ENVIRONMENTAL POLICY AND ECONOMIC GROWTH IN A SIMPLE ENDOGENOUS MODEL

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ABSTRACT

This paper analyzes the economic growth with the environmental asset, and explores the optimal environmental policy instruments in a simple endogenous growth model. It also gives a comparison analysis of different environmental policies. It concludes that environmental quality does affect the growth rates in the ways that depend upon how the environmental quality contributes to the economic system. Appropriate choices of environmental policy contribute to moving the market equilibrium growth to the Pareto optimum. In steady-state growths, technical standard remains constant while the effluent tax is increasing with the growth of capital accumulation, which has implications on policy implementation.

CHAPTER 1. INTRODUCTION: GROWTH AND THE ENVIRONMENT

1.1 Connecting Economic Growth and Environmental Quality

The connections between economic growth and the environment are important to both economists and policymakers. The recent decades have seen the increasing concerns on the environmental and natural resource uses in the world economies. With the development in the growth theory, the relations between environment and the economic growth are explored again after the "Club of Rome" report three decades ago. Recent developments in growth theory shed new lights on the development of models connecting these two respects. This may arises from the large divergence in the per capita GDP, in comparison with big difference in environmental quality among the countries.

Data shows that some countries maintain sustainable increases in national income and at the same time have cleaner environment. While others seem to stay in a vicious cycle of environmental quality and economic growth. Though the interactions between economy and the environment appear to be complex, it would be natural to examine them by addressing such questions as the relations between the environmental quality and long-run growth, how the environmental policy affects the growth rates, and whether it is possible to achieve sustainable growth with the appropriately designed policies.

Efforts in exploring these questions have great implications not only in helping people to better understand on the relations between environment and long-run economic growth, but also in shedding light on improving the environmental policy as an essential part of development policies. Particularly, environmental policy may even become critical in those economies where the economic growth is resource intensive or heavily dependent.

To model the long-term connections between economic growth and the environmental quality, the growth theory with the environmental sector was developed through both theoretical and empirical works.

1.2 Neoclassical vs. New Growth Models

The neoclassical growth theory is mainly referred to as the Solow-Swan growth model. This model uses neoclassical form of production functions with constant returns to scale and diminishing returns to each factor. The elasticity of substitution between inputs is assumed to be positive and smooth. With this specification, the Solow-Swan model predicts that, in the absence of the continuing (exogenous) technological change, the per capita growth rate finally ceases.

The Solow-Swan model is deficient in explaining the actual divergent growth of economies. This arises from its assumption that the technological changes occur exogenously. The long-run per capita growth rate is determined solely by the technological change rate that is outside the model.

Led by Romer (1986, 1990), Lucas (1988), Rebelo (1991) and others, the endogenous growth models were set up intending to explain such questions as the sustainable per capita growth of economies and technological progress. It provides better explanations of the Kaldor's "stylized facts" on economic growth. These models are characterized by the endogenization of technological changes, the increasing returns of physical and human capitals caused by the external effects such as learning-by-doing, and monopolistic competition of new designs.

Two key aspects label this new growth theory as "endogenous model". First, the growth per capita may not cease because the returns to capital accumulation are increasing, rather than diminishing. This may happen because of the knowledge spillovers and external effects of human capital. Second, the new models incorporate the intentional R&D and the related imperfect competition. The technological progress arises from the R&D activities, and the developers could exercise the market power on new goods and inventions.

But the resulting growth rate in the endogenous growth model seems not to be Pareto efficient because of the external effects, spillovers and the monopoly power of new inventions or methods of production. To fix this, government intervention is put in the models, and the long-run growth rate depends on the governmental actions. Taxation on capital, subsidy on researches, patent regulations, provision of public goods are examples of governmental intervention appeared in the endogenous growth models.

Lucas (1988) and Romer (1986) analyzed the socially optimal path and equilibrium path of growth by including the external effects of the average human capital or the spillovers of the total knowledge into the production functions. The new knowledge created by one firm or individual not only improves the productivity but also has positive external effects on the total knowledge in the society, and hence is an externality to other firms. That is, externalities arise from spillovers of knowledge. In their models, the production function shows constant returns to scale in physical capital and internal human capital, if taken the external effects as given. Romer stresses the role of learning-by-doing in capital accumulation, while Lucas examines the human capital production and its external effects.

Romer (1986) also proved that "If the increasing returns to scale are external to the firm, an equilibrium can exist. A finite-valued social optimum exists because of the diminishing returns in the research technology, which imply the existence of a maximum, technologically feasible rate of growth for knowledge."

Romer (1990) and Grossman and Helpman (1991) stress the central role of intentional technological change of profit-maximizing agents in modern economic growth. Technology or knowledge is treated as a non-rival and partially excludable good. The investment of private firms in R&D creates new designs from which the firm could obtain monopolistic rents. On the other hand, it also contributes to the increase of total knowledge stock in the society that would be used for later R&D. Unbounded and endogenous accumulation of new designs or patents evolves exponentially.

As shown above, the public sector and services are necessary to ensure the Pareto efficiency of the decentralized equilibrium. Barro (1990) and Barro and Sala-i-Martin (1992) incorporate the public sector and various public services (expenditure) into the endogenous models by analyzing the growth rates and savings rate impacts of government size and tax policy. They also compare the different effects of income (or output) tax and the lump-sum tax. Other literatures include Rebelo (1991), Jones and Manuelli (1990), and Jones, Manuelli and Rossi (1993), which use the linear or convex endogenous growth models showing that the endogenous growth is sustainable without the externality and increasing returns.

1.3 Models of Growth and the Environment

The examination of empirical relationships between the national income and environmental quality was done in Grossman and Krueger (1995), and Selden and Song (1994), among others. It is shown that environmental quality does not necessarily deteriorate steadily along with economic growth. The environmental quality will deteriorate in the initial phase of economic growth, then improvement follows after a certain phase of growth. Grossman and Krueger (1995) further shows that the income-environment curve displays an inverted bell shape. This implies that the economic activity could be a beneficial source of environmental quality.

However, this empirical relationship is not automatically formed. The economies with higher per capita income and better environmental quality usually have more stringent environmental policy. Therefore, the role of environmental policy can not be overlooked to shape this curve. It is possible that, the very loose environmental regulations in some lower-income countries may take much longer time even never to reach the turning point. Because the national income achieved could be lowered by the polluted environment and exhausted natural resources.

Theoretical studies on the connection between growth and environment were conducted in the frameworks of neoclassical growth model, endogenous growth models and the Overlapping Generations model (OLG). To examine the long-run growth problems, endogenous growth models are used in most previous works.

The works in the neoclassical framework, mainly by d'Arge and Kogiku (1973), Brock (1977), Forster (1973), Stiglitz (1974) and Krautkraemer (1985) explored how the environmental capacity constrains economic growth. The long-run growth rate and the saving rate are exogenously determined outside the model, and the impacts of environmental policy are unable to be dealt with. Therefore, these works can not overcome the deficiency of neoclassical model.

The OLG models (John and Pecchenino 1994; Marini and Scaramozzino 1995; and Olson and Knapp 1997) are appropriate to investigate the short run growth and how the behavior interactions between consecutive two generations shape the income-environment curve. A recent work done by Jones and Manuelli (2000) shows that, if the pollution regulation is made through collective actions, voting over effluent charges rather than over direct regulation of technology would generate an inverted bell shape curve followed by a sustained increase as described in the empirical literature. If instead, when voting is made over direct technical standard, pollution monotonically increases to a bounded level as the economy grows.

Trying to eliminate the deficiency of neoclassical models, recent works have re-examined the growth-environment issues using different approaches, mainly the linear-production model, Lucas (1988) model, Romer (1990) model and the Shumpeterian model (Aghion and Howitt 1992). Though various models are used in the literature, most focused on two aspects of growth-environment interactions. First is the sustainability and conditions of sustainability of economic growth that avoids exhausting environmental quality. The other aspect is the optimal environmental policy and the implications of policy reform on growth if the environmental policy is necessary at all.

Smulders and Gradus (1996) developed conditions under which sustained economic growth is compatible with the preservation of environmental quality, and examined the impacts of increased environmental preference on growth rates. In their model, pollution may hurt the economy, hence reducing the growth rate; while on the other hand, if the economy puts some portion of output to abate the pollution, this may give rise to increased growth.

By splitting physical capital into productive and pollution-abating capital, Cassou and Hamilton (2000) compare the efficiency of command-and-control instrument and the tax (price) control on the reproducible abatement capital. It concludes that price and quantity controls result in the same steady-state level of environmental quality, but under the price control the optimal fiscal policy levies a smaller income tax and leads to a higher economic growth rate. It further concludes that in the long run the price control is superior to quantity control in terms of their effects on output and cumulative utility.

The connections between trade, environment and growth were examined by Elbasha and Roe (1996) based on Romer (1990) and Grossman and Helpman (1991). It examines the interactions between trade, economic growth, and the environmental quality in a small open economy with two traded goods, such as the issues of the growth effects of environmental externalities and trade, the effects of trade on the environment, in both competitive (or decentralized) equilibrium and social optimum. But it does not deal with how the policy would affect the growth rate and welfare. More of significance, it does not set up explicit connections between technological change and pollution, that is, how the pollution policy would promote or retard the "cleaner technology".

Built on the works of Barro (1990) and Barro and Sala-i-Martin (1992), Bovenberg and de Mooij (1997) explored the positive impacts of environmental tax reform on economic growth, pollution and welfare in a second-best world. Stokey (1998) developed a model, which generates an inverted U-shape curve between per capita income and the environmental quality. It concludes that whether the environmental concerns will limit growth or not depends on the effects of pollution abatement on the long-run rates of return. This paper also shows that taxes and vouchers outperform direct regulations in encouraging efficient capital accumulation.

Few previous papers explicitly analyzed the optimal environmental policy in the presence of other distortionary taxes over time except Cassou and Hamilton (2000). The static general equilibrium analysis of double dividend issue is done in Bovenberg and Goulder (1996).

1.4 Purpose of This Paper

This paper extends the previous theoretical researches (especially Cassou and Hamilton (2000), Mohtadi (1996), and Gradus and Smulders (1993)) on the connections between the environment and economic growth, particularly the optimal environmental policy in the long run and the policy effects on growth rates and welfare. Using the endogenous growth model with linear production, the paper examines how environmental policy promotes economic growth in the presence of externality. The environmental tax and technical standard are examined and compared in terms of their long-run effects on growth.

This paper differs from previous analyses in the sense that it deals with two cases. In one case, environmental quality a flow variable that is affected by two types of capital stock, and in the other environmental quality is stock variable affected by two factors: emissions and self-cleaning ability of environment. Environmental quality tends to be positively growing or stay steady in balanced growth of saving and consumption in both cases. The paper considers three cases of how the environmental quality enters the consumption and production functions. However, this type of specification is not used in most literature. So, this paper also uses the production technology where pollution is taken as input and the environmental quality is a stock variable that depends on pollution level.

In order to have reduced forms of solutions, the simple *AK* endogenous growth model is used in the analysis in addition to the general specifications of production technology and preferences. The *AK* model shows constant returns to broad capital. When environmental quality is incorporated, the *AK* model shows increasing returns to scales.

Due to mathematical intricacy of dynamic models, this paper considered only the steady-state analysis, other than also including discussions of transitional dynamics.

The paper concludes that environmental quality curtails or promotes the economic growth depending on its evolution over time. Appropriate choices of environmental policy instruments enable the competitive equilibrium growth to be Pareto optimal, hence improving the social welfare. In this sense, environmental policy is an indispensable component of sustainable growth.

In steady states, the technical standards are constant or independent of the level of economic variables, but the effluent tax is increasing at a rate proportional to the steady-state growth rate of the economy. In addition, both the optimal technical standard and the optimal tax rate are determined by the price level in capital market. This raises the implementation problems of time consistency and lag of environmental policy.

The rest of this paper is structured as following: Chapter 2 first presents the basic model of endogenous growth with the environmental sector, then the *AK* model is used to compare the decentralized competitive equilibrium and social optimal paths in the presence of environmental quality. Chapter 3 analyzes the optimal environmental policies to realize socially optimal growth paths. The efficiency of different policy instruments is examined for environmental tax and technical standard. Chapter 4 examines the optimal environmental tax when environmental quality is treated as stock variable and determined by emissions and self-cleaning. Chapter 5 concludes the paper and puts forward the needs of further research that are of significance.

CHAPTER 2. ENDOGENOUS GROWTH WITH THE ENVIRONMENTAL SECTOR

2.1 Basic Model of Infinite Time Period

A basic point to connect the growth and the environment is that the environment is one indispensable source of growth. The human activities pose negative impacts on the natural environment by exploiting the natural resources and polluting the environmental media such as water and atmosphere. These impacts feed back to the human activities from the environment in the forms of more scarce resource stocks, and poorer quality of services. The polluted environment has two aspects of negative impacts: reducing productivity and reducing amenity value of environment.

The economy to be examined is perfectly competitive in goods market and capital market, and is composed of one representative consumer and one representative firm. There is only one consumable good and one type of capital. Capital is owned by the consumer, and rented to the firm for production. There is no population growth, and the population size is normalized to one, so the aggregate consumption and capital is equivalent to the per capita consumption and capital level.

The qualities of water, air, soil and others are considered in one term as a general variable of environment. Production is the only source of pollution, thus of environmental quality. Being indivisible, the general environmental quality contributes to both consumption and production.

With the above assumptions, the environment-growth model is specified as follows:[1]

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[1] Throughout this paper, the variables final output, capitals, consumption, environmental quality, emission tax, Lagrangian multiplier and instantaneous utility are all functions of time. The time argument, t, is omitted for each of them.

$$Y = F(K_y, E), \tag{1}$$

$$E = E(K_a, K_y), \tag{2}$$

$$\dot{K} = Y - c = F(K_y, E) - c, \tag{3}$$

$$K = K_a + K_y, \tag{4}$$

$$U_0 = \max \int_0^\infty e^{-\rho t} U(c, E) dt. \tag{5}$$

This specification does not differ significantly from most previous works, and it follows the framework of the representative agent model.

Production:

The representative firm uses Cobb-Douglas technology to produce final output Y, using capital stock K_y and environmental quality E as inputs. The environmental quality is exogenous to the firm under competitive equilibrium. $F_{K_y} = \partial F / \partial K_y > 0$, $F_E = \partial F / \partial E > 0$, $F_{K_y E} = \partial F_{K_y} / \partial E \neq 0$.[2]

The growth rate of final product is

$$\frac{\dot{Y}}{Y} = \frac{F_{K_y} K_y}{F} \frac{\dot{K}_y}{K_y} + \frac{F_E E}{F} \frac{\dot{E}}{E}. \tag{6}$$

It is seen that the final product growth rate is determined by the growth rates of capital and environmental quality, and the elasticities of output in capital and environmental quality. In steady states, the balanced growth is ensured by assuming constant elasticities. One example of this type of production is Cobb-Douglas technology.

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[2] Hereafter, the variables with capital, consumption, and environmental quality as subscripts stand for the partial derivatives with respect to them respectively.

One difference of this model from most previous ones is that the pollution control is realized through the using of abatement capital which is a stock variable other than a flow. The rationale for this is the reasonable observations on the production process. Not surprisingly, firms use capital to produce final output. Pollution is a side-product, other than input. Environmental quality, which is determined by the pollution level, contributes to production through affecting the quality of capital. For instance, a firm needing clean water for production needs to treat the polluted water first.

The final output in each period is allocated between consumption for current period and capital saving for future production. The representative individual needs to allocate the capital saving to production and pollution abatement. Under decentralized competitive equilibrium, the capital allocated to the abatement tends to zero, since the environmental quality is exogenous to the agent. This causes environmental quality to decline till its level is below certain critical value. Therefore, the intervention of government in promoting pollution abatement plays crucial role to guarantee sustainable economic growth. How the governmental intervention works is examined in Chapters 3 and 4.

Environment:

The environment is deteriorated due to the emissions as a by-product accompanying production, and emissions are due to the use of polluting capital in the production process. Here pollution is considered as a public "bad" of social production, not as the input of specific firms though there exists a functional relation between output and pollution. The use of dirty capital causes environmental quality to decrease, and the society needs to use abatement capital to stop or prevent this decrease. Environmental quality is considered as a flow variable, which is determined by the productive capital stock K_y and abatement capital stock K_a levels. The society must trade-off the use of capital in order to achieve sustainable growth. On the contrary, the potentially declined environmental quality affects both the consumption and production. The environmental quality function satisfies $E_{K_y} < 0$, $E_{K_a} > 0$, and $E_{K_y K_a} \neq 0$.[3]

By this assumption of environmental quality, if the economy invests zero level in any capital, the environmental quality will be zero. This appears strange, but it is not. Because economic growth is characterized by the changes of consumption, the need of consumption requires production in every period of growth. So, there must be non-zero dirty capital investment. Abatement capital might be zero in a competitive market. If there is no abatement capital, the whole economy disappears. However, this does not happen if at least in one period the society invests abatement activities, because environmental quality is determined by capital stocks, not capital flow. Without capital depreciation, once abatement investment occurs even only for one time, abatement capital will be not zero again in the later periods of growth. On the other hand, the real world is that almost no economy has no abatement activity in the process of industrialization, which requires physical and human capital accumulation. In most economies, the industrialization process is usually accompanied by implementation of environmental regulations. In this sense, our specification is a good approximation of the real world.[4]

The growth rate of environmental quality is

$$\frac{\dot{E}}{E} = \frac{E_{K_a} K_a}{E} \frac{\dot{K}_a}{K_a} + \frac{E_{K_y} K_y}{E} \frac{\dot{K}_y}{K_y}. \tag{7}$$

[3] Taking environmental quality as a flow variable allows the model to be simpler and more tractable. It could also be specified as a stock variable. But the results derived do not change qualitatively, see Smulder and Gradus (1996). We consider this case in Chpater 4.

[4] But the initial abatement investment is zero in the competitive equilibrium. So, there is no optimal competitive equilibrium without government intervention by our specification.

Similarly here, the elasticities of environmental quality with respect to two types of capital play decisive roles in determining growth rate of environment. In steady states, environmental quality may increase, decrease or stay constant, but steady-state growth is only realized when the elasticities are constant.

From the perspective of how the pollution affects environmental quality, we impose a relation between the average environmental quality and average emission level in the economy, that is, $E = D^{-1}$, where D is socially average emission level. In the economy with identical representative agents, D is obviously also the firm's emission level. By this relation, we mean that the marginal contribution of pollution on environmental quality is diminishing.

Consumption:

The representative consumer maximizes the infinite life time utility. In each period, $U(c,E)$ is the instantaneous utility at time t, ρ is the time discount rate which indicates the consumer's impatience. Assume the utility function is strictly concave in consumption c and the environmental quality E, $U_c(c,E) > 0$, $U_E(c,E) > 0$ and $U_{cc}(c,E) = \partial^2 U(c,E)/\partial c^2 < 0$. Consumption and environmental quality are assumed to be weakly separable, $U_{ce} > 0$.

As will be shown later, the elasticity of inter-temporal substitution of consumption must be constant and less than unity to ensure balanced growth. Such class of utility functions is called the isoelastic utility functions.

Sustainable Growth:

We assume that the environmental quality has a lower bound critical value, below which the economy clashes. As discussed before, the decentralized competitive equilibrium tends to deteriorate the environmental quality below the critical value, because the environmental quality is exogenous to the consumer and firm, and there is no incentive for investment in abatement.

By sustainability, one means that the economic growth does not deteriorate the environment in the long term. A benevolent social planner solves the optimal control problem (1)-(5) in order to ensure sustainable growth.

As shown in Smulders and Gradus (1996), by equ. (3), (6) and (7), the feasible conditions for constant positive growth rates of capital and output are:

(1) The elasticities of production in capital and environment are positive;

(2) The elasticity of environmental quality in abatement capital should be larger than or equal to its elasticity in dirty capital;

(3) Saving rate (capital-output ratio) and consumption rate are constant;

(4) The environmental quality level exceeds the critical value.

A steady state in the social optimum for this problem is defined as: the consumption, capital saving and final product evolve according to constant rates; the consumption-output ratio, saving rate, abatement-productive capital ratio, and the environmental quality maintain constant; the environmental quality is above critical value.

From the current-value Hamiltonian, the first order necessary conditions for problem (1)-(5) is obtained:

(a) $U_c = \lambda$, where λ is the co-state variable and the shadow price of capital;

(b) growth rate of consumption $\frac{\dot{c}}{c} = \frac{1}{\Psi}\left(F_{K_y} + E_{K_y}\frac{\partial c}{\partial E} - \rho\right)$, where $\Psi = -\frac{U_{cc}c}{U_c}$ is the elasticity of inter-temporal substitution of consumption;

(c) $\left(\frac{\partial U}{\partial E} + \frac{\partial U}{\partial c}\frac{\partial F}{\partial E}\right)\frac{\partial E}{\partial K_a} = 0$;

(d) $\dot{K} = F(K_y, E) - c$.

The transversality condition for this problem is $\lim_{t\to\infty} e^{-\rho t}\lambda_t K_y(t) = 0$, which implies that asymptotically, either the capital accumulation is trivially small, or the marginal valuation of capital λ is zero.

The first condition says that, in Pareto optimum, the marginal utility of consumption must equal to the shadow price of capital. The second equation indicates that, to ensure positive growth rate, the return to polluting capital plus the return of environment to polluting capital must exceed the social discount rate. The third condition ensures the optimal allocation of capital between production and abatement.

To develop more intuition, we use the discrete-time equations of first-order conditions as

$$r^t U_c(c_t, E_t) - \lambda_t = 0,\ t = 1,\dots,\infty;$$

$$r^t U_{E_t}(c_t, E_t) E_{K_{yt}}(K_{yt}, K_{at}) + \lambda_t [F_{K_{yt}}(K_{yt}, E_t) + 1] - \lambda_{t-1} = 0,\ t = 1,\dots,\infty;$$

$$r^t U_{E_t}(c_t, E_t) E_{K_{at}}(K_{yt}, K_{at}) + \lambda_t [F_{K_{at}}(K_{yt}, E_t) + 1] - \lambda_{t-1} = 0,\ t = 1,\dots,\infty.$$

From these conditions, we have the following propositions:

(1) In Pareto optimum, the ratio of discounted marginal utility between any two time period must equal to the ratio of shadow prices, or $U_c(c_{t-1}, E_{t-1}) / rU_c(c_t, E_t) = \lambda_{t-1} / \lambda_t$.

(2) The marginal utilities of productive capital and abatement capital must equal in each period, or $U_{E_t} E_{K_{yt}} + U_{c_t} F_{K_{yt}} = U_{E_t} E_{K_{at}} + U_{c_t} F_{K_{at}}$.

(3) The shadow price decreases along the growth path, and its change is equal to the discounted sum value of marginal utilities of consumption and environmental quality induced by capital accumulation, or $(\lambda_t - \lambda_{t-1}) = -r^t (U_{E_t} E_{K_{yt}} + U_{c_t} F_{K_{yt}})$.

Further, in the balanced growth, the elasticity of inter-temporal substitution of consumption must be constant. The marginal product of dirty capital plus the return of environment to capital in terms of marginal rate of substitution between consumption and environment must be constant. The saving rate and consumption rate must also be constant.

2.2 Decentralized Equilibrium in the *AK* Model

In order to explicitly characterize the balanced growth in this model, we use the closed forms for environmental quality, production and utility for further analysis.

At any time period, the environmental quality is described as: $E = K_a^{\beta_1} K_y^{-\beta_2}$, where $0 < \beta_1 < 1$, $0 < \beta_2 < 1$, $E_{K_aK_a} < 0$, $E_{K_yK_y} > 0$, $E_{K_aK_y} < 0$.

In this form, β_1 and β_2 are the elasticity of environmental quality in the two types capital stocks respectively. Taking the log of *E*, and differentiating with respect to time,:

$$\frac{\dot{E}}{E} = \beta_1 \frac{\dot{K}_a}{K_a} - \beta_2 \frac{\dot{K}_y}{K_y}.$$

From this relation, the growth rate of environmental quality changes with the growth rates of capital stocks and the elasticities. It is possible that the environment quality displays an inverted U-shape over time if the parameters change. When $\frac{\dot{K}_a / K_a}{\dot{K}_y / K_y} \geq \beta_2 / \beta_1$, the growth rate of environmental quality is non-negative.

The *AK* version of production technology is expressed as $Y = AK_y E^{\gamma}$. It is concave in environmental quality, $0 < \gamma < 1$, where γ is the elasticity of production in environmental quality. When γ tends to zero, the production is unaffected by the environment. The elasticity of substitution between environmental quality and polluting capital is γ.

The growth rate of final output for *AK* technology is as follows:

$$\frac{\dot{Y}}{Y}=\frac{\dot{K}_y}{K_y}+\gamma\frac{\dot{E}}{E}=\gamma\beta_1\frac{\dot{K}_a}{K_a}+(1-\gamma\beta_2)\frac{\dot{K}_y}{K_y}.$$

The steady-state growth exists only if the ratio of growth rates of productive capital and abatement capital is constant. The growth rate of output is determined exogenously by the elasticities of production with respect to environmental quality and capitals. The higher the elasticity γ, the higher the growth rate of output is.

The isoelastic utility function is expressed as

$$U(c,E)=\frac{(c^{\upsilon}E^{\omega})^{1-\sigma}-1}{1-\sigma},\sigma\in(0,1),\upsilon\in(0,1),\omega\in(0,1). \quad (8)$$

The elasticity of inter-temporal substitution of consumption in this function is defined as $\Psi=(1-\upsilon(1-\sigma))$. The elasticity of marginal rate of substitution between consumption and environmental quality is ω/υ. High value of this ratio indicates that consumer has relatively higher preference on environmental quality in utility function, and environmental quality plays more important roles in economic growth. When this ratio is very small, the role of environmental quality in promoting social welfare is negligible.

In a competitive market, the firm's objective is to maximize the profit,

$$\pi=pAK_yE^{\gamma}-r_yK_y-r_aK_a.$$

Normalizing the final output price p to unity. The first-order condition for the firm is

$$r_y=\partial F/\partial K_y=AE^{\gamma},\ r_a=\partial F/\partial K_a=0.$$

From firm's first-order condition, the return to abatement capital is zero. This drives firms to invest zero in pollution abatement. So, under the competitive equilibrium, taking the environmental quality as exogenous, neither the consumer nor the firm has incentives to control the pollution.

By our specification of environmental quality, zero abatement in pollution suddenly causes environment to zero, and utility and production also suddenly vanish to zero.

Therefore, competitive equilibrium without government intervention does not exist. The reason is that environmental quality is a purely public good that is exogenous to firms and consumers, and firms are not rewarded if they invest abatement capital because of the public good property. Without government intervention, there is no abatement capital market, and profit-maximizing firms maximize the use of polluting capital, thus engendering abatement investment to be zero.

This is an extreme case, even though we suppose that a growing economy without regulation finally goes to zero level of everything because of environmental degradation.

2.3 Socially Optimal Growth Path in the *AK* Model

The socially optimal growth paths are achieved by solving the social planner's problem, whose objective is to maximize the representative agent's utility subject to the representative agent's budget constraints. In each period of growth, the social planner needs to choose the optimal level of consumption, capital accumulation and capital allocation between production and abatement in order to realize socially optimal growth rates. That is, the forward-looking social planner needs to decide how much to consume and how much capital needs to abate the pollution in the current period, so that this decision will induce an optimal accumulation of capital for the next period, and so forth. Given the initial capital level at period zero, if consumption and capital allocation in each period are optimized, the socially optimal growth is then realized.

The difference between social optimum and competitive equilibrium is that environmental quality is endogenous and has to be decided inside the social planner's problem, and the optimal consumption and abatement investment are dictated by the benevolent social planner as well. But in the latter circumstance, consumption and abatement investment decisions are determined in the market through price mechanisms; environmental quality change results from the market equilibrium.

The social planner has to trade-off the use of physical capital and environmental quality by choosing the optimal abatement capital level. The total saving at time period t is equal to the final output minus consumption if the capital depreciation is neglected.

This is the transition equation $\dot{K} = Y - c$, which says that the final output of each period is used for two purposes: consumption of current period and saving for production of the next period. All the output which is not consumed will be saved as capital accumulation and move to the next period for further production and pollution abatement.

The production function of the social planner is $Y = AK_y E^{\gamma} = AK_a^{\gamma\beta_1} K_y^{1-\gamma\beta_2}$.

Therefore, the social planner's problem is

$$\max \int_0^{\infty} e^{-\rho t} U(c, E) dt, \tag{12}$$

$$s.t. \quad E = K_a^{\beta_1} K_y^{-\beta_2},$$

$$\dot{K} = AK_y E^{\gamma} - c,$$

$$K \geq K_a + K_y.$$

Using equ. (8) for the utility, the current-value Hamiltonian for this problem is

$$H^c = \frac{[c^{\upsilon} K_a^{\beta_1 \omega} (K - K_a)^{-\beta_2 \omega}]^{1-\sigma} - 1}{1-\sigma} + \lambda [AK_a^{\gamma\beta_1} (K - K_a)^{1-\gamma\beta_2} - c]. \tag{13}$$

The first-order necessary conditions are

$$\frac{\partial H^c}{\partial c} = \upsilon c^{\upsilon(1-\sigma)-1} [K_a^{\omega\beta_1} (K - K_a)^{-\omega\beta_2}]^{1-\sigma} - \lambda = 0; \tag{14}$$

$$\begin{aligned} \frac{\partial H^c}{\partial K_a} &= \omega c^{\upsilon(1-\sigma)} K_a^{\omega\beta_1(1-\sigma)-1} (K - K_a)^{-\omega\beta_2(1-\sigma)-1} [\beta_1 (K - K_a) + \beta_2 K_a] \\ &+ \lambda A K_a^{\gamma\beta_1 - 1} (K - K_a)^{-\gamma\beta_2} [\gamma\beta_1 (K - K_a) - (1 - \gamma\beta_2) K_a] = 0 \end{aligned}; \tag{15}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H^c}{\partial K} = \rho\lambda + \omega\beta_2 (c^{\upsilon} K_a^{\omega\beta_1})^{1-\sigma} (K - K_a)^{-\omega\beta_2(1-\sigma)-1} - \lambda(1 - \gamma\beta_2) AK_a^{\gamma\beta_1} (K - K_a)^{-\gamma\beta_2}; \tag{16}$$

$$\dot{K} = AK_y E^{\gamma} - c. \tag{17}$$

Same as the general case, the first-order conditions here have economic meanings. Condition (14) says that in the optimum the marginal valuation (shadow price) of capital is equal to the marginal utility of consumption in each period. The second condition requires that, in each period, the abatement capital level should be such that the marginal utility of abatement capital equals to the marginal product of abatement capital.

Condition (16) seems not obvious, but if we write out the discrete time case, it is easier to see its meaning:

$$(-\beta_2\omega)\rho^t c_t^{\upsilon(1-\sigma)} K_{at}^{\beta_1\omega(1-\sigma)} (K_t - K_{at})^{-\beta_2\omega(1-\sigma)-1} + \lambda_t A(1-\gamma\beta_2)(K_t - K_{at})^{-\gamma\beta_2} K_{at}^{\gamma\beta_1} - \lambda_{t-1} = 0.$$

It says that in optimum the marginal valuation of capital in the previous period equals to marginal product of total capital plus the marginal utility of total capital in current period.

From the first-order conditions, the optimal consumption level and optimal abatement capital level in each period are obtained, both of them depend on consumption and abatement investment level in future periods because this is a forward-looking process. In finite-time case, the capital level in last period is known and is zero, and we are able to solve the optimal consumption and abatement in each period. But in infinite-time case, we can not do like this.

Further, we get the socially optimal growth rate and the optimal consumption rate as

$$\frac{\dot{c}}{c} = \frac{1}{\psi}\left[AK_a^{\gamma\beta_1} K_y^{-\gamma\beta_2} \frac{\beta_1}{\beta_1 + \beta_2 K_a / K_y} - \rho + \omega(1-\sigma)\left(\beta_1 \frac{\dot{K}_a}{K_a} - \beta_2 \frac{\dot{K}_y}{K_y}\right) \right]; \qquad (18)$$

$$\frac{c}{Y} = \frac{\upsilon[-\gamma\beta_1 + (1-\gamma\beta_2)K_a / K_y]}{\omega(\beta_1 + \beta_2 K_a / K_y)}. \qquad (19)$$

From equ. (18), the growth rate of consumption is decomposed into three parts shown in parenthesis. The first term in the right hand side is the marginal product of polluting capital. The second term is the discount rate, representing the impatience of individuals. The third term is the growth rate of environmental quality. Understanding why the growth rate consists of these three parts is important. Intuitively, since consumption is determined by the output, so the higher the marginal product of capital, the more goods the economy has to increase consumption. Higher value of discount rate shows that people in the economy are more patient to wait for future consumption, which causes lower growth rate of consumption. Though higher marginal product contributes more to the consumption increase rate, it brings more pollution and increasing degradation of environment. Thus, for an optimized level of utility, this trend causes consumption to decrease faster. Therefore, a lower growth rate of environmental quality due to higher change of polluting capital causes slower consumption change. Last, the inter-temporal elasticity of substitution of consumption negatively affects consumption growth rate. If the substitution of consumption is elastic, the consumption grows slower.

Other effects constant, the higher the elasticity of environmental quality in the abatement capital, the higher the growth rate is. The same observation holds for the consumer's preference on the environmental quality.

From policy function equ. (19), we conclude that the optimal consumption level in each period of growth is decided by the capital allocation ratio whose effect on consumption is ambiguous depending on the elasticities of environmental quality, utility and production.

Next, we examine the properties of growth in steady states. The balanced-growth state is defined as one situation of the steady state when growth rates of consumption, capital accumulation, output, and environmental quality are all constant, but not necessarily equal; the ratios of consumption-output, capital-output, and abating-polluting capital are also constant. The balanced growth is just one possible state of social optima.[5] Whether the steady state exists is the problem of stability of steady states. If one steady state is very unstable, we could say that there is no steady state, which also depends on how

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[5] Along the growth paths, the social optimum is not unique in terms of the dynamics of consumption and capital allocation. But the pattern of optimal growth is assumed to be unique.

unstable the state is. This paper does not examine the problem of stability, and assumes the existence of steady states.

In particular, the abating-polluting capital ratio is to be decided in each period endogenously and should be constant in steady state. Let this ratio be $\mu = K_a / K_y$, which is also a function of time, that is, it may change along the growth paths.

To simply the problem and to look at how the environmental quality affects growth in detail, we examine the steady-state growth in three cases where environmental quality enters utility function ($\gamma = 0, \omega \neq 0$), production function ($\gamma \neq 0, \omega = 0$), or both ($\gamma \neq 0, \omega \neq 0$). Most literature deals with the case in which environmental quality only affects utility. In the real world, this case happens when a firm's production does not heavily depend on environmental quality, or firms are insensitive in production decisions no matter how low the environmental quality is. But polluted drinking water, sulfur dioxide in the air and other forms of pollution apparently affect utility. In the second case, consumers' preference of environmental quality is trivially small, while environment is indispensable to production. This may happen when people live far from factories and pollution sources, though it is not easy to find a clear-cut example in the real world. The third case is most general since environment more or less always affects the daily life of people and the production by our commonsense.

Case (1): environmental quality is only a consumable good: $\gamma = 0, \omega \neq 0$.

In this case, the environmental quality does not contribute to production. The growth rates of consumption and output are reduced to

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[\frac{A\beta_1}{\beta_1 + \beta_2 \mu_y} - \rho + \omega(1-\sigma)(\beta_1 \frac{\dot{K}_a}{K_a} - \beta_2 \frac{\dot{K}_y}{K_y})\right], \tag{20}$$

$$\frac{\dot{K}}{K} = \frac{AK_y}{K} - \frac{c}{K}, \tag{21}$$

and the optimal consumption rate becomes

$$\frac{c}{K_y} = \frac{A\upsilon\mu}{\omega(\beta_1 + \beta_2\mu)}. \tag{22}$$

Growth rate of consumption is determined by the inter-temporal elasticity of substitution of consumption, the elasticities of environmental quality with respect to two types of capital, the ratio of capitals μ, the growth rate of environmental quality, and the discount rate. When environmental quality is only a consumable good, the more capital is used for abatement, the slower the economy grows. But abatement capital will not be zero, because the abatement capital contributes to social welfare through affecting environmental quality.

The steady state is defined such that the growth rates of consumption, output, and capital accumulation are constant and equal; and the capital allocation ratio, consumption rate, and saving rate are also constant,

$$g = \frac{\dot{c}}{c} = \frac{\dot{K}}{K} = \frac{\dot{K}_a}{K_a} = \frac{\dot{K}_y}{K_y} = \frac{\dot{Y}}{Y}. \tag{23}$$

Using equ. (20), (21) and (22), we observe that if the steady state exists, then the optimal capital allocation ratio is a constant, and vice versa. Therefore, the sufficient and necessary condition for steady-state growth is that the abating-polluting capital ratio μ is constant. But this does not necessarily require that the elasticities of environmental quality with respect to the two capital stocks are equal.

Combining the above three equations, we get the steady-state growth rate of the economy as

$$\frac{\dot{c}}{c} = \frac{1}{\Psi + \omega(1-\sigma)(\beta_1 - \beta_2)]}\left[\frac{A\beta_1}{\beta_1 + \beta_2\mu} - \rho\right]. \tag{24}$$

The steady-state optimal capital allocation can be obtained through the following result:

$$\frac{1}{\Psi+(1-\sigma)\omega(\beta_1-\beta_2)]}\left[\frac{A\beta_1}{\beta_1+\beta_2\mu}-\rho\right]=\frac{A}{1+\mu}\left[1-\frac{\upsilon\mu}{\omega(\beta_1+\beta_2\mu)}\right]. \tag{25}$$

The abating-polluting capital ratio in equ. (25) is in quadratic form which could be solved (see Appendix A). To ensure a non-negative ratio, restrictions on parameters are needed. From equ. (24), non-negative growth rate of the economy requires $0<\mu\leq\frac{\beta_1(A-\rho)}{\beta_2\rho}$, but it is not clear, from equ. (25), whether the optimal capital allocation satisfies this condition of non-negative growth rate or not. From another perspective, we may need certain parametric restrictions in order to ensure that the optimal capital allocation obtained from equ. (25) is able to satisfy the inequality condition. Whether the economy actually decreases or growth rate is negative in social optimum depends on the parameters, so negative growth might happen, particularly when the elasticity of substitution between consumption and environmental quality in utility function is high.

At least, $A>\rho$ should hold in any case to ensure non-negative growth rates. This means the marginal product in each period must strictly exceeds the discount rate in order to have possible positive growth rates.

Growth rates depend on the elasticities of environmental quality with respect to the two types of capital, so it depends on the growth rate of environmental quality. Environmental quality may increase, decrease or stay constant depending on the change of elastiticities with respect to capitals. This may be used to explain why, in empirical work, environmental quality displays an inverted bell shape in observed patterns of economic growth. The instantaneous utility may grow at a lower or even negative rate if $\beta_2>>\beta_1$ and $\omega>>\upsilon$, that is, if the marginal contribution of polluting capital to environmental quality is high and consumer's preference for environmental quality is high. In detail, when a much larger amount of abatement capital is needed to mitigate the impacts of relatively small polluting capital use, or when elasticity of substitution between environmental quality and consumption is very large, the society has less saving for production. Then the economic growth slows down. However, it is again not easy to examine whether this setting is allowed to happen in steady states as examined above.

When $\beta_1 = \beta_2 = \beta$, this system has the steady-state growth rate

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[\frac{A}{1+\mu} - \rho\right],$$

where μ is the marginal rate of substitution between productive capital and abatement capital for determining the levels of environmental quality. It also determines the environmental quality in the steady state. The Appendix A shows that this ratio is a constant in steady state.

From the above growth rate, the restriction on μ for a positive growth rate is $\mu < \frac{A-\rho}{\rho}$. This sets a limit for the relative level of abatement capital. If this ratio is too high, the society has less resource for further production, which causes the economy to slow down. But this imposes the same question as the case when $\beta_1 \neq \beta_2$.

Case (2): environmental quality is only a productive good: $\gamma \neq 0, \omega = 0$.

From the necessary first-order conditions, we obtain the optimal capital allocation as (see the Appendix A):

$$\mu = \frac{\gamma\beta_1}{1-\gamma\beta_2}. \tag{26}$$

The growth rate of consumption is:

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[AK_a^{\gamma\beta_1}K_y^{-\gamma\beta_2}\frac{\beta_1}{\beta_1+\beta_2\mu} - \rho\right]. \tag{27}$$

When environmental quality affects firm's production only, socially optimal capital allocation ratio is always constant. This arises from the fact that the environmental externality does not affect consumers. It is only necessary for the social planner to earmark certain level of capital for abatement activities to internalize the pollution within the production sector, but this relative level of investment in abatement is constant given the assumed production technology and environmental quality determination.

The steady-state growth rate requires $\beta_1 = \beta_2 = \beta$. Given this condition, there is no transitional dynamics for consumption when environmental quality only affects production. Consumption jumps to a unique steady state instantly provided the initial capital endowment is known. Further, environmental quality also jumps to a constant level instantly.

The steady-state growth rate of consumption is:

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[A(1-\gamma\beta)^{1-\gamma\beta}(\gamma\beta)^{\gamma\beta} - \rho\right]. \tag{28}$$

The growth rate of capital is

$$\frac{\dot{K}}{K} = \frac{A\mu^{\gamma\beta}}{1+\mu} - \frac{c}{(1+\mu)K_y}. \tag{29}$$

The growth rate of capital may be not equal to the growth rate of consumption, with the latter is always constant. In the capital case, growth rate depends on the level of consumption and the level of capital accumulation of each period. Solving equ. (28), we get the policy function of each period which is an exponential function of time, $c(t) = c(0)e^{gt}$, where g is consumption growth rate. Equ. (29) is a differential equation about capital, solving which we get the optimal level of capital in each period. When the two growth rates interact, the economy reaches a steady state.

Setting equ. (28) equal to (29), the steady optimal consumption rate is obtained as:

$$\frac{c}{Y} = \frac{1}{\Psi}\left[(\Psi - 1) + \frac{\rho(1+\mu)}{A\mu^{\gamma\beta}}\right]. \tag{30}$$

It is a constant in steady states. Together with the requirement on elasticity of environmental quality, equ. (26), (30) and the growth rates in equ. (28), (29) show that the steady states exist.

In this case, parameter restrictions are needed to ensure non-negative growth and positive consumption rate, the economic implications of which are not straightforward. The parameter restrictions are $A(1-\gamma\beta)^{1-\gamma\beta}(\gamma\beta)^{\gamma\beta} \geq \rho$ and $\rho(1-\gamma\beta)^{1-\gamma\beta}(\gamma\beta)^{\gamma\beta} / A > \upsilon(1-\sigma)$.

Case (3): environmental quality is both an input and a consumable good: $\gamma \neq 0, \omega \neq 0$.

This is more general than the previous two, representing the most common case in the real world. It is the intersection of the above two cases. When making decisions on optimal consumption and optimal abatement investment in each period, the social planner still needs to trade-off. In one period, the social planner knows capital saving from last period. If more abatement capital is used, environmental quality increases which benefits the consumer more, nevertheless less polluting capital is left for production, thus less output for consumption, which causes utility to decrease. How much the social planner should earmark for production and abatement is addressed through the first-order conditions of optimal control.

From equ. (18), the sufficient and necessary conditions for a steady state of consumption growth to exist are:

(1) The elasticities of environmental quality of the two types of capital stocks are equal;

(2) The abatement-polluting capital ratio maintains constant.

In the steady state, the balanced growth indicates that the growth rates of consumption, capital stocks and output are constant, and the saving rate K/Y, consumption rate c/Y, and capital allocation ratio μ are constant, and environmental quality E maintains constant.

The steady-state growth rates are then:

$$\frac{\dot{c}}{c}=\frac{1}{\Psi}\left[\frac{A\mu^{\gamma\beta}}{1+\mu}-\rho\right]; \tag{31}$$

$$\frac{\dot{K}}{K}=\frac{A\mu^{\gamma\beta}}{1+\mu}-\frac{c}{(1+\mu)K_y}. \tag{32}$$

The consumption rate becomes

$$\frac{c}{Y}=\frac{\upsilon[-\gamma\beta+(1-\gamma\beta)\mu]}{\omega\beta(1+\mu)}. \tag{33}$$

The growth rate of consumption is determined by three terms same as pointed before: inter-temporal elasticity of substitution of consumption, marginal product of capital, and discount rate. It also depends on capital allocation ratio. The marginal rate of substitution between polluting capital and abatement capital in production function is $MRS=(1/\gamma\beta-1)\mu$, so μ reflects the MRS between the two capitals. Its effect on growth rate is ambiguous depending on the elasticity values γ and β. When $A\mu^{\gamma\beta}/(1+\mu)>\rho$, the steady growth rate is positive, which says if the marginal product of capital must exceeds discount rate, the economy grows positively. Larger γ means a higher elasticity of production with respect to environmental quality, and larger β means a higher elasticity of environmental quality with respect to capital. From equ. (31) and (32), we could observe that, if $\mu<1$, the higher the two elasticities, the lower the steady-state growth rate of consumption, thus the slower the growth rate of the economy. This demonstrates that investment in pollution abatement curtails the steady-state economic growth. Further, under the conditions for steady state in this case, it is true that the socially optimal steady-state growth rates of consumption, production, and capital accumulation are all equal and constant, while environmental quality is constant.

Setting equ. (31) equal to (32) and using (33) to solve for c/K, one is able to obtain the optimal steady-state abating-polluting capital ratio μ.[6] From the positive growth condition and the optimal value of μ, the restrictions on parameters is further imposed.[7]

[6] The equation to solve for the optimal μ is non-linear, so it requires simulation, but at least we know μ exists and is constant in steady state.

[7] However one point is not clear. That is, the steady-state μ may not satisfy the condition of non-negative growth rates. Negative steady-state growth may happen, depending on the parameters. This imposes a difficulty when trying to look at how the parameters affect the value of μ, which is actually very important. This problem is beyond this paper.

CHAPTER 3. ENVIRONMENTAL POLICY AND ENDOGENOUS GROWTH

This chapter discusses how optimal environmental policies of internalizing pollution promote the economic growth, and the behavior patterns of the representative agent under regulations.

Under the regime considered in Chapter 2 regarding the evolution of economy and environmental quality, private agents have no incentive to invest the pollution abatement in a competitive environment. Thus the abatement capital accumulation is zero.

In the competitive equilibrium, environmental quality grows at a negative rate, and jumps to zero. So the utility of representative agent also approaches to zero. Certainly, this is not socially optimal because of the existence of externality arising from the use of dirty capital in the process of growth. Particularly, if the consumer's preference for environmental quality is sufficiently large compared to his preference for consumption, the utility goes to zero quickly.

Public choice methods could be used, such as the voting system in the overlapping generation model analyzed in Jones and Manuelli (2000). But the basic regime of governmental intervention is still either the taxes (subsidies) or the technical standards to ensure that the decentralized competitive equilibrium is socially optimal. The objective is to give the private agents incentives in order to replicate socially optimal growth paths under competitive equilibrium.

There are several methods of taxation to improve the investment in the pollution control. Examples include the final output tax, capital tax, emission tax and abatement subsidy. In contrast, the government could also design a technical standard to ensure the investment in pollution abatement.

In addition, one interesting case is to explore whether the tax and technical standard generate the same growth rate in the long run and compare their effects on welfare (utility of representative agents).

Corresponding to the three cases discussed in Chapter 2, the following analysis attempts to examine the competitive equilibrium of an economy with environmental regulations and make it replicate the socially optimal growth path. A dynamic competitive equilibrium is defined as a set of allocations of consumption and capital $\{c, K_y, K_a\}$ given the prices $\{p, r_y, r_a\}$, environmental quality dynamics, the production technology $\{Y : Y = AK_y E^\gamma, K_y \geq 0, E \geq 0\}$ and governmental regulations $\{\mu, \tau, s\}$ along the growth path, under which:

(1) Consumer maximizes the present value of utility under the allocations;

(2) Firm maximizes the current value of profit in each period;

(3) Goods market, abatement capital market and productive capital market clear.

In the above, regulations include technical standards, taxes and subsidies.

If environmental policy is able to make competitive equilibrium replicate social optimum, the policy is said to be a first-best one, meaning that policy improves social welfare by fully internalizing pollution impacts. In a second-best environment, the competitive equilibrium is not Pareto optimal, but may be a Pareto improvement realized through policy implementation.

3.1 Technical Standard

Suppose that the regulator sets a quantitative standard for the investment level of abatement activities in order to curtail environment deterioration. One straightforward way to set the technical standard is to stipulate the dynamic ratio of abatement and polluting capital accumulation. By setting the ratio, the regulator does not need to concern the level of abatement capital required in each period. Assume that the regulator requires the firm to invest some amount of profit in pollution control, and

$\mu = K_a / K_y > 0$. This ratio is set before individuals make consumption and production decisions, so it is a parameter to firms and consumers. Individuals make decentralized decisions given this parameter. In different periods, the technical standard may be changed by the regulator and μ is the function of time.

The firm maximizes profit:

$$\pi = Y - r_y K_y - r_a K_a = (AE^{\gamma} - r_y - r_a \mu) K_y, \tag{34}$$

where the output price is again normalized to unity, r_y is the rental rate of productive capital and r_a is the rental rate of abatement capital. Notice that environmental quality is exogenous now to private agents. The first-order necessary condition is:

$$AE^{\gamma} - r_y - r_a \mu = 0. \tag{35}$$

The firm earns zero profit when maximizing profit. The first order condition indicates that a profit-maximizing firm make the optimal production decisions so that the marginal product value equals to marginal costs. In a linear production technology, the average product value equals to marginal product value, and the marginal cost equals to the average cost.

The representative agent maximizes life-time utility by choosing the optimal consumption and investment plan. Consumer's problem is:

$$\max \int_0^{\infty} e^{-\rho t} U(c, E) dt, \tag{36}$$

$$\text{s.t. } \dot{K} = r_y K_y + r_a K_a - c,$$

$$K = K_y + K_a.$$

The consumer rents capital to firms and firms return interests to the consumer. The first budget constrain comes from the fact that the rest of capital revenue after consumption in one period is saved and moved to the next period, and rent it out to earn capital interest again, and so on.

The current-value Hamiltonian is:

$$H^c = \frac{(c^{\upsilon} E^{\omega})^{1-\sigma} - 1}{1-\sigma} + \lambda(r_y K_y + r_a K_a - c). \tag{37}$$

The first order necessary conditions are

$$\frac{\partial H^c}{\partial c} = \upsilon c^{\upsilon(1-\sigma)-1} E^{\omega(1-\sigma)} - \lambda = 0; \tag{38}$$

$$\frac{\partial H^c}{\partial K_a} = -r_y + r_a = 0; \tag{39}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H^c}{\partial K} = \rho\lambda - \lambda r_y. \tag{40}$$

The first condition is interpreted same as before, the marginal valuation (or shadow price) of capital should be equal to the marginal utility of consumption. The second condition is that if the utility is maximized, the returns to the polluting capital and abatement capital in market must be equal to each other. In other words, the consumer is indifferent in deciding investment in the two capital markets. Otherwise, the consumer sells capital in only one capital market with higher interest rate till the prices in two markets are equal. The third condition says that in optimum the growth rate of marginal valuation of capital equals to the difference between discount and interest rate. In the long run, the marginal valuation of capital is diminishing (as well as the marginal utility of consumption), so it is obvious that in a growing economy, interest rate is no lower than discount rate.

From first-order conditions, we obtain the following optimal growth path of consumption

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[r_y - \rho + \omega(1-\sigma)\frac{\dot{E}}{E} \right]. \tag{41}$$

In a competitive economy, the decentralized growth rate of consumption depends on the inter-temporal elasticity of substitution of consumption, the capital rental rate, the discount rate and the growth rate of environmental quality. What is new in this growth equation is the rental rate of capital. The higher the rental rate, the faster the consumption grows. When interest rate is higher in the market, the consumer obtains higher capital revenue, so consumption increases faster.

Combining firm's and consumer's first-order conditions, in competitive equilibrium the following relation holds:

$$r_y = r_a = \frac{AE^{\gamma}}{1+\mu}. \tag{42}$$

Equ. (42) indicates that the rental rates of two types of capital are equal, that is, in equilibrium, the consumer is indifferent in investing abatement capital market and productive capital market. The rental rate reflects the level of marginal product value of capital. The equal rental rates means that when the economy is in competitive equilibrium the interest rates in all capital market must be equal. If the rental rates in markets are not equal, the economy does not reach equilibrium; but if they are equal, the economy may not be in equilibrium.

From equ. (42), we obtain the optimal technical standard since prices are exogenous. The optimal technical standard is not constant along time, but depends on the prices of capital markets. There is no tractable pattern on how the standard evolves because of the volatility of capital market. However, following usual macroeconomic assumptions, the long-run economy is in equilibrium and the prices do not change. So, in the long term the capital allocation ratio is constant, if the environmental quality holds constant.

The competitive equilibrium growth rate is obtained as

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[\frac{A(K_a^{\beta_1}K_y^{-\beta_2})^{\gamma}}{1+\mu} - \rho + \omega(1-\sigma)(\beta_1\frac{\dot{K}_a}{K_a} - \beta_2\frac{\dot{K}_y}{K_y})\right]. \tag{43}$$

Consider the case where $\beta_1 = \beta_2 = \beta$. This corresponds to the balanced growth conditions in case (2) and (3) of Chapter 2. Then the optimal growth rates of consumption and capital are:

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[\frac{A\mu^{\gamma\beta}}{1+\mu} - \rho\right], \tag{44}$$

$$\frac{\dot{K}}{K} = \frac{A\mu^{\gamma\beta}}{1+\mu} - \frac{c}{(1+\mu)K_y}. \tag{45}$$

The environmental quality growth rate vanishes. Under the condition $\beta_1 = \beta_2 = \beta$, environmental quality is in a level that is indirectly determined by government, only changing with the technical standard.

Set the two growth rates equal we get the steady-state consumption-saving ratio c/K as follows:

$$\frac{c}{K} = \frac{1}{\Psi}[\rho + (\Psi - 1)\frac{A\mu^{\gamma\beta}}{1+\mu}]. \tag{46}$$

When the consumption-capital ratio satisfies equ. (46), the steady-state growth exists. Compare these growth rates with the social optimal growth rates equ. (31) and (32), it is concluded that the competitive equilibrium growth rate replicates the social optimal growth rate when the government sets an abatement-production capital ratio if the consumption-capital ratio satisfies equ. (46) and if the price levels are such that the optimal technical standard in equ. (42) equals to the socially optimal technical standard obtained in equ. (19). In the steady state, there is a unique level of the price set, at which the optimal decentralized growth path repeats the socially optimal growth path. The optimal steady-state standard is obtained from the social optimum, and the unique corresponding price level (at which the two steady-state growth replicates) is obtained by setting equ. (42) to the socially optimal steady-state standard (obtained in the last paragraph of Chapter 2). However, the steady state may not be stable, because the consumption-capital ratio may not be a constant in the long run. There are two problems to be considered.

First, the first-order conditions in the discrete-time case (see the Appendix B) demonstrate that both the optimal consumption and capital in each period are determined by the future levels of consumption and the initial capital endowment, and the interest rates in all future periods. It turns out that the consumption-capital ratio (and consumption rate) may not be constant.

Second, the consumption-capital ratio may not reach the level shown in equ. (46) since it depends on the future values and interest rates which changes in each period. It is also possible that the ratio value is always larger than that in equ. (46) along the growth path. To deal with this problem, we need to put restrictions on the parameters to ensure the existence of the steady state.

In conclusion, in the competitive equilibrium with technical standard, the optimal growth rate of consumption is determined by the technical standard and discount rate. The effect of technical standard on growth rate is ambiguous, depending on the elasticity of environmental quality of capital and the elasticity of production of environmental quality.

The optimal technical standard is determined by the interest rates and again how the standard changes with the change of prices is ambiguous. So, technical standard changes along time.

The appropriate technical standard enables the steady-state growth path in competitive equilibrium to be Pareto optimal, so the policy is first best.

The above analysis is conducted in the first-best economic environment. However, the government usually imposes income tax at a rate of τ_y to cater its exogenous public expenditure. It is necessary to consider what happens if a final output tax is imposed on the representative agent.

The growth rate of consumption under an income tax rate is:

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[\frac{(1-\tau_y)A\mu E^{\gamma}}{1+\mu} - \rho + \omega(1-\sigma)\frac{\dot{E}}{E}\right]. \tag{47}$$

An exogenous income tax curtails the economic growth in the sense that it would reduce both the saving rate and consumption rate of the economy. The imposition of income tax does not affect the conclusion that competitive equilibrium under technical standards replicates the socially optimal growth paths.

3.2 The Pigouvian Environmental Tax

There are several ways to set the optimal tax and subsidy for environmentally sustainable growth of the economy. The following discussions attempt to explore the properties of Pigouvian tax.

There is no explicit pollution in the model previously discussed, though pollution caused by the use of productive capital is assumed the only source of environmental deterioration. Recall that environmental quality is determined by pollution from production, and $E = D^{-1}$ where D is the emission level of identical firms. This indicates that the marginal rate of environment deterioration is diminishing with the pollution amount. Following this postulation, the emission level of the firm is expressed as $D = K_a^{-\beta_1} K_y^{\beta_2}$.

Suppose the regulator wants to put an emission tax with a flat rate τ. At a first look, the tax effect on the growth is ambiguous since, by the emission equation, the tax is on both polluting capital and abatement capital. The following analysis will show whether this intuition is correct.

Under the emission tax, the firm maximizes profit $\pi = Y - r_y K_y - r_a K_a - \tau K_y^{\beta_2} K_a^{-\beta_1}$.

The first order condition for firm is:

$$r_y = AE^{\gamma} - \tau\beta_2 K_y^{\beta_2 - 1} K_a^{-\beta_1}; \tag{48}$$

$$r_a = \tau\beta_1 K_y^{\beta_2} K_a^{-\beta_1 - 1}. \tag{49}$$

Under the emission tax, firms rent capital for production up to a point till the polluting capital rental rate equals the marginal production value of capital abstract marginal tax of polluting capital. In abatement capital market, firms buy the capital with an amount when the rental rate equals to the marginal tax of abatement capital only since firms do not use abatement capital for production.

The consumer's first-order conditions are equ. (38), (39) and (40), same as the case of technical standard.

In the firm's side, the level of capital rented is determined by marginal values of production and marginal tax of polluting capital, while in the consumer's side, optimality requires that the two capitals have the same price. In the competitive equilibrium, equ. (48) equals (49), which gives the optimal emission tax rate and the optimal capital allocation ratio:

$$r_a = \frac{A\beta_1 K_a^{\gamma\beta_1} K_y^{-\gamma\beta_2}}{\beta_1 + \beta_2 (K_a / K_y)}, \tag{50}$$

$$\tau^* = \frac{A K_a^{1+(1+\gamma)\beta_1} K_y^{1-(1+\gamma)\beta_2}}{\beta_1 K_y + \beta_2 K_a}. \tag{51}$$

The capital price is exogenous and changes in each period, so the capital allocation ratio is also changing depending on the capital price. The optimal emission tax rate is the function of capital allocation ratio, hence also depends on capital prices by equ. (51).

Equ. (50) shows how the optimal capital allocation is determined by the capital rent level, which is non-linear in the ratio of capitals. This solution requires the use of approximation methods. For each level of capital rent, there may exist more than one pair of optimal capital allocations depending on the parametric specifications, from which the firm would choose one pair that would maximize profit. From (50), we obtain the optimal capital allocations, then the optimal tax rate is derived by plugging (50) into (51).

From the consumer and firm's optimal conditions, following competitive equilibrium paths are obtained:

$$\frac{\dot{c}}{c}=\frac{1}{\Psi}\left[r_y-\rho+\omega(1-\sigma)\frac{\dot{E}}{E}\right], \tag{52}$$

$$\frac{\dot{K}}{K}=r_y-\frac{c}{K}. \tag{53}$$

In order to look into whether the competitive equilibrium with emission tax is Pareto optimal, and the properties of steady-state equilibrium, next we examine the growth paths and optimal tax policy in three cases as discussed in Chapter 2.

Case (1): environmental quality is only a consumable good: $\gamma=0,\omega\neq 0$.

In this case, the optimal capital allocation and optimal tax rate is respectively:

$$\mu=\frac{\beta_1(A-r_a)}{\beta_2 r_a}, \tag{53}$$

$$\tau^*=\frac{A\mu^{1+\beta_1}K_y^{1+\beta_1-\beta_2}}{\beta_1+\beta_2\mu}. \tag{54}$$

The dynamics of capital allocation ratio and optimal tax rate depends on the capital market price level and the level of capital accumulation. When capital market booms, the increasing capital rental rate causes the capital allocation ratio to decrease, and the tax rate also decreases. If capital price exceeds A, the firm will use zero level of abatement capital and the optimal tax rate also becomes zero. But, we should be clear that the tax rate is given in competitive market, and the optimal capital allocation is determined by the tax rate and capital price, as shown in equ. (49).

With the change of capital allocation between production and abatement, the environmental quality also changes along time. From equ. (52), it is straightforward to derive the condition for steady-state consumption growth rate, which is a constant price level. This is understandable, because, in the long run, the equilibrium price is assumed constant.

When the consumption-saving ratio is constant, there exists steady state with a growth rate as:

$$\frac{\dot{c}}{c} = \frac{\dot{K}}{K} = \frac{r_a - \rho}{\Psi - (1-\sigma)\omega(\beta_1 - \beta_2)}. \tag{56}$$

By the relation between capital allocation ratio and capital price level shown in equ. (53), and compare equ.(56) with equ. (24), it is concluded that, under the emission tax policy, the steady-state growth path in competitive equilibrium replicates the socially optimal growth path if the capital price maintains at a constant level and consumption-capital ratio also maintains at a constant level. In addition, the steady-state competitive equilibrium that is Pareto efficient is unique in the sense that there is only one set of equilibrium prices. The price levels (hence the tax rate) at which the decentralized competitive equilibrium replicates the socially optimal growth path are unique and are such that the optimal capital allocation ratio under decentralized equilibrium is equal to the optimal capital allocation ratio obtained in equ. (25).

In the steady state, the optimal tax rate should increase at a rate proportional to the growth rate of capital accumulation. If $(\beta_2 - \beta_1) < 0$, the tax growth rate is higher than that of the capital accumulation. So, the higher the elasticity of marginal substitution between abatement capital and polluting capital, the higher the optimal tax rate should be, and the faster it grows.

Case (2): environmental quality is only a productive good: $\gamma \neq 0, \omega = 0$.

From equ. (52), we observe that steady-state growth of consumption exists when $\beta_1 = \beta_2$, shown as following:

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}(r_y - \rho). \tag{57}$$

Positive growth rate exists only when interest rate is larger than the discount rate, otherwise, consumers would save more output for future consumption, leading to a negative growth rate.

The steady-state optimal tax rate and optimal capital allocation are:

$$r_a = \frac{A\mu^{\gamma\beta}}{1+\mu}, \tag{58}$$

$$\tau^* = \frac{A\mu^{(1+\gamma)\beta} K_a}{\beta(1+\mu)}. \tag{59}$$

The capital allocation ratio is constant in the sense that capital rent is given. If the consumption-capital ratio maintains constant the steady-state growth rate of capital accumulation exists, and the steady-state growth path exists for the economy in this case.

The growth rate in equ. (57) is same as the growth rate in equ. (27), when capital price is at such a level that it enables the capital allocation ratio obtained in equ. (58) to have the same value as shown in equ. (26). This demonstrates that the competitive equilibrium in this case is Pareto optimal conditionally.

Case (3): Environmental quality is both an input of production and a consumable good $\gamma \neq 0, \omega \neq 0$.

If $\beta_1 = \beta_2 = \beta$, steady-state growth paths in competitive equilibrium are possible to exist, and the optimal capital allocation is same as in the Case (2) shown in equ. (57).

Since the capital rental rate is exogenous to competitive equilibrium, the capital allocation ratio is constant corresponding to each capital rent level. The steady-state tax rate is same as equ. (59) which grows at the same rate as that of capital accumulation..

It is observed again that the steady-state capital accumulation rate exists when the consumption-saving ratio is constant.

Therefore, a properly designed emission tax enables the competitive equilibrium to be Pareto optimal when the equilibrium price reaches certain levels, and in this sense, emission tax is also a first-best policy as the technical standard.

Heuristically, in all the three cases, when the elasticities of environmental quality in the two types of capital are equal, the ratio of two types of capital stock maintains constant in the sense that capital rent is exogenous to this system. Hence steady-state growth rates of consumption and capital exist as in the social optimum. No matter whether the environmental quality affects utility or not, the optimal tax rate in steady state is proportional to the capital stock, which shows that the tax rate also grows at a constant rate same as the economic growth rate. The existence of all the steady state growths of capital depends on the level of consumption-capital ratio. This imposes the stability problem of steady states. If the consumption-ratio is not able to reach a constant level in any period, the economy only has transitional dynamics, and the competitive equilibrium is not Pareto optimal.

For the regulator, the tax policy should be designed such that it grows proportional to or same as the growth rate of capital. Once it observes the capital accumulation level at each period, it could easily calculate a tax rate and does not need to know how the firm would abate the pollution.

If a pre-determined exogenous income tax is put on the consumer or the firm, it is necessary to investigate how the environmental tax responds. For the general situations where the steady state does not concern, it is not easy to see the tax interactions. When the environmental quality is a production input, it appears that the tax interaction is ambiguous.

In Case (1), by adding income tax, we easily obtain the optimal environmental tax:

$$\tau^* = \frac{r_a[\beta_1(1-\tau_y)A-\beta_1 r_a]^{1+\beta_1} K_y^{1+\beta_1-\beta_2}}{\beta_1(\beta_2 r_a)^{1+\beta_1}}. \tag{60}$$

It is perceived that higher income tax causes a lower emission tax, even makes it negative. In other words, taxation of environmental does help to reduce the pre-determined income tax when the environmental quality contributes only to consumption. On the other hand, in the presence of environmental externality, the income tax is not necessarily distortionary. So, the government can use income tax to correct the externality of pollution.

3.3 A Discussion on Tax-subsidy Scheme

In the first-best situation, where the government runs a deficit, a subsidy is able to give private agents an incentive to abate emissions. This is not realistic apparently because the government eventually has to resort to taxes to finance any forms of subsidy. Government may use a tax-subsidy system to promote a capital market of pollution abatement. However, in the competitive market where abatement capital is only used for improving environmental quality and environmental quality is a pure public good, the capital market for abatement investment does not exist. Even the government uses subsidy, firms still uses zero abatement capital. This does not explain the real world that subsidy works in many situations.

Therefore, the use of tax-subsidy scheme requires the introduction the governmental sector to the model. Since the firm may have no incentive to use abatement capital, government can levy the tax on capital income, and play the role of providing public good through investing in the abatement activities using the tax. The technical difficulty to analyze the role of government lies in the specification of inseparable of two types of capital in determining the environmental quality. This is beyond this paper.

3.4 Comparison Between Tax and Standard

From above analysis, we could see that the environmental policy plays an important role in the sustainable growth of economy and environment. Just by imposing environmental regulations on the economy, a Pareto optimal situation is reached through which the social welfare is maximized.

Both the technical standard and environmental tax are able to move the competitive equilibrium growth paths to the socially optimal growth paths. Theoretically, the regulator should be indifferent in choosing any of them since they are equivalent. This is reasonable by the assumptions of AK model, in which the only input is one type capital that is the only distortion. Policies that are able to induce the use of abatement capital are first best.

However, the implementation problems exist in terms of information availability, implementing costs and other aspects.

For the technical standard, a regulator needs to stipulate a ratio of capital allocation which forcefully makes the private agents invest in pollution capital. The problem with this regulation is that in the transitional stages of economic growth, it is technically difficult to calculate the optimal capital allocation ratio. Even more, the regulator has to have complete information about both the environmental evolution and production technology of the firm, thus causing high information seeking cost, because a polluting firm is not willing to reveal its technology to the regulator. In steady states, the regulator is able to obtain the optimal ratio of capitals only if the firm's information is available, particularly the impacts of environmental quality change on both consumption and production, as well as the root sources of environmental quality deterioration. For the case that environmental quality only affects the firm's production, it costs the regulator less to design the optimal ratio.

One caveat for the regulator is that, if the capital ratio is stipulated and implemented based on inaccurate or even wrong information, it may curb the economic growth and the social welfare worsens. Though the problem exists for other instruments as well, the implementation of technical standard needs much more information of individual decisions. In this regard, the economic instruments are preferred.

The Pigouvian tax also has transitional problems that the optimal tax rate is hard to calculate because that requires the level of capital stocks the firm uses to be known. Even in the steady states, the optimal tax rate is not constant, but increases with the same rate as capital accumulation. This causes that the optimal tax rate is not consistent in time.

To the extent that the regulator has to know the firm's capital uses level, the technical standard and Pigouvian tax are equivalent not only in theory but also in implementation.

If we look at the optimal tax rates, it is seen that the optimal tax rate is determined by the price level in capital markets. Here two problems arise: one is the time lag of policy design and implementation, the other is the time consistency of policy. The government may change the tax rate unexpectedly due to the potentially volatile capital market moves. This imposes implementation difficulty of environmental policy.

CHAPTER 4. ANALYSIS WITH ENVIRONMENTAL QUALITY AS STOCK VARIABLE

This chapter briefly considers the case in which environmental quality is a stock variable. As will be shown below, whether environmental quality is taken as flow or as a stock variable does not change the analysis result qualitatively.

Many previous works take environmental quality as a stock variable, such as Aghion and Howitt (1998), and Stokey (2000). In general, the examination of stock pollution enables one to look into the policy designs to control pollutants whose accumulation affects the economic system consecutively. In particular, many real-world pollutants decay very slowly, as requires model specifications different from what we did in the previous chapters.

The indicator of average environmental quality evolves according to the law of transition $\dot{S} = -\delta S + z$, in which δ is a positive parameter characterizing the "cleaning" ability of environment. We can think z as the average net emission in each period, it is determined by the use of capital. Since all the firms are identical in terms of production technology, this average flow variable is also the firm's emission. S could be considered as the pollution stock, and higher value of S indicates a worse environmental quality. Environmental degradation in each period is determined by two factors, one is the self-cleaning of environment, with the other the contribution to degradation of new emissions. It is possible that the self-cleaning ability of environment is very low or even zero when the environment is deteriorated to an extreme level. Here, we assume that the occurrence of an extreme event does not change the ability of self-cleaning of environment.

The firm's production technology is $Y = AKS^{-\gamma}$, where S is negative input as well as the environmental quality. This specification follows Smulders and Gradus (1996).

The net emission is a function of $z = K_y^{\beta} K_a^{-\beta}$ with $0 < \beta < 1$, where K_a is the investment for abatement and K_y is the input of production in the economy. The transition equation for capital accumulation is $\dot{K} = y - c$. In each period, the firm's output is first used for consumption, and the rest is moved to the next period for further production and pollution abatement.

The optimal growth rate of consumption is obtained from the first-order conditions (see Appendix C) as

$$\frac{\dot{c}}{c} = \sigma^{-1}\left[AS^{\gamma}K_y / K - \rho - \xi(1-\sigma)(K_y^{\beta}K_a^{-\beta} / S - \delta)\right]. \tag{61}$$

The steady state is defined when the growth rates of consumption, capital accumulation and output are constant, and the abatement-polluting capital ratio, saving rate and consumption rate are also constant. The pollution stock grows at a zero rate in the steady state. From the optimal conditions, we also get the steady-state abatement-polluting capital ratio.

In the competitive equilibrium, the abatement capital approaches to zero (but can not be zero), then the pollution stock grows at an infinite rate, causing the utility and output to jump to zero levels. Consequently, the competitive equilibrium is not Pareto efficient.

Next, we examine how the emission tax is able to move the competitive equilibrium to social optimum. As in Chapter 3, the regulator levies an emission tax on the pollution at the rate of τ. The firm's first order conditions are same as those in Chapter 3 as following:

$$AS^{-\gamma} - r_y - \tau\beta K_y^{\beta-1}K_a^{-\beta} = 0, \tag{63}$$

$$r_a = \tau\beta K_y^{\beta}K_a^{-\beta-1}. \tag{64}$$

The optimal growth rate of consumption is (see Appendix C)

$$\frac{\dot{c}}{c} = \sigma^{-1}\left[r_y - \rho - \xi(1-\sigma)(K_y^{\beta}K_a^{-\beta} / S - \delta)\right]. \tag{65}$$

Since the returns to the two types of capital are equal in equilibrium, from equ. (63) and (64) we obtain the optimal tax rate as

$$\tau^* = \frac{AS^{-\gamma}}{\beta K K_y^{\beta-1} K_a^{-\beta-1}}. \tag{66}$$

Notice that the optimal tax is equal to the ratio of marginal valuation of pollution stock and marginal valuation of capital, $\tau^* = (-\theta)/\lambda$ (see the Appendix C). Therefore, the competitive equilibrium under the emission tax is Pareto optimal. Whether the environmental quality is a flow or stock variable, the same conclusion is obtained if environmental quality is affected by two types of capital.

Next, I try to show that, if pollution is the input of production, other than determined through the use of capital, we still obtain the similar result. But this case is used in most static models of environmental policy. Environmental quality is still determined as $\dot{E} = \delta E - z$, where z is the average emission level. The environmental quality has a lower bound critical value, $\overline{E}$, below which the economy shuts down. But now, the emission level is an input for production of the firms, so $Y = AKz^{\gamma}$. The emission level is not determined by the use of capital here, but is a productive input, the use of which is determined endogenously.

In this economy, the firms use pollution for production, but pollution control is not rewarded. The emissions from production contribute negatively to the change of environmental quality, and hence affect the utility of consumers by potentially lowering environmental quality.

One problem here is that we implicitly assume that the firm can use as much pollution as it wants to maximize the profit. This is not realistic, because the use of pollution corresponds to the use of other physical inputs that is not free. We mean that the level of pollution in the real world is usually accompanied and constrained by the amount of physical inputs such as the raw materials, as prevents the firm from using as much pollution as it wants for production. To this extent, the assumption of taking emission as the input is not realistic, though it is used widely in literature. But we still follow this assumption for analysis.

The competitive equilibrium is not Pareto efficient, and may not exist. Without environmental policy, to maximize the profit, the firm wants to use any level of pollution for production and reduce the use of capital to a trivial level because of the substitution between two inputs. The emission is arbitrarily large, so the environmental quality finally declines below the critical value.

Technically, to remove this unrealistic non-existence of private equilibrium without regulation, we could impose an ε-tax on firm's emissions. That is, we put an arbitrarily small (of course not optimal) tax on z, so that the firm has a constraint on the use of emissions. Then, a private equilibrium exists, and could be shown not Pareto efficient.

We are interested in the social optimum, as well as the decentralized equilibrium with environmental policy. The social planner chooses the optimal consumption level and the optimal emission level in each period by solving the following problem:

$$\max \int_0^{\infty} e^{-\rho t} U(c,E)dt , \tag{67}$$

$$\text{s.t.} \quad \dot{K} = AKz^{\gamma} - c , \tag{68}$$

$$\dot{E} = \delta E - z . \tag{69}$$

The current-value Hamiltonian is

$$H^c = \frac{(cE^{\xi})^{1-\sigma} - 1}{1-\sigma} + \lambda(AKz^{\gamma} - c) + \theta(\delta E - z) . \tag{70}$$

The first-order conditions are:

$$\frac{\partial H^c}{\partial c} = c^{-\sigma} E^{\xi(1-\sigma)} - \lambda = 0 ; \tag{71}$$

$$\frac{\partial H^c}{\partial z} = \lambda A \gamma K z^{\gamma-1} - \theta = 0 ; \tag{72}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H^c}{\partial K} = \rho\lambda - \lambda A z^\gamma ; \tag{73}$$

$$\dot{\theta} = \rho\theta - \frac{\partial H^c}{\partial E} = \rho\theta - [\xi c^{1-\sigma} E^{\xi(1-\sigma)-1} + \theta\delta] . \tag{74}$$

The interpretations of these conditions are similar to those in Chapter 3. The first condition says that the marginal utility of consumption in each period should equal to the marginal social valuation of capital (shadow price). In the emission part, the marginal product of pollution is equal to the marginal degradation of pollution.

From these conditions, we obtain the socially optimal growth rate of consumption and capital accumulation as

$$\frac{\dot{c}}{c} = \sigma^{-1}\left[Az^\gamma - \rho + \xi(1-\sigma)(\delta - \frac{z}{E}) \right], \tag{75}$$

$$\frac{\dot{K}}{K} = Az^\gamma - c/K . \tag{76}$$

The optimal growth depends on the emission level and environmental quality. The effect of emission level on the growth rate of consumption is ambiguous. From equ. (75), when $E > \xi(1-\sigma)/A\gamma z^{\gamma-1}$, the pollution positively affects consumption growth rate, the higher the emission level, the higher the growth rate is; but when emission grows to a level at which the environmental quality is lower than $\xi(1-\sigma)/A\gamma z^{\gamma-1}$, the consumption starts to grow slower.

From equ. (71) and (72), we have the following relation between the marginal social valuation of capital and the marginal social valuation of environmental quality as

$$\theta = \lambda A\gamma K z^{\gamma-1} = A\gamma K z^{\gamma-1} c^{-\sigma} E^{\xi(1-\sigma)} . \tag{77}$$

Differentiating equ. (77) with respect to time, and using equ. (73), (74) and (76), we obtain the optimal growth rate of pollution as

$$\frac{\dot{z}}{z} = (1-\gamma)^{-1}[(\frac{\xi}{A\gamma z^{\gamma-1}E} - 1)\frac{c}{K} + \delta]. \tag{78}$$

The steady state is defined as a situation when the growth rates of consumption, capital accumulation, and output are constant and equal, and the growth rate of environmental quality is zero. The emission level is also constant in steady state.

The steady-state optimal pollution level is obtained as a constant, further, with a constant emission level, the environmental quality is also known, then the growth rates of consumption and capital accumulation are also constant. So, in the steady state, the environmental quality is $E = z/\delta$. Since emission level maintains constant in the steady state, the growth rate in equ. (78) is zero, and the following relation is obtained:

$$\frac{c}{K} = \delta/(1 - \frac{\xi\delta}{A\gamma z^{\gamma}}). \tag{79}$$

Using the fact that, in the steady state, the growth rates of consumption and capital are equal and constant, the steady-state optimal emission level is obtained by the following equation:

$$(1-\sigma^{-1})Az^{\gamma} + \rho\sigma^{-1} = \delta/(1 - \frac{\xi\delta}{A\gamma z^{\gamma}}). \tag{80}$$

To this step, we have shown that the steady-state growth rates of consumption, capital accumulation and output are constant and equal; and the optimal levels of environmental quality and the emission level are constant.

Government intervention is necessary to curtail the degradation of environmental quality. One obvious method is to tax on the emission so that the firm does not exhaust the environmental quality.

Assume the tax rate per unit of emission is τ, then the firm's first order conditions are $r = Az^{\gamma}$ and $\tau = A\gamma K z^{\gamma-1}$, where r is the return to capital rent. The optimal capital and emission levels are obtained from first order conditions.

The consumer rents capital to the firm and earns capital revenue, and chooses the optimal level of consumption in each period subject to its budget constraint, and the growth rate of consumption in competitive equilibrium is

$$\frac{\dot{c}}{c} = \sigma^{-1}\left[r - \rho + \xi(1-\sigma)(\delta - \frac{z}{E}) \right]. \tag{81}$$

In this growth rate equation, the emission level and environmental quality are exogenous to the consumer, the two variables are adjusted and controlled by the emission tax rate that makes the growth rate of environmental quality and emission level under a competitive equilibrium Pareto optimal.

The growth rate of pollution is obtained from the firm's first-order conditions as

$$\frac{\dot{z}}{z} = (1-\gamma)^{-1}(Az^{\gamma} - c/K - \frac{\dot{\tau}}{\tau}). \tag{82}$$

The socially optimal growth rate of pollution is shown in equ. (78), if we set equ. (82) equal to the socially optimal growth rate of pollution, we obtain the optimal emission tax rate that makes the decentralized competitive equilibrium growth paths be Pareto optimal. Therefore the optimal growth rate of emission tax rate is

$$\frac{\dot{\tau}}{\tau} = Az^{\gamma} - \delta - \frac{\xi}{A\gamma z^{\gamma-1}E}\frac{c}{K}. \tag{83}$$

In the steady state, the growth rate of tax grows at the same rate as capital. The optimal tax rate is obtained from the firm's first order condition $\tau = A\gamma K z^{\gamma-1}$, where the emission level is obtained from equ. (80). If the optimal tax rate is set such that $\tau^{*} = A\gamma K(z^{*})^{\gamma-1}$, where z^{*} is from equ. (80), then the steady-state level of emission is equal to z^{*}. So, the steady-state growth of the economy in the competitive equilibrium is Socially optimal.

From the optimal tax rate, we see that this rate grows at the same rate of capital in the steady state. This conclusion is also what is obtained in Chapter 3.

In the steady state, we could also use the technical standard to regulate pollution, the regulator could simply impose the allowed amount of emission on the firm's production. The optimal technical standard is just the optimal emission level obtained in equ. (80). So, the technical standard in steady state is constant, as is the same conclusion as what we reached in Chapter 3.

One possible extension of this model is to look into the transitional paths in equilibrium through numerical simulations. As the private equilibrium relies on more emissions, it may become costly to jump directly to the steady state as described in equ. (80). It may be interesting to examine the transitional dynamics of tax rate, pollution, output, capital and consumption in equilibrium.

CHAPTER 5. CONCLUSIONS

This paper explores the interactions of environmental quality and social welfare in the process of economic growth, using a simple endogenous growth model. It argues that environmental quality curtails the economic growth if there is no abatement activity in the society. Environmental policy is shown to be conducive to economic growth other than curtailing growth given that it is properly designed and implemented. In the long run, both the technical standards and tax instruments are able to move the market equilibrium growth paths to socially optimal ones.

The Pareto efficient competitive equilibrium under the technical standard and emission tax is unique. That is, there exist unique sets of prices and tax rates at which the growth paths and optimal capital allocation ratios in decentralized equilibrium are same as those in the socially optimal growth paths.

However, there exist several problems and difficulties in implementing the proposed environmental policies in this paper. Each of them has both merits and shortcomings. The technical standard and the emission tax are equivalent in promoting the economic growth, so the regulator should be indifferent in choosing either instrument. But, the technical standard requires that the government know the full information of the economy, e.g. the determinants of environmental quality, the consumption behavior and production technology. The collection of related information is costly which may be high such that the technical standard is not optimally implemented.

The optimal emission tax rate grows at the same rate as that of the economy in the steady state, the regulator needs to observe the capital accumulation in order to set the optimal tax rate.

The use of tax-subsidy to internalize pollution along the economic growth rate requires the introduction of governmental sector to the model to provide provision of public good.

Two notable issues not addressed in this paper are worthy pointing out. First, the decentralized competitive equilibrium may not exist without governmental intervention.

This does not agree with the real world because firms in reality usually control certain amount of emissions. One technical way to overcome this situation is to put an arbitrary ε-tax on emissions to avoid corner solutions of competitive economy.

Another issue is about transitional dynamics. Instead only examining the steady-state properties of the economy and environmental policy, it may be more interesting to look at the transitional paths of economic variables by using numerical techniques. In particular, we may want to know how the optimal environmental policy affects the change of emission level transitionally.

There are some aspects of the issue discussed in this paper that are not examined in this paper, two of which should be pointed out.

First, the technological change is not embodied in the analysis. This is a significant omission, since the new knowledge contributes much of pollution control in economies. The models of research and development in endogenous growth theory are due to Romer (1990), and Aghion and Howitt (1992) in two arenas. One interesting question is how the environmental policy could contribute to the incentives of technological changes that are environmentally sound.

Second, one point of endogenous growth models is that (positive) externalities in economy are part of engine of economic growth, such as knowledge spillovers. In recognizing that pollution is a negative externality, how the environmental externality and positive externalities interact in economic growth is another significant problem to be examined. This could be done in the framework of Uzawa-Lucas two sector model. The critical issue is how the environmental quality enters the model.

The interactions between environment and economic growth is complex and the understanding of economists on this issue is far from satisfactory, in part because the physical interactions are still not fully recognized in natural sciences, such as the CO_2 emissions. More works need to do to explain more realistically the various issues of sustainable growth of environment and economy.

APPENDIX A

A1. Socially Optimal Paths

The production function of the social planner is $Y = AK_y E^{\gamma} = AK_a^{\gamma\beta_1} K_y^{1-\gamma\beta_2}$.

The social planer's problem is to maximize the following utility by choosing the optimal consumption, environmental quality and abatement capital input:

$$Max \int_0^{\infty} e^{-\rho t} U(c(t), E(t)) dt \tag{a1}$$

$$s.t. \qquad E(t) \equiv E(K_a, K_y) = K_a^{\beta_1} K_y^{-\beta_2} \tag{a2}$$

$$\dot{K} = AK_y E^{\gamma} - c \tag{a3}$$

$$K \geq K_a + K_y \tag{a4}$$

To simplify this problem, we substitute equ. (a1) for the environmental quality in the utility function (a3), and use $K_y = K - K_a$ in social optimum.

The current-value Hamiltonian is:

$$H^c = \frac{[c^{\upsilon} K_a^{\beta_1 \omega} (K - K_a)^{-\beta_2 \omega}]^{1-\sigma} - 1}{1-\sigma} + \lambda [AK_a^{\gamma\beta_1} (K - K_a)^{1-\gamma\beta_2} - c].$$

The first-order necessary conditions are equ. (a3) and the following:

$$\frac{\partial H^c}{\partial c} = \upsilon c^{\upsilon(1-\sigma)-1} [K_a^{\omega\beta_1} (K - K_a)^{-\omega\beta_2}]^{1-\sigma} - \lambda = 0 \tag{a5}$$

$$\begin{aligned} \frac{\partial H^c}{\partial K_a} &= \omega c^{\upsilon(1-\sigma)} K_a^{\omega\beta_1(1-\sigma)-1} (K - K_a)^{-\omega\beta_2(1-\sigma)-1} [\beta_1 (K - K_a) + \beta_2 K_a] \\ &+ \lambda A K_a^{\gamma\beta_1 - 1} (K - K_a)^{-\gamma\beta_2} [\gamma\beta_1 (K - K_a) - (1 - \gamma\beta_2) K_a] = 0 \end{aligned} \tag{a6}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H^c}{\partial K} = \rho\lambda + \omega\beta_2 (c^{\upsilon} K_a^{\omega\beta_1})^{1-\sigma} (K - K_a)^{-\omega\beta_2(1-\sigma)-1} - \lambda(1 - \gamma\beta_2) AK_a^{\gamma\beta_1} (K - K_a)^{-\gamma\beta_2} \tag{a7}$$

From condition (a5) and (a6), we obtain the optimal relations among the output, consumption, abatement capital and dirty capital:

$$\frac{c}{Y}=\frac{\upsilon[-\gamma\beta_1 K_y+(1-\gamma\beta_2)K_a]}{\omega(\beta_1 K_y+\beta_2 K_a)} \tag{a8}$$

From conditions (a5) and (a7), we obtain the socially optimal growth rates of consumption and output and capital:

$$\frac{\dot{c}}{c}=\frac{1}{\Psi}\left[\frac{AK_a^{\gamma\beta_1}K_y^{-\gamma\beta_2}\beta_1}{\beta_1+\beta_2 K_a/K_y}-\rho+\omega(1-\sigma)(\beta_1\frac{\dot{K}_a}{K_a}-\beta_2\frac{\dot{K}_y}{K_y})\right] \tag{a9}$$

$$\frac{\dot{Y}}{Y}=\gamma\beta_1\frac{\dot{K}_a}{K_a}+(1-\gamma\beta_2)\frac{\dot{K}_y}{K_y} \tag{a10}$$

$$\frac{\dot{K}}{K}=\frac{AK_a^{\gamma\beta_1}K_y^{1-\gamma\beta_2}}{K}-\frac{c}{K} \tag{a11}$$

Case (1): environmental quality is only a consumable good.

In this case, the environmental quality will not affect production, thus $\gamma=0$, the necessary first-order conditions become:

$$\lambda=\upsilon c^{\upsilon(1-\sigma)-1}(K_a^{\omega\beta_1}K_y^{-\omega\beta_2})^{1-\sigma} \tag{a5'}$$

$$\omega c^{\upsilon(1-\sigma)}K_a^{\omega\beta_1(1-\sigma)-1}(K-K_a)^{-\omega\beta_2(1-\sigma)-1}[\beta_1(K-K_a)+\beta_2 K_a]-\lambda A=0 \tag{a6'}$$

$$\dot{\lambda}=\rho\lambda+\omega\beta_2(c^{\upsilon}K_a^{\omega\beta_1})^{1-\sigma}(K-K_a)^{-\omega\beta_2(1-\sigma)-1}-\lambda A \tag{a7'}$$

$$\dot{K}=A(K-K_a)-c \tag{a3'}$$

Substitute equ.(a5′) into (a6′), we get the optimal ratios of capitals and consumption:

$$\frac{K_a}{c}=\frac{\omega}{A\upsilon}(\beta_1+\beta_2\frac{K_a}{K_y}),\ \text{or}\ \frac{c}{K_y}=\frac{A\upsilon(K_a/K_y)}{\omega(\beta_1+\beta_2 K_a/K_y)}. \tag{a8'}$$

Substitute equ.(a6′) into (a7′), and take derivative of equ.(a5′) with respect to time, after some simplification, we get the socially optimal growth rate of consumption.

$$\frac{\dot{c}}{c}=\frac{1}{\Psi}\left[\frac{A\beta_1}{\beta_1+\beta_2\mu}-\rho+\omega(1-\sigma)(\beta_1\frac{\dot{K}_a}{K_a}-\beta_2\frac{\dot{K}_y}{K_y})\right] \quad \text{(a9′)}$$

The condition for a balanced growth is the abatement-dirty capital ratio is constant, but does not require $\beta_1=\beta_2$. This also enables the growth rates are all constant and equal, except that the environmental quality grows at a none-zero rate.

Then the steady-state growth rate of consumption is:

$$\frac{\dot{c}}{c}=\frac{1}{\Psi+(1-\sigma)\omega(\beta_1-\beta_2)}\left[\frac{A\beta_1}{\beta_1+\beta_2\mu}-\rho\right] \quad \text{(a12)}$$

From equ. (a3′), we could get the growth rate of capital accumulation:

$$\frac{\dot{K}_y}{K_y}=\frac{\dot{K}_a}{K_a}=\frac{A}{1+\mu}-\frac{c}{(1+\mu)K_y}. \quad \text{(a13)}$$

From equ. (a14), (a8′) and (a13), we could obtain the optimal abatement-dirty capital ratio by the following equation:

$$\frac{1}{\Psi+\omega(1-\sigma)(\beta_1-\beta_2)}\left[\frac{A\beta_1}{\beta_1+\beta_2\mu}-\rho\right]=\frac{A}{1+\mu}\left[1-\frac{\upsilon\mu}{\omega(\beta_1+\beta_2\mu)}\right] \quad \text{(a14)}$$

The steady-state optimal capital allocation is then obtained:

$$\mu=\frac{A[\beta_1-\Delta(\beta_2-\upsilon)-\rho(\beta_1+\beta_2)]\pm\sqrt{A^2[\beta_1-\Delta(\beta_2-\upsilon)-\rho(\beta_1+\beta_2)]^2+4\rho\beta_1\beta_2[A(1-\Delta)-\rho]}}{2\rho\beta_2},$$

where $\Delta=\Psi+\omega(1-\sigma)(\beta_1-\beta_2)$.

When $\beta_2=\beta_1=\beta$, the balanced growth rate is:

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[\frac{A}{1+\mu} - \rho\right]. \tag{a15}$$

When $\mu < \frac{A-\rho}{\rho}$, the growth rate is positive.

The optimal capital allocation is

$$\mu = \frac{A[\beta(1-\Psi-2\rho)+\Psi\upsilon] \pm \sqrt{A^2[\beta(1-\Psi-2\rho)+\Psi\upsilon]^2 + 4\rho\beta^2[A(1-\Psi)-\rho]}}{2\rho\beta}.$$

The optimal abatement-dirty capital ratio is obtained from (a13), from which the parameter restrictions could be obtained together with $\mu < \frac{A-\rho}{\rho}$.

Case (2): environmental quality is only a productive good.

In this case, $\omega = 0$, the first-order necessary conditions are:

$$\upsilon c^{\upsilon(1-\sigma)-1} - \lambda = 0 \tag{a5''}$$

$$\lambda A K_a^{\gamma\beta_1 - 1}(K-K_a)^{-\gamma\beta_2}[\gamma\beta_1(K-K_a) - (1-\gamma\beta_2)K_a] = 0 \tag{a6''}$$

$$\dot{\lambda} = \rho\lambda - \lambda(1-\gamma\beta_2)AK_a^{\gamma\beta_1}(K-K_a)^{-\gamma\beta_2} \tag{a7''}$$

$$\dot{K} = AK_y E^{\gamma} - c. \tag{a3''}$$

From equ. (a6''), we obtain the optimal capital allocation ratio:

$$\mu = \frac{K_a}{K_y} = \frac{\gamma\beta_1}{1-\gamma\beta_2} \tag{a16}$$

The growth rate of consumption is:

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[AK_a^{\gamma\beta_1}K_y^{-\gamma\beta_2}\frac{\beta_1}{\beta_1 + \beta_2 K_a / K_y} - \rho\right]. \tag{a17}$$

The steady-state growth rate requires $\beta_1 = \beta_2$, but the optimal capital allocation is always constant which will guarantee steady-state growth. So the steady-state growth rate is:

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[A(1-\gamma\beta)(\frac{\gamma\beta}{1-\gamma\beta})^{\gamma\beta} - \rho\right]. \tag{a18}$$

The growth rate of capital accumulation is:

$$\frac{\dot{K}_y}{K_y} = \frac{\dot{K}_a}{K_a} = \frac{A\mu^{\gamma\beta}}{1+\mu} - \frac{c}{(1+\mu)K_y} \tag{a19}$$

Setting equ. (a18) equal to (a19), we obtain the steady state consumption-capital ratio as

$$\frac{c}{K_y} = \frac{1}{\Psi}\left[A\mu^{\gamma\beta}(\Psi-1) + \rho(1+\mu)\right], \tag{a20}$$

and the steady-state consumption rate as

$$\frac{c}{Y} = \frac{1}{\Psi}\left[(\Psi-1) + \frac{\rho(1+\mu)}{A\mu^{\gamma\beta}}\right]. \tag{a21}$$

Case (3): environmental quality is both an input of production and a consumable good.

For a balanced growth rate exists, the elasticities of environmental quality in the two types capital stocks have to be equal, or $\beta_1 = \beta_2 = \beta$. Further, the ratio of two types of capital stock must maintain constant in steady state. $g = \frac{\dot{c}}{c} = \frac{\dot{K}}{K} = \frac{\dot{K}_a}{K_a} = \frac{\dot{K}_y}{K_y} = \frac{\dot{Y}}{Y}$, then the balanced growth rates are:

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[\frac{A\mu^{\gamma\beta}}{1+\mu} - \rho\right]; \tag{a22}$$

$$\frac{\dot{K}_y}{K_y} = \frac{\dot{K}_a}{K_a} = \frac{A\mu^{\gamma\beta}}{1+\mu} - \frac{c}{(1+\mu)K_y}. \tag{a23}$$

The consumption rate in this case is

$$\frac{c}{Y} = \frac{\upsilon[-\gamma\beta + (1-\gamma\beta)\mu]}{\omega\beta(1+\mu)}. \tag{a24}$$

A2. Basic Discrete-time Model

The Social planner's problem is to maximize the social welfare by choosing the optimal level of consumption, capital accumulation, and the capital allocation in each period. Assume that there is no capital depreciation.

$$\text{Max} \sum_{t=1}^{\infty} r^t U(c_t, E_t) \tag{a25}$$

$$\text{s.t. } K_{t+1} = F(K_{yt}, E(K_{yt}, K_{at})) - c_t + K_t,\ t = 1,\ldots,\infty. \tag{a26}$$

The Lagrangian is:

$$L = \sum_{t=1}^{\infty} r^t U(c_t, E(K_{yt}, K_{at})) + \sum_{t=1}^{\infty} \lambda_t [F(K_{yt}, E(K_{yt}, K_{at})) - c_t + K_t - K_{t+1}]. \tag{a27}$$

So, the first-order necessary conditions are:

$$\frac{\partial L}{\partial c} = r^t U_c(c_t, E_t) - \lambda_t = 0,\ t = 1,\ldots,\infty; \tag{a28}$$

$$\frac{\partial L}{\partial K_{yt}} = r^t U_{E_t}(c_t, E_t) E_{K_{yt}}(K_{yt}, K_{at}) + \lambda_t [F_{K_{yt}}(K_{yt}, E_t) + 1] - \lambda_{t-1} = 0,\ t = 1,\ldots,\infty; \tag{a29}$$

$$\frac{\partial L}{\partial K_{at}} = r^t U_{E_t}(c_t, E_t) E_{K_{at}}(K_{yt}, K_{at}) + \lambda_t [F_{K_{at}}(K_{yt}, E_t) + 1] - \lambda_{t-1} = 0. \tag{a30}$$

A3. The Socially Optimal Paths—Discrete-time Case

The social planner's problem is

$$\max \sum_{t=0}^{\infty} r^t \frac{(c_t^{\upsilon} E_t^{\omega})^{1-\sigma} - 1}{1-\sigma} \tag{a31}$$

s.t. $$E_t = K_{at}^{\beta_1} K_{yt}^{-\beta_2}, \ t = 0,\ldots,\infty; \tag{a32}$$

$$K_{t+1} \le AK_{yt} E_t^{\gamma} - c_t, \ t = 0,\ldots,\infty; \tag{a33}$$

$$K_t \ge K_{at} + K_{yt}, \ t = 0,\ldots,\infty. \tag{a34}$$

It is hard to use dynamic programming in this problem because there are two control variables (consumption and abatement capital), we try to use the usual Lagrangian method:

$$L = \sum_{t=0}^{\infty} r^t \frac{(c_t^{\upsilon} K_{at}^{\beta_1\omega} (K_t - K_{at})^{-\beta_2\omega})^{1-\sigma} - 1}{1-\sigma} + \sum_{t=0}^{\infty} \lambda_t [A(K_t - K_{at})^{1-\gamma\beta_2} K_{at}^{\gamma\beta_1} - c_t - K_{t+1}]. \tag{a35}$$

The first-order necessary conditions are for $t = 0,\ldots,\infty$,

$$\frac{\partial L}{\partial c_t} = r^t \upsilon c_t^{\upsilon(1-\sigma)-1} (K_{at}^{\beta_1\omega} (K_t - K_{at})^{-\beta_2\omega})^{1-\sigma} - \lambda_t = 0; \tag{a36}$$

$$\begin{aligned} \frac{\partial L}{\partial K_{at}} &= r^t \omega c_t^{\upsilon(1-\sigma)} K_{at}^{\omega\beta_1(1-\sigma)-1} (K_t - K_{at})^{-\omega\beta_2(1-\sigma)-1} [\beta_1 (K_t - K_{at}) + \beta_2 K_{at}] \\ &+ \lambda_t A K_{at}^{\gamma\beta_1 - 1} (K_t - K_{at})^{-\gamma\beta_2} [\gamma\beta_1 (K_t - K_{at}) - (1-\gamma\beta_2) K_{at}] = 0 \end{aligned}; \tag{a37}$$

$$\frac{\partial L}{\partial K_t} = (-\beta_2\omega) r^t c_t^{\upsilon(1-\sigma)} K_{at}^{\beta_1\omega(1-\sigma)} (K_t - K_{at})^{-\beta_2\omega(1-\sigma)-1} + \lambda_t A(1-\gamma\beta_2)(K_t - K_{at})^{-\gamma\beta_2} K_{at}^{\gamma\beta_1} - \lambda_{t-1} = 0. \tag{a38}$$

From these conditions, we get the policy function as

$$c_t = \frac{A\upsilon K_{at}^{\gamma\beta_1} (K_t - K_{at})^{1-\gamma\beta_2} [(1-\gamma\beta_2) K_{at} - \gamma\beta_1 (K_t - K_{at})]}{\omega[\beta_2 K_{at} + \beta_1 (K_t - K_{at})]}. \tag{a39}$$

But the value of abatement capital is not known yet. Now plug the above policy function on consumption into the second condition (a37), then plug the second condition (a37) into the third condition (a39) to cancel out λ_t and λ_{t-1}. We obtain a difference equation that only has one unknown term K_{at}, solving it we get the optimal abatement capital. In the abatement capital equation, there is abatement capital of previous period, it is a highly non-linear difference equation. Plug the abatement capital back into policy function, we get the optimal consumption level in each period of growth.

We only show how this works in the Case (3) where $\gamma \neq 0, \omega \neq 0$. From the continuous time model, the condition $\beta_1 = \beta_2 = \beta$ should hold so that we may find steady state. In this case, we obtain the optimal abatement-polluting capital through the following complex equation:

$$\mu_t^{\omega\beta(1-\sigma)+\gamma\beta\Delta}(1+\mu_t)^{1-2\Delta}[(1-\gamma\beta)\mu_t-\gamma\beta]^{-\Psi}=(Ar)^{-1}(K_t/K_{t-1})^{\Psi}\mu_{t-1}^{\omega\beta(1-\sigma)-\gamma\beta\Psi}(1+\mu_{t-1})^{2\Psi}[(1-\gamma\beta)\mu_{t-1}-\gamma\beta]^{-\Psi},$$

where $\Delta = \upsilon(1-\sigma), \Psi = 1-\Delta$.

APPENDIX B

In a competitive economy, the representative agent maximizes life-time utility by choosing the optimal consumption and investment plan as following:

$$\max \int_0^\infty e^{-\rho t} U(c,E)dt \tag{b1}$$

$$\text{s.t. } \dot{K} = r_y K_y + r_a K_a - c \tag{b2}$$

$$K = K_y + K_a \tag{b3}$$

Under technical standard, the current-value Hamiltonian is:

$$H^c = \frac{(c^\upsilon E^\omega)^{1-\sigma} - 1}{1-\sigma} + \lambda(r_y K_y + r_a K_a - c) \tag{b4}$$

The first-order necessary conditions are:

$$\frac{\partial H^c}{\partial c} = \upsilon c^{\upsilon(1-\sigma)-1} E^{\omega(1-\sigma)} - \lambda = 0; \tag{b5}$$

$$\frac{\partial H^c}{\partial K_a} = -r_y + r_a = 0; \tag{b6}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H^c}{\partial K} = \rho\lambda - \lambda r_y. \tag{b7}$$

From equ. (b5) and (b7), we obtain the competitive equilibrium path:

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[r_y - \rho + \omega(1-\sigma)\frac{\dot{E}}{E} \right]. \tag{b8}$$

B1. Technical Standard

The firm's optimal condition is $AE^\gamma - r_y - r_a \mu = 0$.

From (b6) and the firm's optimal condition, we obtain the following equilibrium relation:

$$r_y = r_a = \frac{AE^{\gamma}}{1+\mu}. \tag{b9}$$

Plug equ. (b9) into (b8), and use $\frac{\dot{E}}{E} = \beta_1 \frac{\dot{K}_a}{K_a} - \beta_2 \frac{\dot{K}_y}{K_y}$, we obtain the competitive equilibrium growth rate of consumption as following:

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[\frac{A(K_a^{\beta_1} K_y^{-\beta_2})^{\gamma}}{1+\mu} - \rho + \omega(1-\sigma)(\beta_1 \frac{\dot{K}_a}{K_a} - \beta_2 \frac{\dot{K}_y}{K_y})\right]. \tag{b10}$$

The steady-state growth of consumption requires that $\beta_1 = \beta_2 = \beta$. This corresponds to the balanced growth conditions in case (3) of Appendix A. Then the steady-state growth rate is

$$\frac{\dot{c}}{c} = \frac{1}{\Psi}\left[\frac{A\mu^{\gamma\beta}}{1+\mu} - \rho\right]. \tag{b11}$$

The steady-state growth rate of capital is:

$$\frac{\dot{K}}{K} = \frac{A\mu^{\gamma\beta}}{1+\mu} - \frac{c}{(1+\mu)K_y}. \tag{b12}$$

B2. Standard in the Second Best

From B1, it is apparent that the steady-state growth rates in competitive equilibrium under an income tax are as follows:

$$\frac{\dot{c}}{c} = \frac{1}{1-\upsilon(1-\sigma)}\left[\frac{(1-\tau_y)pA\mu^{\gamma\beta}}{1+\mu} - \rho\right]; \tag{b13}$$

$$\frac{\dot{K}}{K}=\frac{(1-\tau_y)A\mu^{\gamma\beta}}{1+\mu}-\frac{c}{K}. \tag{b14}$$

B3. Environmental Tax

By choosing the optimal capitals K_y and K_a, the firm maximizes profit $\pi = Y - r_y K_y - r_a K_a - \tau K_y^{\beta_2} K_a^{-\beta_1}$, where $Y = AK_y E^{\gamma}$.

The first order condition is:

$$r_y = \frac{\partial Y}{\partial K_y} - \tau \frac{\partial D}{\partial K_y} = AE^{\gamma} - \tau\beta_2 K_y^{\beta_2 - 1} K_a^{-\beta_1}; \tag{b15}$$

$$r_a = \frac{\partial Y}{\partial K_a} - \tau \frac{\partial D}{\partial K_a} = \tau\beta_1 K_y^{\beta_2} K_a^{-\beta_1 - 1}. \tag{b16}$$

By condition (b6), under an emission tax, in equilibrium the returns of production and abatement capitals would be equal, so that the consumer will be indifferent in making investment decisions. This implies that:

$$\tau(\beta_1 K_y + \beta_2 K_a) = AK_a^{1+(1+\gamma)\beta_1} K_y^{1-(1+\gamma)\beta_2}. \tag{b17}$$

The consumer's problem is still equ. (b1), (b2) and (b3), and the first-order necessary conditions are equ. (b5), (b6) and (b7). Plug equ. (b15) into equ. (b8), we obtain the competitive equilibrium growth rates of consumption with an emission tax as:

$$\frac{\dot{c}}{c}=\frac{1}{\Psi}\left[AE^{\gamma} - \tau\beta_2 K_y^{\beta_2-1} K_a^{-\beta_1} - \rho + \omega(1-\sigma)(\beta_1 \frac{\dot{K}_a}{K_a} - \beta_2 \frac{\dot{K}_y}{K_y})\right] \tag{b18}$$

$$\frac{\dot{K}}{K} = r_a - \frac{c}{K}. \tag{b19}$$

Set this growth rate in equ. (b18) equal to equ. (a9) to obtain the optimal emission tax rate, then:

$$\tau^* = \frac{AK_a^{1+(1+\gamma)\beta_1} K_y^{1-(1+\gamma)\beta_2}}{\beta_1 K_y + \beta_2 K_a}. \tag{b20}$$

But from equ. (b17) we could derive the same optimal tax rate. This proves that the competitive equilibrium in an economy with emission tax rate set in equ. (b20) realizes the Pareto optimal growth rate.

Plug the optimal tax rate back into the firm's first-order conditions, we obtain:

$$r_a = \frac{A\beta_1 K_a^{\gamma\beta_1} K_y^{-\gamma\beta_2}}{\beta_1 + \beta_2 (K_a / K_y)}. \tag{b21}$$

Next turn to the steady states for three cases of the functions of environmental quality in the economy.

Case (1): environmental quality is only a consumable good.

In this case, the optimal tax rate and optimal capital allocation is respectively:

$$\mu = K_a / K_y = \frac{\beta_1 (A - r_a)}{\beta_2 r_a}; \tag{b22}$$

$$\tau^* = \frac{A\mu^{1+\beta_1} K_y^{1+\beta_1-\beta_2}}{\beta_1 + \beta_2 \mu}. \tag{b23}$$

The equilibrium growth rate of consumption is:

$$\frac{\dot{c}}{c} = \frac{1}{1-\upsilon(1-\sigma)} \left[r_a - \rho + \omega(1-\sigma)(\beta_1 - \beta_2)\frac{\dot{K}_y}{K_y} \right]. \tag{b24}$$

When the consumption-saving ratio is constant, there exist steady state with a growth rate as:

$$\frac{\dot{c}}{c} = \frac{\dot{K}}{K} = \frac{r_a - \rho}{1-(1-\sigma)[\upsilon + \omega(\beta_1 - \beta_2)]} \tag{b25}$$

Case (2): environmental quality is only a productive good.

Set $\omega = 0$, the steady-state optimal tax rate and optimal capital allocation are same as that of Case (3) shown in equ. (b27) and (b28). But the steady state growth of consumption exists without requiring $\beta_1 = \beta_2$:

$$\frac{\dot{c}}{c} = \frac{r_a - \rho}{1 - \upsilon(1-\sigma)} \tag{b26}$$

When $\beta_1 = \beta_2$ as in Case (3), the capital allocation ratio is constant in the sense that capital rent is given. If consumption-capital ratio maintains constant the steady-state growth rate of capital accumulation exists shown in equ. (b19).

Case (3): environmental quality is both an input of production and a consumable good.

From equ. (b18) and (b20), if $\beta_1 = \beta_2 = \beta$, steady-state growth paths in competitive equilibrium are possible to exist. To show this, look at equ. (b21), we obtain the following equation:

$$r_a = \frac{A(K_a / K_y)^{\gamma\beta}}{1 + (K_a / K_y)}. \tag{b27}$$

The steady-state tax rate is:

$$\tau^* = \frac{A\mu^{1+(1+\gamma)\beta} K_y}{\beta(1+\mu)}. \tag{b28}$$

From equ. (b19), it is observed that the steady-state capital accumulation rate exists when consumption-saving ratio is constant.

B4. Competitive Equilibrium—Discrete-time Case

Technical standard

The firm's first-order condition is written as $AE_t^{\gamma} - r_{yt} - r_{at}\mu_t = 0$.

Consumer's problem is

$$\max \sum_{t=0}^{\infty} r^t \frac{(c_t^{\upsilon} E_t^{\omega})^{1-\sigma} - 1}{1-\sigma} \tag{b42}$$

s.t. $$K_{t+1} \le K_t + r_{yt} K_{yt} + r_{at} K_{at} - c_t,\ t = 0,\ldots,\infty; \tag{b43}$$

$$K_t \ge K_{at} + K_{yt},\ t = 0,\ldots,\infty. \tag{b44}$$

The Lagrangian is

$$L = \sum_{t=0}^{\infty} r^t \frac{(c_t^{\upsilon} E^{\omega})^{1-\sigma} - 1}{1-\sigma} + \sum_{t=0}^{\infty} \theta_t [K_t + r_{yt} K_{yt} + r_{at} K_{at} - c_t - K_{t+1}] \tag{b45}$$

First-order necessary conditions are

$$\frac{\partial L}{\partial c_t} = \upsilon r^t c_t^{\upsilon(1-\sigma)-1} E_t^{\omega(1-\sigma)} - \theta_t = 0,\ t = 0,\ldots,\infty; \tag{b46}$$

$$\frac{\partial L}{\partial K_t} = \theta_t (1 + r_{yt}) - \theta_{t-1} = 0,\ t = 1,\ldots,\infty; \tag{b47}$$

$$\frac{\partial L}{\partial K_{at}} = \theta_t (r_{at} - r_{yt}) = 0,\ t = 0,\ldots,\infty. \tag{b48}$$

Two derived conditions are obtained for the consumer:

$$r_{at} = r_{yt}; \tag{b49}$$

$$c_t = \left[\frac{\theta_t}{\upsilon r^t E_t^{w(1-\sigma)}} \right]^{\Psi} = \left[\frac{\theta_0}{[(1+r_{yt})(1+r_{y(t-1)}) \cdots (1+r_{y0})] \upsilon r^t E_t^{w(1-\sigma)}} \right]^{\Psi}. \tag{b50}$$

The optimal level of consumption in period t depends on the initial consumption level c_0. But the initial consumption is arbitrary, which can not be solved in the consumer's problem.

However, combining the first-order conditions for firm and consumer, the following relation should be satisfied in equilibrium:

$$r_{yt} = r_{at} = \frac{AE_t^{\gamma}}{1+\mu_t}. \tag{b51}$$

At period *t*, the optimal consumption-capital ratio and consumption rate depends on the initial level of consumption and capital.

In each period, the optimal abatement is $K_{at} = [\mu_t/(1+\mu_t)]K_t$.

Emission tax

Firm's first-order conditions are

$$r_{yt} = AE_t^{\gamma} - \tau_t \beta_2 K_{yt}^{\beta_2 - 1} K_{at}^{-\beta_1}; \tag{b52}$$

$$r_{at} = \tau_t \beta_1 K_{yt}^{\beta_2} K_{at}^{-\beta_1 - 1}. \tag{b53}$$

APPENDIX C

C1. Social Planner's Problem

The social planner's problem is

$$\max \int_0^\infty e^{-\rho t} U(c,S)dt \tag{c1}$$

$$\text{s.t.} \qquad \dot{K} = AK_y S^{-\gamma} - c \tag{c2}$$

$$\dot{S} = -\delta S + K_y^{\beta} K_a^{-\beta} \tag{c3}$$

The current-value Hamiltonian is

$$H^c = \frac{(cS^{-\xi})^{1-\sigma} - 1}{1-\sigma} + \lambda(AK_y S^{-\gamma} - c) + \theta(-\delta S + K_y^{\beta} K_a^{-\beta}). \tag{c4}$$

The first-order conditions are:

$$\frac{\partial H^c}{\partial c} = c^{-\sigma} S^{-\xi(1-\sigma)} - \lambda = 0; \tag{c5}$$

$$\frac{\partial H^c}{\partial a} = -\lambda A S^{-\gamma} - \theta\beta K K_y^{\beta-1} K_a^{-\beta-1} = 0; \tag{c6}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H^c}{\partial K} = \rho\lambda - [\lambda A S^{-\gamma} + \theta\beta K_y^{\beta-1} K_a^{-\beta}; \tag{c7}$$

$$\dot{\theta} = \rho\theta - \frac{\partial H^c}{\partial S} = \rho\theta - [-\xi c^{1-\sigma} S^{-\xi(1-\sigma)-1} - \lambda A\gamma K_y S^{-\gamma-1} - \theta\delta]. \tag{c8}$$

From equ. (c6), we get the relation between marginal social value of capital and marginal social value of environment as

$$\frac{\theta}{\lambda} = -\frac{AS^{-\gamma}}{\beta K K_y^{\beta-1} K_a^{-\beta-1}}. \tag{c9}$$

Plug (c9) into (c7), and combine equ. (c5) and c(7), we obtain:

$$\frac{\dot{c}}{c} = \sigma^{-1}\left[AS^{\gamma}K_y / K - \rho - \xi(1-\sigma)(K_y^{\beta}K_a^{-\beta} / S - \delta)\right]. \tag{c10}$$

And the growth rate of capital is

$$\frac{\dot{K}}{K} = \frac{AK_y S^{-\gamma}}{K} - \frac{c}{K}. \tag{c11}$$

Steady state: Let $\mu = K_a / K_y$, differentiating equ. (c9) with respect to time, and combining equ. (c7) and (c8) we obtain the steady-state growth rate of capital accumulation as:

$$\frac{\dot{K}}{K} = \frac{\dot{\theta}}{\theta} - \frac{\dot{\lambda}}{\lambda} = \delta - [\frac{\beta\xi}{A} S^{\gamma-1}\mu^{-\beta-1}(1+\mu)^2]\frac{c}{K} - \gamma\beta S^{-1}\mu^{-\beta-1}(1+\mu) + AS^{-\gamma}K_y / K. \tag{c12}$$

Set equ. (c12) equal to equ. (c11), we obtain the steady-state consumption-capital ratio expressed with abatement-polluting capital ratio.

By the steady state conditions, set equ. (c10) and (c11) equal and use the obtained consumption-capital ratio, we obtain the steady-state abatement-polluting capital ratio.

C2. Competitive Equilibrium

The consumer's problem is

$$\max \int_0^{\infty} e^{-\rho t} U(c,S)dt \tag{c12}$$

$$s.t. \qquad \dot{K} = r_y K_y + r_a K_a - c \tag{c13}$$

The current-value Hamiltonian is

$$H^c = \frac{(cS^{-\xi})^{1-\sigma} - 1}{1-\sigma} + \lambda(r_y K + r_a K_a - c). \tag{c14}$$

The first-order conditions are

$$\frac{\partial H^c}{\partial c} = c^{-\sigma} S^{-\zeta(1-\sigma)} - \lambda = 0\,; \tag{c15}$$

$$r_y - r_a = 0\,; \tag{c16}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H^c}{\partial K} = \rho\lambda - \lambda r_y\,. \tag{c17}$$

The growth rate of consumption is

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\left[r_y - \rho - \xi(1-\sigma)(K_y^{\beta} K_a^{-\beta} - \delta)\right]. \tag{c18}$$

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