Entry Deterrence by Non-Horizontal Merger

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Abstract

We study when and how pure non-horizontal mergers, whether cross-product or vertical, can deter new entry. Organizational mergers implicitly commit firms to more aggressive price competition. Because heightened competition deters entry, mergers can occur in equilibrium even when, absent entry considerations, they do not. We show that, in order to prevent a flood of entrants, mergers arise even when a marginal merger costs incumbent firms more than does a marginal entrant.

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<u>I. Introduction</u>

   This paper studies pure non-horizontal mergers, whether vertical or cross-product. Vertical mergers combine input and output producers in a single firm. Cross-product mergers combine sellers of different products in a single firm -- a "superstore" vs. a mall, for example. Unlike horizontal mergers, neither of these combinations integrates producers of the same output good. The question we ask is: Do such non-horizontal mergers deter the entry of rival producer / retailer chains. And if so, do such entry-deterrence considerations help explain when and why such mergers occur?

   Multi-product retailing is pervasive in contemporary economies, whether in supermarkets, "big box" superstores, malls or bundled products (e.g., computer hardware and software). Because "shopping" -- the process of searching for desired products and purchasing them -- is a time-consuming activity, there are economies of retailing many products at one location, whether in physical, product or cyber space (Dudey, 1990). A key question for positive economics is: how might one expect this multi-product retailing to be organized -- for example, in a mall of independent outlets or an integrated (merged) "big box" store? Overall, the literature has given relatively little attention to this question. However, Beggs (1994) offers one possible thesis for the emergence of "malls." By spurring marginal consumers to shop elsewhere, a higher price of one outlet's product can reduce demand at another store; merged outlets, accounting for this cross-store externality, charge lower prices (ceteris paribus).[1] Although this practice is profit-enhancing given the pricing regimes of rival retailers, it can prompt rivals to lower their prices as well. In view of this effect, a multi-product retailer can effectively pre-commit to higher prices by organizing

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[1]Some recent papers (Pashigian, 1998; Bruekner, 1993) consider how mall contracts (and store rents in particular) can help internalize inter-store externalities in the choice of store space (square footage). This work is particularly useful in showing how mall developers can maximize and reap mall profits. Other papers study the economics of multi-product retailing (e.g., Bliss, 1988; Klemperer, 1992; Armstrong, 1999; Girard-Heraud, et al., 2003; and many others). However, neither of these literatures considers the choice between alternative organizational forms (mall vs. superstore) and the pricing externalities at the heart of this choice.

itself as a mall of independent outlets. Because rivals respond with higher prices, this pre-commitment can be advantageous.

In this paper, we take this logic a step further by considering the role of mergers in altering entry incentives. Precisely because mergers lead to lower market prices -- their disadvantage in a model without any prospective entry -- they discourage entry of new rivals. When entry is costly to extant retailers, mergers can thus be advantageous for retail groups, essentially because their anti-competitive effect (limiting entry) can dominate their pro-competitive effect (lowering prices for a given set of competitors). Moreover, this can be true even when mergers are "very costly" to retailers -- indeed, when marginal mergers are more costly than is marginal entry. The reason is that mergers are then needed not only to deter marginal entry, but to prevent an "opening of the floodgates" that prompts the operation of many more retailers than would operate if mergers were made.

Similar logic applies in the case of vertical mergers, a topic much more exhaustively treated in the literature. A well-known advantage of vertical separation in differentiated product markets (Bonnano and Vickers, 1988) is that separated output producers and input suppliers can sign contracts that precommit them to above-cost wholesale prices, which in turn prompt the output producers to charge higher output prices (ceteris paribus). Because rival output prices are strategic complements, rivals respond to the contractual precommitment by charging higher prices themselves -- the strategic benefit of vertical separation. Here, this logic implies that vertical separation, by prompting higher equilibrium prices, will encourage entry. Conversely, vertical integration -- by voiding the opportunity for contractual precommitments -- deters entry.[2]

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[2]There is an extensive literature on optimal contracts in vertically separated markets; a very small sample -- with apologies for omissions -- is Rey and Stiglitz (1988), Bolton and Bonnano (1988), Wiinter (1993), Perry and Besanko (1991), Blair and Lewis (1994). To our knowledge, the entry-enhancing effect of vertical contracts -- and the attendant entry-deterrence motive for vertical integration identified here -- have not yet been studied. The vertical restraint literature implicitly offers some other motives for vertical integration that are absent here. For example, contracts may be unable to achieve desired outcomes due to their unobservability (O'Brien and Shaffer, 1992) or a principle's limited commitment ability (McAfee and Schwartz, 1994).

This entry-deterrence motive for vertical integration differs from the recent literature on "vertical foreclosure," wherein vertical integration also has the effect of limiting downstream competition. The dominant theme in this work is that vertically integrated firms can exploit their control of an input supplier to raise input prices to which competitors are subject (see, for example, Salinger, 1988; Ordover, Saloner and Salop, 1990; Hart and Tirole, 1990; Bolton and Whinston, 1991; Reiffen, 1992; Riordan, 1998; Economides, 1998; Chen, 2001; Chemla, 2003).[3] A necessary ingredient to these "raising rivals' costs" arguments (Salop and Scheffman, 1987) is that the input market not be perfectly contestable. In this paper, we void such motives for vertical integration by assuming that input markets are perfectly contestable (with inputs produced at a common constant marginal cost in a free-entry industry).

The balance of the paper is organized as follows. Section II presents the general argument for entry-deterring non-horizontal mergers, deriving sufficient conditions for an equilibrium to maximally deter entry. Section III evaluates these conditions in two applications, one a generalized version of Beggs' (1994) model of mergers and malls, and the other a simple model of vertical chains. In both cases, we show that entry-deterring mergers occur despite traditional (contractual precommitment) motives for organizational separation. Section IV concludes.

II. The General Model

Let N denote the number of independent retail units, where a retail unit may be a vertical supplier / producer chain (as in Bonnano and Vickers, 1988) or a multi-product seller (as in Beggs, 1994). Each retail unit may either be merged or not merged. For example, a supplier / producer chain can be vertically integrated (merged) or vertically separated (not merged). Similarly, a multi-product seller can retail each product with

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[3]See also related papers on entry deterrence by exogenously vertically integrated firms that can affect rivals' input supplies (Song and Kim, 2000; Reiffen, 1998; Weisman, 1995) and on supply assurance motivations for vertical integration (Bolton and Whinston, 1993). Chemla (2003) presents an extreme version of the raising rivals' cost argument, wherein vertical integration by an upstream monopolist forecloses downstream competition that would otherwise arise due to exogenous limits on the monopolist's contractual bargaining power.

independent firms (the case of an unmerged mall) or retail all products in one firm (the case of a merged super-store). The number of merged retail units (or groups) will be denoted by M.

For conceptual clarity -- and consonant with the vertical foreclosure literature -- we do not consider horizontal mergers in this paper. Hence, each retail unit is distinct and cannot preempt entry with added "units" (superstores or vertical chains) of its own. For example, when the equilibrium number of retail units is sufficiently small, anti-trust laws preclude horizontal mergers (Hay and Werden, 1993) due to their well-known economic costs (e.g., Farrell and Shapiro, 1990; Werden and Froeb, 1998; Spector, 2003; Cabral, 2003).[4] Section IV discusses implications of horizontal mergers for the paper's conclusions.

Given N and M, let $\pi^u(N,M)$ and $\pi^m(N,M)$ denote the equilibrium profit that is earned by an unmerged and merged group, respectively. (We will turn to specific characterizations of these profit functions shortly, but begin by studying their implications for the merger-cum-entry equilibrium.) The following assumption will be maintained (and verified for specific model frameworks) in what follows:

Assumption 1: $\pi^u(N,M)$ and $\pi^m(N,M)$ are decreasing in M and non-increasing in N

for $0 \leq M \leq N$ and $N \geq 2$.

Due to logic that is well known (e.g., Bonnano and Vickers, 1988; Beggs, 1994), merged groups are more price-competitive, leading to lower equilibrium profits for rival retailers. Because entry can also increase competition, profits fall (or do not rise) with N.

Let E denote the cost for a group to enter the market.[5] Entry occurs sequentially with group 1 entering and merging (or not), group 2 entering and merging (or not) next, and so on. Prices, production and trade -- and the realization of profits -- occur after all entry

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[4]See also recent papers on horizontal (spatial) preemption, including Hadfield (1991), Reitzes and Levy (1995) and Eaton and Schmitt (1994). Ziss (1995) considers horizontal mergers in a vertically separated market.

[5]We assume that E is the same, whether an unmerged group enters or a single merged group enters. We thus avoid an obvious motive for merger -- the saving of setup/entry costs.

decisions have been made. For simplicity, we adopt the following tie-breaking conventions: (i) if a group is indifferent between entering or not, it does not enter; and (ii) if a group is indifferent between merging or not, it merges. In this sequential game, an entry / merger equilibrium is constrained to be subgame perfect.

Because mergers reduce profits (Assumption 1), entry is "more deterred" when more mergers occur. Hence, if entry is profitable despite (N-1) mergers, with

(1) $E < \overline{E}(N) \equiv \max(\pi^u(N,N\text{-}1), \pi^m(N,N))$,

then at least N groups will enter. Likewise, if marginal entry is not profitable when N mergers occur,

(2) $E \geq \overline{E}(N+1) \equiv \max(\pi^u(N+1,N), \pi^m(N+1,N+1))$,

then N mergers will deter entry by the marginal (N+1) group. At this juncture, the following observation is useful, permitting us to focus on *marginal* entry deterrence without loss in generality:

> Lemma 1. If marginal entry (to (N+1) groups) is deterred at $N \geq 2$, then all further entry is deterred.
>
> *Proof.* It suffices (by induction) to show that if marginal entry to (N+1) firms is deterred,

$$\max(\pi^u(N+1,M), \pi^m(N+1,M+1)) \leq E,$$

then entry to (N+2) firms is deterred:

$\max(\pi^u(N+2,M), \pi^m(N+2,M+1)) \leq \max(\pi^u(N+1,M),\pi^m(N+1,M+1))\ (\leq E)$, and

$\max(\pi^u(N+2,M+1), \pi^m(N+2,M+2)) \leq \max(\pi^u(N+1,M),\pi^m(N+1,M+1))\ (\leq E)$.

Considering the two possible cases, $\pi^u(N+1,M) \geq \pi^m(N+1,M+1)$ and $\pi^m(N+1,M+1) \geq \pi^u(N+1,M)$, the latter inequalities can be seen to follow directly from Assumption 1. QED.

By Assumption 1, the right-hand-side of equations (1) and (2) falls with the number of groups (N), $d\overline{E}(N)/dN \leq 0$. Hence, if an equilibrium involves *maximal entry deterrence* -- minimizing the number of groups subject to entry constraints -- then the equilibrium number of groups is determined by the size of the entry cost E. For example, if $E \in [\overline{E}(3)$,

$\overline{E}(2)$), then entry of the third group can be deterred by mergers, and maximal entry deterrence will lead to the operation of just two groups. Similarly, if E is lower, $E \in [\overline{E}(4)$, $\overline{E}(3)$), then entry of the fourth group (but not the third) can be deterred, and maximal entry deterrence will lead to the operation of three groups. More generally, we can define the unique number of groups that operates with maximal entry deterrence:

$$(3) \qquad\qquad N^* \equiv N: E \in [\overline{E}(N+1) , \overline{E}(N)).$$

In order to avoid dwelling on the monopoly case -- wherein merger is trivially optimal for the one group that operates -- we assume:

Assumption 2. $N^* \geq 2$.

Within the interval, $[\overline{E}(N+1) , \overline{E}(N))$, different entry costs will require different numbers of mergers in order to deter marginal entry. For example, we can have

$$(4) \qquad \overline{E}(N+1) \ \leq \ E < \max(\pi^u(N+1,N-1), \pi^m(N+1,N)) < \overline{E}(N) ,$$

in which case N mergers are needed to deter further entry. If E is higher (but still less than $\overline{E}(N)$), then fewer than N mergers will deter entry. We will let M* denote the minimum number of mergers needed to achieve maximal entry deterrence (N*).

Knowing N* and M*, consider the choice problem of an entering group that is *pivotal* to entry deterrence in the following sense:

Definition: A group is *pivotal* if, given the merger decisions of prior entrants, and anticipating mergers by all subsequent entrants, marginal entry (to (N*+1) groups) will be deterred if and only if the present group merges.

If a pivotal group merges, it anticipates the payoff $\pi^m(N^*,M^*)$, with further entry deterred. However, if it does not merge, then further entry will occur and the group will obtain at most $\pi^u(N^*+1,0)$. (By Assumption 1, the unmerged group will obtain less if either more entry occurs or some mergers take place.) Hence, a sufficient condition for maximal entry deterrence to take place is:

Condition 1. (Strong Entry Effects.) Due to relatively large costs of entry (vs. merger) in lost profit, groups are better off if they merge and thereby deter marginal entry, than if they do not merge and thereby accommodate marginal entry:

$$\pi^m(N^*,N^*) \geq \pi^u(N^*+1,0).$$

If Condition 1 holds, then any pivotal group will merge, knowing that subsequent entrants are also pivotal and thus will prefer the entry-deterring merge strategy. Thus, we have:

Proposition 1. If Condition 1 holds, then an equilibrium yields maximal entry deterrence, with exactly $N^* \geq 2$ groups operating.

However, for some of the cases of interest in this paper -- those for which mergers would not occur absent entry considerations -- groups may prefer not to merge and instead accommodate a marginal unmerged entrant. That is, Condition 1 will be violated. The reason is simple: While unmerged entry depletes profits, the cost of entry-deterring merger -- due to heightened price competition -- can be even greater.

Despite this motive to accommodate unmerged entry, we nevertheless find that groups will often merge to deter entry. The reason is that the groups cannot accommodate an unmerged entrant without either (1) prompting merger by the entrant or another rival (in order to deter yet further entry) or (2) prompting further entry still (opening the floodgates as it were). Merger of a rival negates the only possible advantage of accommodating entry, namely, avoiding the price competition -- and attendant profit cost -- that mergers impart.

To develop these points, consider the following counterpart to Condition 1:

Condition 2. (Strong Merger Effects.) Due to relatively large costs of merger (vs. entry) in lost profit, groups are better off with one less merger, even when this means one more entrant:[6]

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[6]A weaker version of Condition 2 requires equation (5a) and either equation (5b) or that mergers be disadvantageous (absent entry considerations): $\pi^u(N,M-1) \geq \pi^m(N,M)$ for $1 \leq M \leq N^*-1$, $N^* \leq N \leq 2N^*$. All that follows applies with the weaker version of Condition 2. For simplicity -- and because the weaker condition is not required when we come to the applications (in Section III below) -- we state the simpler (stronger) Condition 2 here.

(5a) $\pi^u(N+1,M-1) \geq \pi^u(N,M)$ for $1 \leq M \leq N^*-1$, $N^* \leq N \leq 2N^*$;

(5b) $\pi^m(N+1,M-1) \geq \pi^m(N,M)$ for $1 \leq M \leq N^*$, $N^* \leq N \leq 2N^*$.

Lemma 2. If Condition 2 holds, then:

(I) At least (N^*-j) mergers are needed to deter entry to (N^*+j) groups for $j \leq N^*-1$. Formally, for $j \in \{1,..., N^*-1\}$,

(6) $\max(\pi^u(N^*+j,N^*-1-j), \pi^m(N^*+j,N^*-j)) \geq \max(\pi^u(N^*,N^*-1), \pi^m(N^*,N^*)) > E$,

where the last inequality is due to the definitions of N^* and $\bar{E}(N)$ in (3) and (1).

(II) If equation (4) holds at $N=N^*$ (so that N^* mergers are required to obtain maximal entry deterrence), then at least (N^*-j) mergers are needed to deter entry to (N^*+j+1) groups. Formally, for $j \in \{1,..., N^*-1\}$,

(7) $\max(\pi^u(N^*+1+j,N^*-1-j), \pi^m(N^*+1+j,N^*-j))$

$\geq \max(\pi^u(N^*+1,N^*-1), \pi^m(N^*+1,N^*)) > E$,

where the last inequality is due to equation (4).

(III) $\pi^u(N^*+j,N^*-1-j)$ (for $j \leq N^*-1$) and $\pi^u(N^*+j,N^*-j)$ ($j \leq N^*$) increase with j.

Proof. (I) Condition 2 implies (by induction)

(8a) $\pi^u(N^*+j,N^*-1-j) \geq \pi^u(N^*,N^*-1)$; and

(8b) $\pi^m(N^*+j,N^*-j) \geq \pi^m(N^*,N^*)$.

Equation (8) implies (6). (II) follows from identical logic. (III) follows (by induction) from Condition 2, equation (5a). QED.

Let us suppose that Condition 2 holds. By Lemma 2(I), at least (N^*-1) mergers are then needed to support an N^* (maximal entry deterrence) equilibrium. Hence, there are two cases to consider: (1) when $M^*=(N^*-1)$ mergers deter entry to (N^*+1) groups, and (2) when equation (4) holds at $N=N^*$, so that $M^*=N^*$ mergers are necessary and sufficient to deter entry to (N^*+1) groups.

Turning to the first $(M^*=N^*-1)$ case, consider the choice problem of a pivotal group. If the group does not merge, then (because the group is pivotal), further entry will occur. Indeed, by Lemma 2(I), the "renegade" (non-merging) group can anticipate at best

an equilibrium with (N*+j) operating groups and (N*-j-1) mergers for some j≤ N*-1. Moreover, among these potential equilibria, Lemma 2(III) (and Assumption 1) imply that the renegade prefers the one with the highest j (j=N*+1); hence, the best that the renegade can hope for is a maximal amount of entry (with N=2N*-1) and no mergers at all, yielding a payoff equal to π^u(2N*-1,0). If, on the other hand, the group merges, then it deters further entry and obtains π^m(N*,N*-1). The pivotal group thus compares a no-merge strategy that ideally yields "mega-entry" (to (2N*-1) groups) to a merger that deters *all* further entry. The following condition is sufficient for the group to prefer the entry deterrence strategy, despite large costs of merger (Condition 2):

> Condition 3. (Costs of Mega Entry.) Groups prefer to merge, and thereby deter entry, than not merge and thereby accommodate "mega-entry" that nearly doubles to number of groups: π^m(N*,N*-v) ≥ π^u(2N*-v,0) for v ∈ {0,1}.

If Condition 3 holds, then any pivotal group will merge, knowing that subsequent entrants are also pivotal and thus will prefer the entry-deterring merge strategy. Hence, maximal entry deterrence will be achieved in the first (M*=N*-1) case.

The second case -- when mergers by all N* groups are needed to deter further entry -- is similar. Then, by Lemma 2(II), a renegade can anticipate at best an equilibrium with (N*+j) operating groups and (N*-j) mergers for some j≤ N*. Again, by Lemma 2(III) (and Assumption 1), the renegade's most preferred among these potential equilibria, is one with N=2N* (j=N*) and no mergers. If Condition 3 holds, the pivotal group would rather merge -- and thereby obtain the entry-deterrence payoff π^m(N*,N*) -- than elicit mega-entry (to N=2N*) with a no merge strategy. Hence, we have:

> Proposition 2. If Conditions 2 and 3 hold, an equilibrium yields maximal entry deterrence, with exactly N*≥2 groups operating and at least (N*-1) mergers. If equation (4) holds at N=N*, or π^m(N*,N*) > π^u(N*,N*-1), then there will be N* mergers in equilibrium.

III. Applications

We consider two models of an N-firm differentiated product market, one with multi-product retailing and the other with vertical (upstream and downstream) retail chains.

A. Mergers and Malls

Beggs (1994) shows that multi-product retailers may prefer to organize themselves as malls -- with each product sold by an independent firm -- rather than as merged firms (or super-stores) that sell all of the multiple products. We now generalize Beggs (1994) in the simplest possible way to illustrate effects of entry on multi-product mergers. In doing so, we adapt the foregoing results to show that, even when no mergers would occur absent entry-deterrence considerations, the prospect of entry can prompt groups to merge.

Let us suppose that N groups sell two products each. Each group is either merged, selling both products as one firm, or separated, selling the two products as two independent firms. Both goods are produced at constant and zero marginal cost. The demand for product i of group j takes the form,

$$(9) \qquad D_i^j = (\lambda/N) \{ a - b [(p_1^j + p_2^j) - \bar{P}^j]\}, \lambda{>}0, a{>}0, b{>}0,$$

where p_1^j and p_2^j are group j's prices for its two products, and $\bar{P}^j = (N-1)^{-1} \sum_{h \neq j} (p_1^h + p_2^h)$ is the average price of group j's competitors.[7]

This demand structure implicitly envisions a Hotelling-type setting with two properties. First, as is standard in the product variety literature (e.g., Kuhn and Vives, 1999; Dixit and Stiglitz, 1977; Spence, 1976), potential products/firms are different from one

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[7]Beggs (1994) allows for distinct coefficients on own price, coefficient b (on $P^j = p_1^j + p_2^j$), and competitors' price, coefficient d (on $\bar{P}^j$). Assuming that own-price effects dominate (b≥d), Beggs (1994) shows that the incentive not to merge is greatest when b=d. We focus on the latter (b=d) case for three reasons: simplicity, because it follows from a Hotelling-type specification, and because we wish to show that mergers arise -- due to entry-deterrence considerations -- when they otherwise would not. We also note, as in Beggs (1994), that the demands of equation (9) arise as the limiting case for a class of quadratic preferences; specifically, if preferences over the composite products of N producers are u(x,y)=(1/2)(x-A(N))'B(N)$^{-1}$(x-A(N))+ηy, where x is the N-vector of composite demands, y is consumption of a numeraire good, A(N) is an N-vector with elements (λa/N), and B(N) is an (NxN) matrix with diagonal elements (-λb/_N) and equal off-diagonal elements, λ(1-ε)b/_N(N-1), then (9) gives the consumer demands that prevail in the limit as ε goes to zero.

another in an exogenous and symmetric way.[8] Second, each firm engages in head-to-head

competition for customers against all other firms in the market; that is, for any two firms,

there are consumers who find these firms' products to be the "best" on offer and, hence,

who choose between them.[9] This second property is violated in standard spatial

differentiation models wherein consumers and firms have fixed locations on the same line

(e.g., Eaton and Wooders, 1985) or circle (e.g., Klemperer, 1992) and, hence, firms compete

for consumers only with proximate neighbors.[10] Consider instead the following "random

preference Hotelling" model. There is a consumer population of size λ^*. Each consumer

demands one unit of each product (inelastically), although this demand may be broken up

between different firms. Consumers are "located," and uniformly distributed, on a circle of

circumference C. The composite products of the N firms are located at N equally spaced

points on the circle. However, any possible ordering of the N firms between these locations

occurs with equal relative frequency. For example, if there are three firms, then there are six

possible orderings of these firms between the three "locations" on the circle, and each

ordering occurs with a relative frequency of one-sixth. A consumer's "transport" or

"preference" cost of purchasing from a particular product location is proportional to her

"distance" from that location (traveling on the circle). This model gives rise to the demand

in equation (9), with $\lambda=(\lambda^*/t)$, t=unit transport cost, a=t, and b=1.[11]

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[8]Our premise of exogenous and symmetric market location for different firm products also parallels recent work on spatially differentiated markets (e.g., Girard-Heraud, Hamudi and Mokrane, 2003). Allowing for endogenous product location decisions (as is the focus of Heywood, et al., 2001, for example) would dramatically complicate our analysis, but is unlikely to qualitatively alter our conclusions. Non-horizontal mergers will intensify price competition among operating sellers, but deter entry and thereby expand the market segment that each seller serves.

[9]While this second property is quite realistic, it is also useful for our analysis because it permits us to derive firm decisions that are a function of market-wide merger activity.

[10]For example, suppose that there are four products, A, B, C, and D. Suppose further than consumer 1 most prefers A and B, and consumer 2 most prefers A and C. Then, in the standard model, consumer 3 cannot most prefer A and D, and firm A only directly competes with firms B and C.

[11]The number of customers served by retail group j in this model (and, hence, the demand for product i in group j's "store") equals $\sum_{k \neq j} q_k^j d_k^j$, where $q_k^j = 2/(N-1)$ is the relative frequency with which firm j has firm k as a neighbor, and $d_k^j = \frac{1}{2Nt}(t+P^k-P^j)$ is firm j's consumer demand on the arc between firms j and k when

We call this structure Model 1. For this model, we first determine properties of equilibrium prices and group profits for a given number of operating groups N, and a given number of merged groups, M≤ N.

Let p^u and P^m denote, respectively, equilibrium prices of an unmerged firm (for its one product) and a merged firm (for its two products). Then we can define the average competitor prices for merged (m) and unmerged (u) groups respectively:

(10) $\qquad \overline{P}^m = (\alpha-k) P^m + (1-\alpha+k)\, 2\, p^u,$

(11) $\qquad \overline{P}^u = \alpha P^m + (1-\alpha)\, 2p^u,$

where $k=k(N)=(N-1)^{-1}$ and $\alpha=Mk=M/(N-1)$. Unmerged firms maximize profits as follows (where $i,h \in \{1,2\}$, $h \neq i$):

(12) $\qquad \max_{p^j_i} (\lambda/N)\ \{ a - b\,[(p^j_i + p^j_h) - \overline{P}^u]\}\, p^j_i \ \Rightarrow\ p^j_i = \{a - b p^j_h + b\overline{P}^u\}/2b.$

Similarly, merged firms maximize profits,

(13) $\qquad \max_{P^j} (\lambda/N)\ \{ a - b\,[P^j - b\,\overline{P}^m]\}\, P^j \ \Rightarrow\ P^j = \{a + b\overline{P}^m\}/2b.$

Solving (10)-(13) yields the equilibrium prices,

(14) $\qquad p^u = ab(2+k)/Q\ \ ,\ \ P^m = ab(3+2k)/Q,$

where $Q = (2+\alpha+k)$. We define the attendant equilibrium group profits,

(15a) $\ \pi^u(N,M) = (\lambda/N)\ \{ a - b\,2p^u + b\,\overline{P}^u\}\, 2p^u\ = \{\lambda a^2 2(2+k)^2\}/[NbQ^2],$

(15b) $\ \pi^m(N,M) = (\lambda/N)\ \{ a - b\, P^m + b\,\overline{P}^m\}\, P^m\ = \{\lambda a^2(3+2k)^2\}/[NbQ^2].$

Several properties of the equilibrium merit note:[12]

> Proposition 3. In Model 1, with M mergers and N operating groups, (a) merged prices are lower, $P^m < 2p^u$; (b) merged profits are higher, $\pi^m > \pi^u$; (c) prices and

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the two are neighbors, with $P^h = p^h_1 + p^h_2$ denoting the composite firm h product price. Substitution yields equation (9) with the indicated parameter values. The demand structure in equation (9) also arises in a generalized Hotelling setting wherein (1) each consumer demands one unit of each product (inelastically); (2) consumers lie on line segments between each of the N groups, with each line segment (or strand) containing the same fraction of the total consumer population and consumers distributed uniformily within each strand; and (3) "transport" or "preference" costs of purchasing from a particular group are proportional to distance to that group (traveling on line segments). In this case, the proportional constant of equation (9), (λ/N), derives from a population size, per line segment, equal to $(\lambda^*/s(N))$, where s(N)=number of line segments = N!/2(N-2)!. The demand constant thus equals $\lambda^*(N-1)/2s(N)t = \lambda/N$, with $\lambda=\lambda^*/t$, a=t and b=1.
[12]Proofs of Proposition 3 and other Section III results are contained in the Appendix.

profits fall when the number of merged groups rises, $\partial p^u/\partial M < 0$, $\partial P^m/\partial M < 0$, $\partial \pi^u/\partial M < 0$, and $\partial \pi^m/\partial M < 0$; and (d) for $N \geq 2$, an increase in the number of operating firms leads to lower profits, $\partial \pi^u(N,M)/\partial N < 0$ and $\partial \pi^m(N,M)/\partial N < 0$.[13]

Corollary 1. Assumption 1 holds in Model 1.

A merged group considers the adverse effect of an increase in one product's price on the other product's demand. Because an unmerged firm does not consider this effect, it sets a higher price (Proposition 3(a)) at the cost of lower group profit (Proposition 3(b)). Although a merger is directly beneficial to a group, it also heightens price competition by rivals; faced with lower merged group prices, rivals lower their prices, leading to lower market prices and profits (Proposition 3(c)). If the latter (adverse) effect of merger dominates the former (beneficial) effect of merger, groups do not merge in equilibrium (ignoring the entry concerns at issue in the present paper). This is Beggs' (1994) key point, which he develops in a variant of the present model when N=2. Indeed, for our model, when N=2 and there is no threat of entry, Beggs (1994) shows that neither retail group merges. Generalizing this result to allow for more than two firms, we have: [14]

Proposition 4. Given N operating retail groups in Model 1 -- without prospect for further entry -- no firm will merge if $N \leq 5$ and all firms will merge if $N \geq 6$.

The strategic benefit that any given firm derives from organizational separation (a mall structure) wanes as the number of retail groups rises; the reason is that each firm's choice of a mall structure -- and the attendant pre-commitment to higher prices -- has a smaller impact on average rival prices when there are more firms in the market. As a result, when N gets

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[13]The curious reader may also note a few other properties of equilibrium in Model 1. For example, an increase in the number of operating firms leads to higher prices for $M \geq 1$ and the same prices when M=0: $\partial p^u/\partial N \geq 0$ and $\partial P^m/\partial N > 0$ for $M \geq 1$. However, fixing the proportion of merged groups ($\alpha = Mk(N)$), an increase in the number of operating groups leads to (a) lower prices for $\alpha \in (1,1+k)$, and the same prices for $\alpha \in \{0,1+k(N)\}$: $\partial p^u/\partial N \leq 0$, $\partial P^m/\partial N \leq 0$; and (b) lower profits, $d\pi^u(N,\alpha(N-1))/dN < 0$ for $\alpha \in [0,1+k)$, and $d\pi^m(N,\alpha(N-1))/dN < 0$ for $\alpha \in [0,1+k(N)]$.

[14]The outcomes described in Proposition 4 are unique subgame perfect equilibria in the sequential move (merge) game modeled in this paper. They are also Nash equilibria in a simultaneous move game. When $N \leq 3$ or $N \geq 6$, the simultaneous move equilibria are unique. However, when N=4 or N=5, there are two simultaneous move equilibria, all-merge and all-not-merge, with the latter yielding higher firm profits.

sufficiently large (six in our model), the direct benefits of a merger exceed the strategic benefits of a mall. However, for sufficiently few firms, the strategic benefits of separation dominate, ignoring entry considerations. These are the cases of interest here.

Even in these cases, we find that mergers will occur when the threat of entry is taken into account. Specifically, an absence of merger -- precisely because it raises prices and profits -- encourages entry of new competitors, which depletes profits (Proposition 3(d)). Conversely, mergers deter costly entry. Despite entry deterrence benefits of mergers, *marginal* entry is always less costly than a *marginal* merger in our Model 1. That is, Condition 1 fails to hold. Nevertheless, we have the following:

Proposition 5. In Model 1, Conditions 2 and 3 hold. Hence, an equilibrium yields maximal entry deterrence, with exactly $N^* \geq 2$ groups operating and at least (N^*-1) mergers.

B. Vertical Integration vs. Vertical Separation

Bonnano and Vickers (1988) identify strategic advantages of vertical separation in differentiated product markets. By separating, a vertical chain can sign a contract that stipulates an above-cost wholesale price, thus implicitly pre-committing the downstream retailer to charge higher prices (ceteris paribus). Because rivals respond to higher competitor retail prices by themselves charging higher prices, the contractual pre-commitment is advantageous to the vertically separated chain. We now reconsider this logic in the simplest possible model of entry with vertical merger decisions. In doing so, we adapt the results of Section II to show that the prospect of entry will prompt firms to vertically integrate when they otherwise would not.

Consider again N retailers, now with each selling only one product. Products are supplied by upstream firms. The demand for retailer j's product takes the Hotelling form:

$$(17) \qquad D^j = (\lambda/N) \{ a + b (\overline{P}^j - P^j) \}$$

where P^j is retailer j's product price and $\overline{P}^j = (N-1)^{-1} \sum_{h \neq j} P^h$ is the average price of group j's rivals. Unit costs of production and marketing are constant and zero. A vertically integrated (merged) retailer thus faces a zero wholesale price. For vertically separated (unmerged) retailers, we follow Bonnano and Vickers (1988) (and others) in restricting attention to observable linear contracts between upstream suppliers and downstream retailers.[15] Contracts thus stipulate a fixed transfer and a wholesale price that can be positive, zero or negative (positive in equilibrium). As described in Section II, retailers (or vertical chains) enter and merge (or not) sequentially. After entry, unmerged chains sign vertical contracts. Finally, retailers set prices, and production and trade occurs.

We call this structure Model 2, and begin by characterizing equilibrium prices and profits that prevail with a given number of retail chains (N) and merged groups (M≤N):

Lemma 3. Given M and N in Model 2, unmerged chains sign vertical contracts with the wholesale price,

(18) $w^* = \{ak(2+k)\}/\{b(2+k(1+\alpha))\} > 0,$

yielding the equilibrium prices and profits,

(19) $P^m = (a/b)\{2+2k+k^2\}/\{2+k+\alpha k\}$, $P^u = (a/b)\{2+3k+k^2\}/\{2+k+\alpha k\},$

(20a) $\pi^m(N,M) = (a^2\lambda/Nb)\{2+2k+k^2\}^2/\{2+k+\alpha k\}^2,$

(20b) $\pi^u(N,M) = (a^2\lambda/Nb)\{2+3k+k^2\}(2+k)/\{2+k+\alpha k\}^2.$

Proposition 6. In Model 2, with M mergers and N operating groups, (a) merged prices are lower, $P^m < P^u$; (b) merged profits are higher, $\pi^m > \pi^u$; (c) prices and profits fall when the number of merged groups (M) rises; and (d) for N≥ 2, entry lowers profits, $\partial\pi^u(N,M)/\partial N < 0$ and $\partial\pi^m(N,M)/\partial N < 0$.

Corollary 2. Assumption 1 holds in Model 2.

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[15]See, for example, Ziss (1995), Shaffer (1991), Mathewson and Winter (1984), Hamilton (2003). For analyses of unobservable contracts, see O'Brien and Shaffer (1992) and McAfee and Schwartz (1994). For a critique of two-part contracts in vertically separated markets, see Fershtman, Judd and Kelai (1991).

Vertically separated groups precommit to positive wholesale prices that raise their own retail prices (Proposition 6(a)). The benefit of this pre-commitment is that it prompts rivals to raise their retail prices as well. Because vertically integrated firms charge lower prices (ceteris paribus) -- with rivals competing by lowering their prices as well -- prices and profits fall as more groups integrate (Proposition 6(c)). Indeed, this cost of mergers makes them strictly disadvantageous:[16]

> Proposition 7. Given N operating retailers in Model 2 -- without prospect for further entry -- no firm will vertically integrate in equilibrium. Formally, $\pi^u(N,M) > \pi^m(N,M+1)$ for $0 \le M \le N-1$, $N \ge 2$.

Here, however, vertical integration can be advantageous, precisely because it lowers prices and profits, thereby deterring costly entry (Proposition 6(d)).

> Proposition 8. In Model 2: (a) Condition 1 holds and, hence, an equilibrium yields maximal entry deterrence, with exactly $N^* \ge 2$ retailers operating; and (b) if $N^* \ge 3$, then at least (N^*-1) mergers occur in order to deter further entry.

C. A Note on Welfare in a Random Preference Hotelling Model. A final issue that we raise here is whether mergers, by deterring entry, raise or deplete economic welfare. The answer is ambiguous. Consider the random preference Hotelling model. Because consumer demands are perfectly inelastic, the only economic margin that is relevant for economic welfare is the number of firms (N). On one hand, increasing the number of firms reduces consumer "transport" costs. On the other, it raises set-up / entry costs. Formally, let $L=(C/N)$ denote the length of the arc between any two product locations (where C is the circumference of the market circle); and let d denote "distance" to a product location. Then total consumer transport costs equal:

$$T = N \, 2 \int_0^{L/2} td \, dd \, (\lambda^*/C) = \frac{tC\lambda^*}{4N},$$

_____

[16]The logic of Bonnano and Vickers (1988) implies that this property is general. A vertically separated chain can achieve the same outcome as an integrated firm by contracting for a zero wholesale price; however, the vertically separated retailer prefers to select a positive wholesale price, implying (by revealed preference) that integration is strictly disadvantageous.

where t is unit transport cost and λ^* is the total consumer population. An optimal number of firms, N^{**}, minimizes the sum of transport costs T and entry costs, EN:

$$N^{**} = [tC\lambda^*/4E]^{1/2}.$$

What is notable here is that the *optimal* number of firms depends upon the dispersion of consumer preferences (the parameter C), whereas product demands -- and hence, the *equilibrium* number of firms -- do not depend at all upon this dispersion parameter. Therefore, mergers -- by limiting entry -- may improve welfare when there is little dispersion in consumer preferences (so that C and N^{**} are small) and may deplete welfare when there is a great deal of consumer dispersion (so that C and N^{**} are large).

IV. Conclusion

This paper considers how a non-horizontal merger can deter new entry and, for this reason, emerge in equilibrium. Organizational mergers, by implicitly committing firms to more aggressive price competition, can deter entry. By modeling these entry-deterrence effects, this paper shows that non-horizontal mergers can occur in equilibrium even though, absent entry considerations, they do not. We also find that mergers can arise under some rather surprising conditions. One might expect that the likelihood of equilibrium mergers would turn on the relative weight of their pro-competitive (price-dampening) and anti-competitive (entry-deterring) effects. However, we find that mergers occur even when the cost of a marginal merger (to firms) is greater than the cost of a marginal entrant. The reason is that marginal entry cannot be accommodated, and a marginal merger avoided, without prompting a veritable flood of new rivals.

To the extent that entry or exit is possible in multiple-good retail markets, we thus find a strategic motive for the emergence of the "big box" superstores so prevalent in contemporary retail markets. For example, there are many complaints that Walmart promotes the exit of local competitors (PBS, 2001); this paper suggests that Walmart's merged structure may find motive in precisely this effect. Of course, we ignore some considerations that may also affect the equilibrium choice between malls and superstores.

Malls may be advantageous when retail products require customer service, with decentralized ownership permitting better incentives for service provision. On the other hand, there may be cost economies that favor superstores.

This paper likewise identifies a new strategic motive for vertical integration. While the extant literature on vertical integration is extensive, this phenomenon is explained by either cost economies, or benefits of avoiding double-marginalization, or incomplete contracts for relationship-specific investments (Grossman and Hart, 1986), or incentives to prevent downstream horizontal integration (Conagelo, 1995), or attempts at vertical foreclosure -- when a chain can deny its rivals access to upstream production (e.g., Hart and Tirole, 1990). For conceptual clarity, none of these forces operate in the present paper. Instead, we find that vertical integration can occur because it credibly deters the entry of prospective rivals. The logic of this result requires that vertically separated chains can sign mutually advantageous contracts that have the intended effect of limiting competition. If anti-trust laws preclude vertical restraints, then our entry-deterrence motive for vertical integration evaporates. To our knowledge, contemporary anti-trust laws in the developed world do not prevent the two-part vertical contracts considered here. However, as a matter of policy, one might ask: *Should* such vertical restraints be limited? Alternately, should vertical integration be proscribed?

In the context of our model, the answers to such questions are ambiguous. If consumer demand is perfectly inelastic (as in our Section III applications), then the only effect of mergers on overall economic welfare is due to their effect on the number of firms. More firms imply greater product diversity, reducing consumers' "transport" or "preference" costs of obtaining their most-desired products. Greater product diversity comes at the cost of higher set-up (entry) expenditures. An optimal number of firms trades off these two economic margins. In general, this optimum will depend upon the extent of dispersion in consumer preferences over the product domain, with greater dispersion yielding greater "transport" costs of obtaining a given set of product offerings and thus

favoring more product diversity (i.e., more firms). However, whether mergers are allowed or not, the equilibrium number of firms need not be driven at all by the extent of dispersion in consumer preferences. Dispersion affects consumers' costs, but need not affect the calculus of firms' price competition or, therefore, firm profits. Hence, when preferences are "very disperse," an equilibrium will likely yield too few firms, and mergers will thus be welfare-reducing. Conversely, when preferences are quite concentrated, an equilibrium will likely yield too many firms, and mergers will be welfare-enhancing. Because the welfare effects of mergers are ambiguous without any efficiency effects of product pricing (when product demands are inelastic), they are also ambiguous in general (when product demands may be elastic).

Perhaps the greatest limitation of our analysis is its neglect of any possible horizontal coordination. However, our arguments are likely to be robust. Consider, for example, an incumbent monopolist who can set up N "firms" (as subsidiaries) before entry takes place (a standard assumption in the horizontal merger literature). It will be advantageous for the incumbent to deter further entry. This can be done in two ways. First, the incumbent can sign horizontal franchise contracts with subsidiaries that implicitly coordinate the franchisees in the absence of entry, but permit open decentralized competition in the event of entry (Hadfield, 1991). Second, in order to achieve maximal entry deterrence, the franchisees can be merged (vertically or across products) in order to implicitly precommit them to more aggressive competition when entry occurs. In sum, because they limit entry, non-horizontal mergers continue to arise.

Appendix

Proof of Proposition 3. (a)-(b) follow directly from (14)-(15). (c) follows from $\partial Q/\partial\alpha=1>0$, $\alpha=Mk$, and (14)-(15). To establish (d), we can use (15) to obtain

(A1) $\partial\pi^u(N,M)/\partial N \stackrel{s}{=} \Delta^u(N,M) \equiv -(2+k)Q(N-1)^2 - 2NQ + (2+k)2N\{M+1\}$,

where $Q=(2+\alpha+k)$ and "$\stackrel{s}{=}$" denotes "equals in sign." Now note:

(A2) $\partial\Delta^u(N,M)/\partial M = 2N+1 > 0$.

By (A1)-(A2), it suffices to sign $\Delta^u()$ when M=N-1 (so that Mk=1).

(A3) $\Delta^u(N,N-1) = (2+k)(N-1)\{2-N\} - 2N < 0$ for $N\geq 2$.

Turning next to $\pi^m()$,

(A4) $\partial\pi^m(N,M)/\partial N \stackrel{s}{=} \Delta^m(N,M) \equiv -(3+2k)Q(N-1)^2 - 4NQ + (3+2k)2N\{M+1\}$,

Now note:

(A5) $\partial\Delta^m(N,M)/\partial M \stackrel{s}{=} Z \equiv \{(3+2k)(N^2-1)-4N\} > 0$ for $N\geq 2$,

where the inequality is due to Z>0 at N=2 and $\partial Z/\partial N = 6N-2 > 0$. By (A4)-(A5), it suffices to sign Δ^m when M=N. Substituting for Q and k in (A4) and rewriting, we find $\Delta^m(N,N)<0$ for N≥2. We thus have $\Delta^u(N,M)<0$ and $\Delta^m(N,M)<0$ for all M≤ N and all

N≥ 2, implying part (d) (by (A1) and (A4)). QED.

Proof of Proposition 4. Define the unilateral benefit of separation (not merge):
    $\pi^u(N,M)-\pi^m(N,M+1) \stackrel{s}{=} \Delta(k,\alpha) \equiv -4+12k+31k^2+20k^3+4k^4 -$

$[4\alpha+\alpha^2(1+4k+2k^2)]$

where $k=(N-1)^{-1}$, $\alpha=Mk$, and 0≤ M≤ N-1. If $\Delta>0$ for all $\alpha\in[0,1]$, as is easily verified for

N≤ 3, no firms merge in a subgame perfect equilibrium. Likewise, if $\Delta<0$ for all $\alpha\in[0,1]$,

as is easily verified for N≥ 6, then all firms merge in a subgame perfect equilibrium. This

leaves two cases, N=4 and N=5. For N=4, we have $\Delta>0$ for $\alpha\in\{0,1/3,2/3\}$, and $\Delta<0$ for

$\alpha=1$. For N=5, we have $\Delta>0$ for $\alpha=0$, and $\Delta<0$ for $\alpha\in\{1/4,1/2,3/4,1\}$. In both cases, if we

construct the extensive forms for the sequential move (merge) game, and solve by backward

induction, these Δ values -- combined with Corollary 1 / Assumption 1 -- imply a unique

equilibrium in which no firm merges. For example, for N=4, let us order the firms by order

of play from F1 to F4. Except when all prior firms merge (action I for "integrated"), F4 does not merge (action U for "unintegrated") because $\Delta>0$. Hence, if F3 chooses U, it obtains at least $\pi^u(4,M)$, with $M\leq 2$ denoting prior firm mergers. If F3 chooses I, it obtains at most $\pi^m(4,M+1)$ ($>\pi^m(4,M+2)$ by Corollary 1). With $\alpha\leq 2/3$ and, hence, $\Delta>0$, F3 thus chooses U, as does F4. Anticipating U by F3 and F4, F2 and F1 also choose U (because $\alpha\leq 1/3$ and, hence, $\Delta>0$). Similar logic applies to N=5. QED.

Proof of Proposition 5. Beginning with Condition 2, equation (5a), we have

$$\pi^u(N+1,M-1)-\pi^u(N,M) \overset{S}{=} (2N+1)\,^2N(2N+M-1)^2 - (2N-1)^2(N+1)(2N+M)^2$$

$$\equiv X^u(N,M)= 8\,N^3M + 4\,N^3 + 4\,N^2M^2 + 8\,N^2M - 4\,N^2 + 4\,NM^2 - 6\,NM + N - M^2$$

Now note:

$$\partial X^u/\partial M = 8N^3+8N^2M+8N^2+8NM-6N-2M > 0 \text{ (with } M\leq N\text{)}.$$

Hence, it suffices to show that $X^u()$ is positive at M=1:

$$X^u(N,1) = 12N^3+8N^2-N-1 > 0 \quad \text{for} \quad N\geq 1.$$

Turning next to Condition 2, equation (5b), we have

$$\pi^m(N+1,M-1)-\pi^m(N,M) \overset{S}{=} (3N+2)\,^2N(2N+M-1)^2 - (3N-1)^2(N+1)(2N+M)^2$$

$$\equiv X^m(N,M)=18N^3M - 3N^3 + 9N^2M^2 + 12N^2M - 8N^2 + 9NM^2 - 12NM + 4N - M^2$$

Now note:

$$\partial X^m/\partial M = 18N^3+18N^2M+12N^2+18NM-12N-2M > 0 \text{ (with } M\leq N\text{)}.$$

Hence, it suffices to show that $X^m()$ is positive at M=1:

$$X^m(N,1) = 15N^3+13N^2+N-1 > 0 \quad \text{for} \quad N\geq 1.$$

Finally, to establish Condition 3, we have from equation (15):

$$\pi^m(N,N-1)-\pi^u(2N-1,0) \overset{S}{=} (3N-1)\,^2(2N-1) - 2N\,(3N-2)^2 = 3N^2-1 > 0,$$

$$\pi^m(N,N)-\pi^u(2N,0) = 0.$$

Proposition 5 now follows from Proposition 2. QED.

Proof of Lemma 3. For reasons that will become clear shortly, we partition the groups into those that are merged (m), an unmerged chain (j), and unmerged groups other than j (u). Denoting the corresponding retailer prices by $\{P^m, P^j, P^u\}$, respective average competitor prices are:

(A6) $\quad\quad\quad \overline{P}^m = (\alpha-k) P^m + (1-\alpha) P^u + k P^j, \quad \overline{P}^u = \alpha P^m + (1-\alpha-k) P^u + k P^j,$

$$\overline{P}^j = \alpha P^m + (1-\alpha) P^u,$$

where, as before, $k=k(N)=(N-1)^{-1}$ and $\alpha=Mk=M/(N-1)$. Denoting contractual wholesale prices for group j and other unmerged groups (u) by w^j and w^u, respectively, unmerged firms maximize profits as follows:

(A7a) $\quad\quad \max_{P^h} (\lambda/N) \ \{ a + b (\overline{P}^h - P^h)\}(P^h - w^h) \ \Rightarrow \ P^h = \{a+b(\overline{P}^h+w^h)\}/2b.$

for $h\in\{j,u\}$. Similarly, merged firms maximize profits,

(A7b) $\quad\quad \max_{P^m} (\lambda/N) \ \{ a + b (\overline{P}^m - P^m)\}P^m \ \Rightarrow \ P^m = \{a+b\overline{P}^m\}/2b.$

Solving (A6)-(A7) for $\{P^m, P^j, P^u\}$ gives:

(A8a) $\quad\quad\quad P^j(w^j, w^u) = \{a(2+k)+bw^j(1+k)+bw^u(1-\alpha)\}/\{b(2+k)\}$

(A8b) $\quad\quad\quad P^u(w^j, w^u) = \{a(2+k)+bkw^j+bw^u(2-\alpha)\}/\{b(2+k)\}$

(A8c) $\quad\quad\quad P^m(w^j, w^u) = \{a(2+k)+bkw^j+bw^u(1-\alpha)\}/\{b(2+k)\}$

The vertically separated chain j selects its contractual wholesale price w^j to maximize its integrated profit as follows:

(A9) $\quad\quad\quad \max_{w^j} (\lambda/N) \ \{ a + b (\overline{P}^j - P^j(w^j, w^u))\}P^j(w^j, w^u),$

where $\overline{P}^j = \alpha P^m(w^j, w^u) + (1-\alpha) P^u(w^j, w^u)$. Solving (A9) and setting $w^j = w^u$ (by symmetry) yields the equilibrium wholesale price, for vertically separated chains, given in equation (18). Substituting into price and profit functions gives equations (19-(20). QED.

Proof of Proposition 6. (a)-(c) follow directly from equations (19)-(20), with $\alpha=Mk$. For (d), beginning with $\pi^u()$ (evaluated at $\alpha=Mk$), we have:

(A10) $\quad \partial\pi^u()/\partial N = (A+B+C+D)(\pi^u/(N-1)^2) \ , \ A = -(N-1)^2/N < 0 \ , \ B = -1/(2+k) < 0$

$$C = -(3+2k)/(2+3k+k^2) < 0 \ , \ D = 2(1+2\alpha)/(2+k+\alpha k) > 0.$$

Noting that $\partial D/\partial\alpha > 0$ and $\alpha \leq 1$ (with $M \leq N-1$ when there is an unmerged firm), it suffices to show that $\partial\pi^u()/\partial N < 0$ at $\alpha = 1$. Moreover, at $\alpha = 1$,

(A11) $\qquad A+D \overset{s}{=} -(N-1)^2(2+2k)+6N \leq 0 \quad$ for $\quad N \geq 4$.

(A11) implies that $\partial\pi^u()/\partial N$ in (A10) is negative for $N \geq 4$. For the remaining cases ($N=2$ and $N=3$), we have (evaluating (A10) at $\alpha = 1$)

$\qquad \partial\pi^u(2,M)/\partial N \leq -(1/6)\,\pi^u() < 0 \quad , \quad \partial\pi^u(3,M)/\partial N \leq -(1/5)\,\pi^u() < 0$

Turning next to $\pi^m()$, we have (with A and D as in (A10)):

(A12) $\qquad \partial\pi^m()/\partial N = (A+D+E)(\pi^m/(N-1)^2) \quad , \quad E = -(4+4k)/(2+2k+k^2) < 0.$

Noting that $\partial D/\partial\alpha > 0$ and $\alpha \leq 1+k$, we have (evaluating (A12) at $\alpha = 1+k$),

(A13) $\qquad (A+D+E) \leq A + 2/\{2+2k+k2\} \overset{s}{=} -2(N-1)^2+1 < 0 \quad$ for $\quad N \geq 2$.

(A13) implies that $\partial\pi^m()/\partial N$ in (A12) is negative for $N \geq 2$. QED.

Proof of Proposition 7. Using equation (20) and the definition of α, we have

$\qquad \pi^u(N,M)-\pi^m(N,M+1) = \pi^u(N,\alpha(N-1)-)\pi^u(N,(\alpha+k)(N-1))$

$\qquad\qquad \overset{s}{=} (2+3k+k^2)(2+k)(2+k+\alpha k+k^2)\,2 - (2+2k+k^2)^2(2+k+\alpha k)^2$

$\qquad\qquad = 4k^2+k^3(16-4\alpha) + k^4(21-2\alpha-3\alpha^2) + k^5(15-3\alpha^2) + k^6(6-\alpha^2)+k^7 > 0,$

where the inequality is due to $\alpha \leq 1$ (with $M \leq N-1$ when there is an unmerged firm). QED.

Proof of Proposition 8. Using equation (20), we have

$\qquad \pi^m(N,N)-\pi^u(N+1,0) \overset{s}{=} N^{-1} - \{(2N2+3N+1)/[(N+1)N(2N+1)]\} = 0.$

Hence, Condition 1 is satisfied and, by Proposition 1, maximal entry deterrence occurs.

$\qquad$ (b) It suffices to show (for $N=N^*$):

(A14) $\qquad \pi^u(N+1,N-2) \geq \pi^u(N,N-1) = \max(\pi^u(N,N-1),\pi^m(N,N)) > E,$

where the equality follows from Proposition 7 and the last inequality follows from the definition of N^*. By the inequality in (A14), (N^*-2) mergers is insufficient to deter entry to (N^*+1) firms; hence, at least (N^*-1) mergers is required to support a maximal entry deterrence equilibrium. To establish (A14) for Model 2, equation (20) gives us:

(A15) $\quad \pi^u(N+1,N-2)-\pi^u(N,N-1) \overset{s}{=} N(N-1)(2N+1)(2N^2+3N+1)(3N-2)^2$

$\qquad\qquad\qquad\qquad\qquad - (N+1)(2N-1)^2(2N^2+2N-2)^2$

$$= 20N^7 - 44N^6 - 31N^5 + 68N^4 - 9N^3 - 24N^2 + 16N - 4 > 0 \quad \text{for} \quad N \geq 3. \quad \text{QED.}$$

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