the trials as a basis, see Table 5.4. Of 36 spring barley trials, 24 were designed with five levels of fertilizer nitrogen ranging from 0 to 160 kg per ha with equal increments of 40 kg. In the remaining 12 trials a sixth application of 200 kg nitrogen per ha was added. The average

Table 5.4 Average MSE in Spring Barley Trials with Varying N-Applications

24 Trials with 5 N-application levels			12 Trials with 6 N-application levels		
Function	Avg. MSE	t-test	Function	Avg. MSE	t-test
Linear	17.76	A	Linear	23.76	A
Mitsch./plateau	10.19	B	Cobb-Douglas	11.46	B
LRP-Liebig	9.40	B	Mitsch./plateau	10.24	B
Cobb-Douglas	8.99	B	LRP-Liebig	9.96	B
Quadratic polynom.	8.07	B	Sq. root polynom.	9.92	B
Cobb-Douglas/plat.	8.06	B	Mitscherlich	9.92	B
Quad./plat.- top/left	8.02	B	Quad./plat.- top/left	9.53	B
Quad./plateau - top.	7.83	B	Cobb-Douglas/plat.	9.32	B
Mitscherlich	7.78	B	Quad./plateau - top.	9.29	B
Cubic polynomial	7.76	B	Quadratic polynom.	9.28	B
Sq. root polynom.	7.74	B	Cubic polynomial	8.73	B

* Means with same letter are not significantly different (95% level)

MSE is lower for all models when only five levels of nitrogen are considered. The ranking differs to a certain extent between the two groups of trials. It was expected that the higher number of nitrogen applications would favor the plateau models, and the table does generally display an improved ranking. The square root polynomial, which scored best in the group with five N-levels, falls to number seven when six N-levels are applied. However, in the light of the small differences between means in the models, not much emphasis can be attached to changed ranking between the two groups of trials.

Across the models, the more narrow domain of nitrogen application results in average figures for the yield maximizing nitrogen amount that are lower compared with averages for trials with 6 nitrogen applications. The same is true regarding the calculated nitrogen application corresponding to the economic optimum. By going from six to five observations the decrease in yield is generally relatively modest. This plateau tendency does not comply well with models like the symmetric quadratic functional form.

Removing the sixth observation improves the fit of the quadratic curve. At the same time, the curve becomes narrower and more sharply peaked and the calculated optimum nitrogen application falls. The same effect is seen for the plateau versions of the quadratic model. In the case of the square root model the removal of part of the "plateau" from the observation data points effects the shape of the fitted function so that calculated optimum nitrogen application increases dramatically.

Jonsson (1974) found similar tendencies when fitting quadratic and square root functions to Swedish cereal trials when he deleted the upper or the two upper out of total seven application levels. Sparrow (1979) and Cerrato and Blackmer (1990) report similar effects. When the number of nitrogen applications increases and the highest level of nitrogen increases, greater variation among model fit is observed. It is therefore important that the domain of nitrogen application and the number of individual levels be sufficiently large to span the yield-maximizing domain of N. Similarly, it is important that a sufficient number of observations allow for evaluation of curvature at the lower end of the scale. This is especially true when attempting to predict the functional relationship to the left of the lowest nitrogen application where only soil sources of nitrogen are represented.

An evaluation of how trial design affected results was also attempted on the basis of the sample of 27 wheat trials with climate data (see Table 5.5). All 27 trials were treated with 6 levels of nitrogen fertilizer ranging from 0 to 250 kg per ha in equal increments of 50 kg. The effect of trial design was assessed by removing the highest nitrogen level observations, fitting the functions, and repeating the goodness-of-fit calculation on the basis of the truncated data sets.

Table 5.5 Average MSE in 27 Winter Wheat Trials with Varying N-Applications

5 N-application levels, without 250 kg			6 N-application levels as in actual trials		
Function	Avg.	t-test	Function	Avg. MSE	t-test
Linear	37.64	A	Linear	50.53	A
LRP-Liebig	15.32	B	LRP-Liebig	17.31	B
Cobb-Douglas	14.41	B	Cobb-Douglas	15.73	B
Cobb-Dougl./plateau	14.35	B	Quadratic polynom.	13.76	B
Mitsch./plateau	13.29	B	Square root	13.49	B
Square root	13.14	B	Mitsch./plateau	13.16	B
Mitscherlich	12.67	B	Quad./plat.- top/left	12.95	B
Quadratic polynom.	12.51	B	Cobb-Dougl./plateau	12.84	B
Quad./plat.- top/left	12.35	B	Mitscherlich	12.62	B
Cubic polynomial	12.34	B	Cubic polynomial	12.36	B
Quad./plateau - top	12.27	B	Quad./plateau - top	12.34	B

* Means with same letter are not significantly different (95% level)

As was the case with spring barley in Table 5.3, fewer observations lead to a decrease in average MSE for almost all functional forms. Also, a general narrowing of the difference between functions takes place. A few shifts occur in the ranking of functions, but no firm conclusion about the effect of changed trial design can be made on the basis of average MSE data. Additional calculations of the general impact on maximizing and optimizing nitrogen amounts by truncating the nitrogen application domain display the same tendencies as were explained above in relation to spring barley.

Goodness-of-fit as defined on the basis of MSE is an important, but not the only, criterion of selecting the best functional form(s). A few of the examined functions, e.g. LRP-Liebig and Cobb-Douglas, tend to be ranked low. However, in general they can not be distinguished from other functions, which are very close regarding statistical fit to observed data. Other behavioral characteristics like the ability to yield adequate estimates of yield maxima, economic optima, and mineralized nitrogen are considered in the following sections to broaden the basis for comparing the different functional forms.

5.1.2 Some Behavioral Features of Different Functional Forms

The quadratic function is generally well behaved. The estimated coefficient, $\hat{\beta}_2$, (the quadratic effect) is always negative indicating concavity for all trial data sets. In three cases, where the observed data approach linearity, $\hat{\beta}_2$ was not significant at the 95 percent level, and the calculated yield maximizing and yield optimizing nitrogen application –N_{max} and N_{opt} – seem unrealistically high. The same is the case with estimated total nitrogen mineralization, NS, which is based on the negative N-axis intercept. In general, the regularity of the quadratic function makes it possible to establish realistic estimates of N_{max}, Y_{max} (maximum yield), N_{opt}, Y_{opt} (yield at economic optimum), and NS. The cubic function is more complex as was explained in Chapter 4. In only 22 of the total of 84 trials did the cubic coefficient $\hat{\beta}_3$ obtain a negative sign.[7] Local stationary values did occur in all situations so that a growth segment could be displayed in the domain of observed data. Only five of the 22 cases displayed an inflection point found in the domain of total nitrogen supply (including the estimated mineralization

[7] To reveal a classic three-stage text book pattern with decrease as N gets larger requires $\hat{\beta}_3$ to be negative. However, if $\hat{\beta}_3$ is positive it is still possible to have Stage II and Stage III revealed. Good fits to data may be obtained in both situations, but the fitted function is not necessarily well behaved in the vicinity of the observed data domain.

domain) so that a three-stage production function could be revealed. In the other 17 cases the local minimum was located at high negative nitrogen axis values so that strict concavity prevailed in the nitrogen supply domain. In seven of the 22 trials with negative cubic coefficient there was no intercept with the negative part of the nitrogen axis, and an estimate of total mineralization was therefore not displayed. If a three-stage performance were the relevant response pattern, this could possibly be revealed with additional data points at the low level of the nitrogen application scale.

In the other 62 trials, where the estimated cubic coefficient was positive, the fitted cubic function had no local stationary values in 27 situations so that no N_{max} and Y_{max} values could be estimated. Economic optimum based on present price conditions (scenario 2 in Table 4.6) could not be established in 18 out of the mentioned 27 trials. This demonstrates that, although the cubic function has great flexibility and fitting ability in the domain of observed data, it often does not comply well with agronomic or economic theory. The cubic function was not satisfactory in many cases and a good deal of caution must be exerted when attempting inferences about functional relationships outside the data domain. This does not preclude that appropriate N_{max} and N_{opt} results can be obtained in those situations where a cubic specification is meaningful. This is probably the reason why the cubic function has been the model of choice for purposes of setting N-application policy in Denmark.

When fitting the square root polynomial, parameter signs are significant and consistent with concavity in all trials. In about half of the trials the fitted square root polynomial displays estimated N_{max} values, which – in many cases dramatically – exceed maximum nitrogen application. This influences the function's ability to yield adequate estimates of other key variables like N_{opt}, on a consistent basis. It is reasonable to expect that the experts by design of trials have defined nitrogen applications in a way so that yield optimum and maximum can normally be found within the highest level of nitrogen application. The steep descent of the square root curve to the left results in very low estimates of mineralization. In 26 out of 46 trials with information about plant available mineralized nitrogen at the beginning of growing season (N1), the estimated mineralization during the growing season (N2) is negative. Negative net mineralization can occur under certain circumstances, e.g. by extreme climate situations or when the bacterial erosion of large amounts of organic material binds free nitrogen. However, given the climatic data, soil type, and crop rotation negative mineralization is unlikely in any of the selected Danish field trials. Therefore, the square root function is deemed a less adequate model for the purposes of the present analysis.

By definition, the Cobb-Douglas without plateau displays no yield maximum. As a result calculated N_{opt} tends to be unrealistically high in many trials, as was the case with the square root function. Further, negative N2 values were found in 37 out of 46 trials for which N1 can be established. Taken together with the relatively poor statistical fit (see Section 5.1.1) it can be concluded that the Cobb-Douglas function does not satisfy the general requirements.

With the Mitscherlich function, which asymptotically approaches a maximum yield, there is no N_{max}. The level of the asymptote is higher – although not significantly higher – than the yield maxima found with the cubic and quadratic polynomials and the plateau functions. Estimates of N_{opt} and Y_{opt} values for the Mitscherlich are also on the high side but in most cases not significantly different from the averages provided by the polynomial and plateau functions. The Mitscherlich function's steep descent to the left results in relatively low NS estimates, and even a small number of negative N2 estimates. The Mitscherlich function displays relatively good fits to data with statistically significant parameter estimates throughout. Certain modifications of the basic formula, e.g. as suggested by Burt (1995), who introduced polynomials of the independent variable instead of the simple independent variable in the exponent specification, could possibly correct for its somewhat high estimates of optimum values and low N2 estimates.

In nine out of ten trials the Mitscherlich plateau version yields estimates of optimal values and mineralized nitrogen that are comparable with results of the Mitscherlich function without plateau. However, in the plateau version the estimated N_{max} at the knot point assumes values that in almost all trials exceed the maximum nitrogen application dramatically. This indicates that the plateau feature does not add to the overall performance of the general Mitscherlich function. The Mitscherlich with plateau does not provide additional relevant information.

The LRP-Liebig function displays statistically significant parameter estimates in all trials but compared with other functional forms the MSE fit is relatively poor, especially in wheat trials. Further, linearity of the growth segment results in calculated nitrogen axis intercepts, which are considerably lower than in the curvilinear functions, i.e. the estimated nitrogen mineralization is higher. Residual plots for the LRP-Liebig curve, which is shown together with all other functions in Appendix B, Section B4, displays a tendency to overestimate yield around the calculated optimum. Yield is underestimated in the middle section of the growth segment. Similar deviations were found in the plot for a strictly linear function. For polynomials and curvilinear plateau models the deviation pattern is more evenly distributed on negative and positive deviations over the whole nitrogen application domain. When forcing linearity upon apparent curvilinear observation patterns, the result is systematic lower values of yield and profit maximizing nitrogen application levels. The results do not suggest that LRP-Liebig is superior to other (plateau) functions when fitted to barley trials. When fitted to winter wheat data the LRP-Liebig is inferior to other functional forms.

The Cobb-Douglas with plateau displays fairly reasonable results in most analyses. Profit maximizing quantities of applied nitrogen tend to be lower than those found in other curvilinear plateau functions. Also, estimated N2 figures are relatively low although higher than in the Cobb-Douglas function without plateau.

The quadratic with plateau yield ranks well in terms of statistical fit. The plateau model with forced onset of the plateau at the estimated parabola maximum, QP1, displays estimates of yield maximizing and profit maximizing quantities of applied nitrogen that in most cases are smaller than and never exceed corresponding estimates based on the simple quadratic function. As expected, the yields corresponding with N_{max} and N_{opt} are only moderately lower, and consistently never higher than in the simple quadratic model. Estimated total availability of mineralized nitrogen is also lower in the plateau version because the estimated parabola becomes narrower, when the plateau captures observations in the flatter curve segment.

In the other quadratic/plateau version, QP2, where the plateau can start to the left of the estimated parabola maximum, N_{max} and N_{opt} are generally lower than in QP1. The estimated parabola is generally wider. As a result, estimated availability of mineralized nitrogen tends to be higher than in QP1. Estimated yield maximum and yield at economic optimum are very similar in the two plateau models. In QP2, as in QP1, the coefficient of the squared independent variable is negative in all trials indicating concavity in the growth segment of the function. However, in 28 trials this coefficient is not statistically significant for QP2, i.e. the function fit approaches that of the LRP-Liebig form. In almost all of these cases the plateau onset is well to the left of the parabola top. Therefore, the domain of the growth segment typically only comprises three data points and only the steepest segment of the parabola, which adds to the uncertainty of the curvature estimate. QP2 also produces a number of key variable values, which fall outside the nitrogen application domain. A larger number of N-applications in the domain of yield growth could possibly improve the statistical fit. On this background more emphasis will be placed on QP1 when making inferences based on quadratic/plateau models.

5.1.3 Estimation of Optimal Nitrogen Application

Prediction of economically optimal nutrient application is a main goal for yield response research. Table 5.6 shows the average nitrogen application estimated by fitting different functional forms to all 84 field trials. A distinction between crops is necessary because of general differences in yield. Winter barley is omitted in the comparison because of few observations. The Mitscherlich with plateau, the Cobb-Douglas, and the square root functions are not considered here for reasons mentioned in Section 5.1.2. Outliers – defined as averages exceeding 120 percent of the highest level of nitrogen application in each trial – are not considered. If the estimated optimal nitrogen amount is negative a zero is assigned to the pertinent trial. The numbers in brackets next to function names show how many individual trial results are included in each average calculation. Optimal nitrogen application is based on Scenario 2 in Table 4.6, that is price levels of

Table 5.6 Estimated Optimal Nitrogen Application, All Trials
Kg Nitrogen per Ha

Spring Barley, 36 Trials		Winter Wheat, 40 Trials	
Function	N_{opt}, Kg/Ha	Function	N_{opt}, Kg/Ha
Quadratic (34)	139	Mitscherlich (29)	214
Quad./Plateau 1 (34)	129	Quadratic (38)	201
Cobb-D./Plateau	126	Quad./Plateau 1 (38)	188
Mitscherlich (24)	122	Cubic (28)	186
Cubic(29)	121	Quad./Plateau 2 (39)	178
Quad./Plateau 2 (35)	117	Cobb-D./Plateau (39)	174
LRP-Liebig (36)	103	LRP-Liebig (40)	131

Note: Numbers in parenthesis are lower than the total number of trials due to omission of outliers or no solutions.

nitrogen and cereals as determined by markets and policy instruments in the period following full implementation of the 1993-Reform of the common agricultural policy (CAP) in the European Union.

The LRP-Liebig displays the lowest optimal amount of nitrogen in both spring barley and winter wheat trials. In spring barley trials the quadratic function yields an average figure, which is significantly higher than results of the other functions. Mitcherlich is highest for winter wheat trials. The ranking of individual functions is somewhat different between barley and wheat trials. The cubic function tends to be in the middle with QP1 showing a higher and QP2 a lower nitrogen optimum. To reduce uncertainty caused by a possible yield trend over the time period covered by the analysis an alternative calculation was based on the smaller sample of trials for which information about N1 was available. These trials were executed during the last half of the total period 1987-1998, see Section 4.2.1. The results in Table 5.7 show that the ranking of functional

Table 5.7 Estimated Optimal Nitrogen Application, Trials with N1 Data
Kg Nitrogen per Ha

Spring Barley, 10 Trials		Winter Wheat, 30 Trials	
Function	N_{opt}, Kg/Ha	Function	N_{opt}, Kg/Ha
Quadratic (10)	173	Mitscherlich (23)	217
Mitscherlich (6)	169	Quadratic (29)	199
Quad./Plateau 1 (10)	166	Quad./Plateau 1 (29)	185
Cubic(8)	151	Cubic (21)	185
Quad./Plateau 2 (10)	142	Quad./Plateau 2 (30)	177
Cobb-D./Plateau	141	Cobb-D./Plateau (30)	172
LRP-Liebig (10)	124	LRP-Liebig (30)	126

Note: Numbers in brackets are lower than total number of trials due to omission of outliers or no solutions.

forms does not change between the barley and wheat calculations except for a switch of positions between Mitscherlich and QP1. The absolute level of nitrogen optimum in later years is considerably higher than the average for the whole period in the case of spring barley. Only 10 out of 36 barley trials refer to the last part of the period, and the figures do indicate an increase over time in N-optimum. The same effect is not apparent for winter wheat trials. Further details are available in Appendix C.

Similar estimations from some of the studies that were cited in Chapter 3 are shown in Table 5.8. Even if different crops and geographic sites etc. prevent a meaningful comparison of absolute figures between the individual examinations, it is interesting to compare the ranking of functions with the results of the present study. There is a general tendency for the quadratic model and the Mitscherlich model to yield high optimum nitrogen amounts. The quadratic with plateau was also relatively high in Cerrato and Blackmer's (1990) and Bullock and Bullock's (1994) examinations, but closer to average in Olesen's (1999) study. The cubic form, which was included in Olesen's study, had a medium to high placement. The LRP- Liebig model invariably displays the lowest estimate. The square root function, which was tested in three of the quoted examinations, results in very high N-optimum in Cerrato and Blackmer's (1990) study and it was also in the top of the scale according to Olesen (1999). Anderson and

Table 5.8 Calculated Economically Optimal Nitrogen Application, Kg per Ha

Model:	Andersson et al. 1975 Corn	Boyd et al. 1976 Barley	Cerrato et al. 1990 Corn	Frank et al. 1990 Corn	Bullock et al. 1994 Corn	Olesen 1999 W. Wheat
Quadratic	166	138	225	235	211	149
Quad./Plat.			184		182	125
Square root	139		379			147
LRP-Liebig	102	95	128	112		108
Mitscherl.		160	252	178		146
Cubic						141

Nelson's (1975) calculations based on corn trials placed the square root towards the middle of the scale. Taken as a whole the results found in the present study are consistent with findings of previous studies.

Olesen's (1999) winter wheat results in Table 5.8 suggest a considerably lower optimal-N than those found in the present study (Table 5.6 and 5.7) despite the fact that the same source of empirical information was used. The main reason for the difference is probably that Olesen used a broader sample of winter wheat trials, which includes soil types of lower quality. Further, Olesen includes trials with preceding crops other than cereals, which reduced the need for fertilizer-nitrogen application.

Plantedirektoratet[8] (1999) has fixed the Danish maximum norm at 183 kg nitrogen per ha in winter wheat grown after cereals crops on the same soil type as used in this study. The figure is based on optimum minus ten percent found by fitting the cubic function to field trials that were executed over the last ten years. The calculated optimum is therefore 203 kg per ha. The corresponding result in this study is lower, about 185 kg per ha. The reason for this difference is most likely a less strict correction for outliers in the official norm calculation than in this study. If a calculated optimum exceeds the highest nitrogen application level by more than the difference between the two highest application levels, the optimum is set at the limit in the official norm calculation. In the present study such observations are omitted. Also, in certain cases where it is not meaningful to fit a cubic function, the quadratic may be chosen as a substitute. Since the

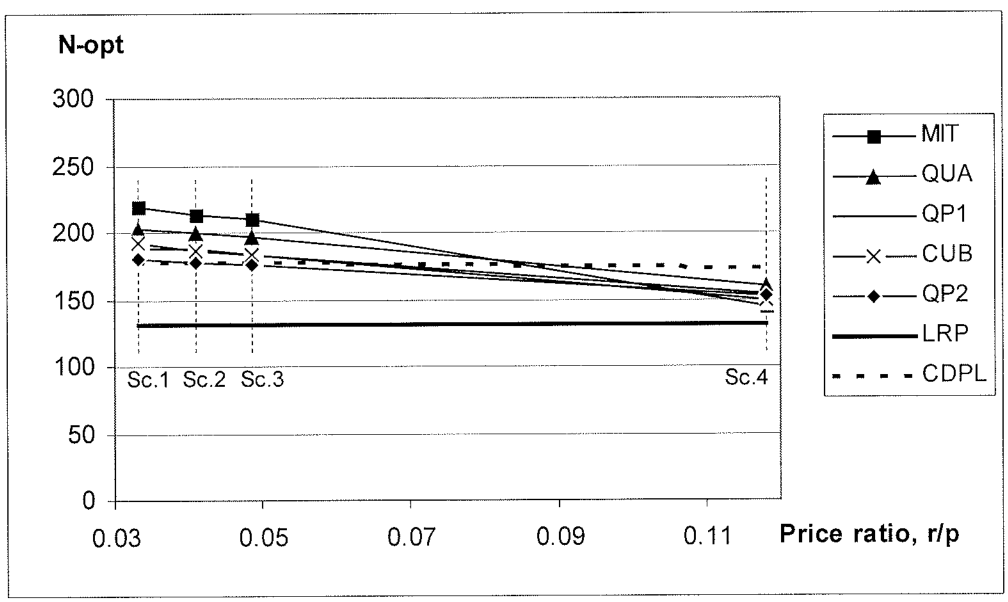
[8] Plantedirektoratet, which is an agency of The Ministry of Food and Agriculture, is responsible for administration and control of various legislative environmental measures in relation to crop production.

quadratic tends to result in relatively high N_{opt} estimates this procedure can result in a certain upward correction of optimum.

Clearly the choice of model has a significant impact on the calculated optimum nitrogen application. This is an important issue for recommendation of fertilization norms. Figure 5.1 shows the sensitivity of the optimum nitrogen application solution for different functional forms when the unit price ratio, r/p, between nitrogen and grain varies. The four price scenarios are taken from Table 4.6 and, in the example, the figures are averages from fitting the different functions to all 40 winter wheat trials. In Scenario

Figure 5.1 Nitrogen Application at Economic Optimum, Kg Nitrogen per Ha

By Functions and Nitrogen/Cereal Price Ratios
Functions Fitted to all 40 Winter Wheat Trials



4, where a significant nitrogen levy is assumed, the calculated optimal nitrogen application is quite similar for all functional forms. LRP-Liebig displays a lower N-optimum than the other functions. LRP-Liebig is unaffected by price ratio changes between the different scenarios, because the optimum solution is stable at the knot-point of the two linear curve segments. The Cobb-Douglas with plateau displays the same feature as the LRP-Liebig because the growth segment is relatively steep. The quadratic/plateau model with possible onset of plateau to the left of the parabola maximum is the least price sensitive of the other functions. The optimal nitrogen amount changes considerably more in the Mitscherlich function, and the other functional forms fall in between. The price ratio representing the nitrogen levy alternative is far removed from the non-levy alternatives, which reflect changes in cereals policy in the European Union over the last decade. The trend in N-optimum as the r/p ratio increases, which is seen over the whole range of price ratios in Figure 5.1, is discernible also over the narrower domain of Scenarios 1 through 3. Scenario 4 is pertinent to farmers who do not register and respect environmental legislation involving maximum norms for nitrogen application. Unlike complying farmers they must pay a nitrogen levy of DKR 5 per kg nitrogen applied.

Cerrato and Blackmer (1975) compared their models fitted to corn trial data under the assumption of different price ratios as shown in Table 5.9. In the LRP-Liebig model optimum nitrogen application is stable at the knot point for all price ratios because the slope of the pre-plateau curve exceeds price ratios. Quadratic and quadratic with plateau show significant sensitivity by variation of the r/p price ratio. At a low nitrogen-corn price ratio the square root model and the Mitscherlich model result in unrealistically high nitrogen optima, which is caused by modest curvature of the response curve in the segment where its slope is near the numerical value of the inverse price ratio. The

Table 5.9 Predicted Optimum Fertilization of Corn, Kg Nitrogen per Ha
Source: Cerrato and Blackmer (1975)

Nitrogen-corn price ratio:	LRP-Liebig	Quad./Plateau	Quadratic	Mitscherlich	Square root
2	128	190	233	296	490
4	128	182	221	237	347
6	128	174	208	203	261
8	128	165	196	179	205
10	128	157	184	160	163

quadratic and the quadratic with plateau have steeper pre-maximum slopes, which yields a less dramatic change in calculated nitrogen optimum when the price ratio changes. These results are generally in concert with the results of this study.

5.1.4 Estimated Mineralization of Soil Nitrogen

Estimates of plant available soil nitrogen are established on the basis of function intercepts with the nitrogen axis as explained in Chapter 4. This procedure involves predictions outside the observed data domain. Therefore, the estimates are encumbered with uncertainty, which should be borne in mind when evaluating the calculation results (see also the caveat about this question in Chapter 4, Section 4.1.2). Lack of relevant information justifies this attempt to generate mineralization data. The results also serve as a basis for comparing the general behavioral characteristics of different functional forms. For the whole sample of 84 trials, estimates were made of total soil nitrogen availability (NS) for each trial. In 46 trials dating from the last half of the period, available mineralized nitrogen at the beginning of the growing season (N1) is measured in soil samples. An average for the whole trial site represents all plots in the trial. Mineralized nitrogen during the growing season (N2) is estimated as the difference between the two (N2 = NS – N1). Both NS and N2 are considered in the following. Special interest is attached to 41 of the 46 trials. For the 41 trials both N2 and information about climatic conditions can be established. Chapter 6 considers, whether a connection can be made between N2 fluctuations and variation in climatic conditions.

Table 5.10 shows that estimates of plant available soil nitrogen vary considerably among functional forms. Except for the LRP-Liebig, which displays average figures far above all other functions, averages are similar when comparing spring barley and winter wheat trials. The square root, the Cobb-Douglas and the Cobb-Douglas with plateau functions yield low estimates. Further information about NS-estimates by crops and functions is available in Appendix D, including a simultaneous test of the average differences between functions.

Table 5.10 Estimated Total Available Mineralized Nitrogen, Kg/Ha
Total Sample of Field Trials

Spring Barley, 36 Trials		Winter Wheat, 40 Trials	
Function	NS = N1 + N2	Function	NS = N1 + N2
LRP-Liebig	164	LRP-Liebig	134
Quadratic	93	Quadratic	91
Quad./Plateau 2	91	Quad./Plateau 2	87
Quad./Plateau 1	84	Quad./Plateau 1	82
Cubic	73	Cubic	72
Mitsch./Plateau	55	Mitsch./Plateau	57
Mitscherlich	55	Mitscherlich	57
Square Root	33	Cobb-D./Plateau	31
Cobb-D./Plateau	30	Square Root	29
Cobb-Douglas	12	Cobb-Douglas	13

Note: Estimates reflect total availability of mineralized soil nitrogen as a sum of available nitrogen at beginning of growing season and further mineralization during the growth season.

Mineralization during the growing season is shown in Table 5.11 in comparison with the estimated total availability of mineralized soil nitrogen for the trials where N1 information is available. The NS averages for winter wheat are only slightly lower than in Table 5.10 when disregarding the LRP-Liebig. In contrast, estimates based on the 10 spring barley trials are considerably higher than average for all barley trials in Table 5.10 and also higher than the estimates for wheat trials. This difference may be caused by special circumstances related to spring barley trials from the last part of the analyzed period. The Cobb-Douglas model shows negative average mineralization during the growing season (N2) for both barley and wheat trials. Also the results of the square root and the Cobb-Douglas with plateau models are low and even negative in wheat trials. The LRP-Liebig suggests very high estimates of nitrogen mineralization during the growing season. Recall that the LRP-Liebig scored relatively low for statistical fit of the observed

Table 5.11 Estimated Mineralization during Growing Season, Kg N/Ha
Trials with N1 Information

Spring Barley, 10 Trials			Winter Wheat, 30 Trials		
Function	NS	N2	Function	NS	N2
LRP-Liebig	191	155	LRP-Liebig	121	92
Quad./Plateau 2	121	85	Quadratic	85	55
Quadratic	110	73	Quad./Plateau 2	80	50
Quad./Plateau 1	103	67	Quad./Plateau 1	76	46
Cubic	85	47	Cubic	66	37
Mitsch./Plateau	83	47	Mitsch./Plateau	52	22
Mitscherlich	74	38	Mitscherlich	52	22
Cobb-D./Plateau	55	19	Cobb-D./Plateau	27	-3
Square Root	43	7	Square Root	26	- 4
Cobb-Douglas	22	-14	Cobb-Douglas	10	-19

Note: Estimates reflect total availability of mineralized soil nitrogen and mineralization during the growing season.

trial data. The two Mitscherlich functions yield almost identical results, which are relatively low compared with the quadratic functions and the cubic function.

Which functions are more reliable when it comes to determining the extent of soil nitrogen uptake can only be assessed through more extensive studies of physiological theories and scientific response experiments. For comparison, recall that Jarvis et al. (1996) reported from Scottish experiments an uptake of soil nitrogen in spring barley of 57 kg per ha. Combining Danish Farmers' Union's (1997) estimate of total nitrogen content in the soil and Olesen's (1999) estimate of annual mineralization rate yields 100 – 150 kg per ha, part of which will leach during the winter season. For the analysis of climatic influence on mineralization in Chapter 6 the number of calculation alternatives was narrowed down based in part on the ability of the various functions to provide reasonable estimates of N2.

5.2 Summary

No single functional form is clearly superior to the others when comparing goodness of statistical fit. Functions with quadratic growth specification have the highest number of first and second placements in a ranking of all functions based on MSE. The square root, the cubic, and the Mitscherlich functions are close behind the quadratic specifications. The average MSE values for curvilinear plateau models and polynomials are not significantly different when fitted to all 84 field trial data sets. The Cobb-Douglas and the LRP-Liebig are at the low end of the ranking scale. A strictly linear growth pattern, which was included for comparison, is inferior to the other functional forms.

Besides the statistical fit to observed data, the functions were also evaluated on the basis of how well they complied with agronomic theory and knowledge. The quadratic function consistently displayed decreasing MPP and it generally exhibited reasonable values for yield and profit maximizing nitrogen levels. It also displayed good ability to estimate mineralization of soil nitrogen based on its negative nitrogen-axis intercept. However, its symmetric shape with a peak does not comply well with a general expectation of relatively stable yield after maximum is reached – at least over a certain nitrogen domain. This causes the values of Y_{max}, N_{max} and N_{opt} to be on the high side. Despite its flexibility and good statistical fit the cubic function performed poorly in a number of cases, where no – or at best, unrealistic – yield and profit maximizing nitrogen application were established. Further, in a number of cases the negative nitrogen-axis intercept was unrealistically low or non-existing.

The square root model in many cases displayed unrealistically high N_{max} and N_{opt} estimates. Likewise, its mineralization estimates were often so low that the N2 estimates became negative. The same inadequacy was displayed by the Cobb-Douglas function. The Cobb-Douglas function's poor performance was underlined by its generally poor statistical fit and the fact that it exhibits infinite growth so that, by definition, it can display no maximum yield. The Cobb-Douglas with plateau function behaves better. However, its estimation of nitrogen mineralization appears low. The Mitscherlich function was generally well behaved. Its yield asymptote and its N_{max} and N_{opt} estimates were found within a reasonable range in most cases, although on the high side. Mineralization estimates resulted in negative N2 values in a few cases. The plateau version of the Mitscherlich model added no new information and did not improve the overall performance of the Mitscherlich function. The plateau versions of the quadratic models yielded slightly lower N_{max} and N_{opt} values than the quadratic function itself. All things considered they were generally found well behaved and in good concert with knowledge about growth patterns. The quadratic with possible plateau onset before the parabola maximum was fitted with some uncertainty because of the low number of observations in its growth domain. There was found no evidence to sustain a linear growth pattern in general. This together with a poor statistical fit to observed data resulted in a low evaluation of the LRP-Liebig model's general performance.

On balance, the quadratic plateau model had many attractive features among all the tested functional forms. The function's statistical fit, the adequacy of its general behavior in relation to hypothesized response patterns, and the estimates of N_{max}, N_{opt} and N2 were all taken into account.

Chapter 6

Influence of Climatic Variation on Mineralization

Chapter 5 gave indications about which functional forms best describe the relationship between cereal yield and applied nitrogen. The curve fitting to trial data also enabled estimation of nitrogen supply from soil resources at each trial site. In this chapter these estimates are combined with information about climatic conditions at the trial sites, which is available beginning in 1993. The aim is to examine the influence of climate variations on N2, that is mineralization during the growing season.

6.1 The Nature of Climate Effects

Chapter 2 established that among other factors soil temperature and soil moisture conditions influence the amount of nitrogen made available through mineralization. Climate also influences yield for a given amount of nitrogen. The photosynthetic process of building organic material in plants hinges on the amount of solar radiation. And temperature is known to influence the speed or efficiency of chemical processes that take place inside the plants. Burt (1995) showed that water supply interacts with nitrogen in determination of yield but in general, little information is available about the functional forms and estimated coefficients that describe the relationship between climate factors on one side and mineralization and yield on the other. This is partly due to a lack of coherent and relevant empirical observations about these matters. This study is concerned only with an evaluation of climatic influence on the extent of mineralization.

Figure 6.1 shows schematically how changes in mineralization caused by climatic fluctuations can influence the revealed relationship between yield and NF (applied fertilizer). The "true" response function based on NT (total nitrogen supply) is represented by the curve in its entirety in the three panels. Depending on the extent of mineralization – normal, high, or low – the estimated yield/NF relationship as revealed by the solid-line curve segment is located in different NT domains. The shape of the general yield/nitrogen curve is unchanged because plant available nitrogen from different sources has the same value. However, mineralization fluctuation causes a similar change in the actual need for fertilizer nitrogen to ensure economic optimum. Extreme situations such as exhibited in the graphic example can obviously affect the possibilities of estimating the "true" yield/NT relationship based on yield/NF observations.

In the examined trials climate induced fluctuations are believed to be less dramatic, and the observed yield/NF segment – even with different locations in the total N-supply domain – is more likely to produce appropriate and consistent estimates of the yield/NT function. The extent to which mineralization could be predicted based on variation of climate indicators was analyzed. It was assumed that climatic influence on the general shape of the productivity function would not affect the estimation of N2. It remains to be examined whether this assumption is valid. The present study therefore can only be regarded as a first step toward a more comprehensive analysis of climatic influence on yield. The examination was also a means of evaluating whether available climate data adequately reflect relevant climate factors.

Figure 6.1 Response Curves and Mineralization Fluctuations