Modeling Time Varying Volatility and Skewness

by

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STATEMENT BY AUTHOR

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I would like to thank my parents Harry and Nancy for their continued love and support. Also I would like to thank my advisor Dr. Aradhyula as well as my other committee members Dr. Dahlgran and Dr. Thompson

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ABSTRACT

Financial returns have been traditionally assumed to be normally distributed. However returns have been shown to possess thick tails as well as being leptokurtic and asymmetric. Accurate modeling of these features is of the upmost importance for financial investors. This study makes use of the skewed generalized error distribution for capturing asymmetries and thicker tails in the returns distribution. In addition, both volatility and skewness are modeled as time varying. Using daily returns data, we find evidence of time varying skewness in both S&P 500 and oil spot price. Also we find that variance is greater in days after a weekend/holiday for both sets of data, but there is no such holiday effect in skewness.

CHAPTER 1

INTRODUCTION

The financial literature has known for a long time that financial asset returns are non-normal. Returns traditionally have fat-tails and are more leptokurtic than the normal distribution (Mandelbrot 1963; Fama, 1965). It is also known that financial returns are distributed asymmetrically (Mittnik and Paolella 2000).

One of the most widely used frameworks for modeling such returns is a GARCH model. This modeling extends from the ARCH framework, introduced by Engle (1982), which allowed for variances in inflation rates to change over time. Bollerslev (1987) allowed for past conditional variances to be used in the current conditional variance equation with the GARCH model. Bollerslev *et al*. (1992) later found that the parsimonious GARCH(1,1) specification works best for most applications. Sadorsky (2006) also found that a GARCH(1,1) model generated the best results when modeling the returns of oil futures.

Different non-normal distributions have been used over the years for capturing the distinct features financial returns exhibit. Nelson (1991), Taylor (1994), Lee *et al.* (2001) all use the generalized error distribution to capture the excess kurtosis. While this is able to account for the excess kurtosis, it is not able to capture the asymmetry of the returns. In order to model this aspect, more flexible distributions, which can allow for skewness, are needed. One model which can allow for skewness, and has the normal distribution as a special case, is the skew-normal distribution. This is unable to allow for excess kurtosis however. The skewed Student’s t distribution is also able to allow for skewnes, and is a popular distribution for this purpose, having been used by Hansen (1994), Bond and Patel

(2003), Jondeau and Rockinger (2003), and Lambert and Laurent (2002). Another distribution, which allows for skewness, is the skewed generalized error distribution (SGED).

Developed by Theodossiou (2001), the SGED has also been used by Bali (2007), and Lee and Pai (2010) for modeling interest rate and Real Estate Investment Trust (REIT) returns respectively. In modeling REIT returns, Lee and Pai (2010) found that a GARCH model with SGED errors outperformed GARCH models with normal errors and with skew-t errors.

The modeling of time varying skewness and kurtosis in financial returns is a relatively new procedure. Hansen (1994) was the first to use the ARCH framework to do so. Using a skewed Student's t distribution, Hansen (1994) proposed quadratic laws of motion; modeling conditional skewness and kurtosis as functions of lagged errors and lagged errors squared. Later, Harvey and Siddique (1999) began to model time varying skewness in a GARCH framework as a function of lagged cubed errors and lagged skewness. Using this framework with a non-central t distribution Harvey and Siddique (1999) found that time varying skewness is present in daily, weekly, and monthly stock returns.

There have been a number of different distributions used when trying to model time varying skewness. Among them are, the (generalized) skewed Student's t (Hansen 1994; Bond and Patel 2003; Jondeau and Rockinger 2003; Lambert and Laurent 2002), Pearson IV (Brannas and Nordman 2003; Premaratne and Bera 2000), and Normal Inverse Gaussian (Jensen and Lunde 2001;Wilhelmsson 2009).

Results for time varying skewness are somewhat mixed. Brannas and Nordman

(2003) found evidence both for and against time varying skewness in NYSE data depending on the distribution used. When using a Log-generalized gamma they found evidence of time varying skewness, however when using a Pearson IV distribution, skewness was not found to be time varying. Premaratne and Bera (2000) also found a lack of time varying skewness using daily NSYE returns. On the other hand, Bond and Patel (2003), Harvey and Siddique (1999), Hansen (1994), and Wilhelmsson (2009) all concluded that skewness was, in fact, time varying.

In this study, we model S&P 500 returns and oil prices using SGED distribution. Taking advantage of the nature of the distribution, we explicitly allow both volatility and skewness to be time varying using GARCH type specification.

Recent events have highlighted the increased uncertainty over oil prices, as well as the importance of oil in the economy, making accurate modeling of oil prices very important. To our knowledge this is the first study to study asymmetries and kurtosis in oil prices.

The plan for the rest of this paper is as follows. Chapter 2 will discuss the SGED distribution. Chapter 3 will discuss the data and the estimation procedure. Chapter 4 will present the empirical results. Chapter 5 concludes.

CHAPTER 2

THE SKEWED GENERALIZED ERROR DISTRIBUTION

The skewed generalized error distribution (SGED) is a flexible distribution, which encompasses other distributions such as the standard normal and Laplace as special cases. Unlike the normal distribution, the SGED allows for skewness and excess kurtosis.

Compared to the most popular distribution used to model skewness and kurtosis, the generalized (skewed) t, developed by Hansen (1994), the SGED is more flexible. The generalized t distribution assumes a mean of 0 and a variance of 1, whereas the SGED allows the mean and variance to be parameters of the distribution. Another drawback of the generalized t distribution is it is not defined for all combinations of skewness and kurtosis[1].

The density of the SGED is as follows.

(2.1) $f(y|\mu,\sigma,k,\lambda) = \frac{C}{\sigma} exp\left(-\frac{|y-\mu+\delta\sigma|^k}{[1-sign(y-\mu+\delta\sigma)\lambda]^k\theta^k\sigma^k}\right)$

where

(2.2) $C = \frac{k}{2\theta}\Gamma\left(\frac{1}{k}\right)^{-1}$

(2.3) $\theta = \Gamma\left(\frac{1}{k}\right)^{\frac{1}{2}}\Gamma\left(\frac{3}{k}\right)^{-\frac{1}{2}}S(\lambda)^{-1}$

(2.4) $\delta = 2\lambda AS(\lambda)^{-1}$

(2.5) $S(\lambda) = \sqrt{1+3\lambda^2-4A^2\lambda^2}$

(2.6) $A = \Gamma\left(\frac{2}{k}\right)\Gamma\left(\frac{1}{k}\right)^{-\frac{1}{2}}\Gamma\left(\frac{3}{k}\right)^{-\frac{1}{2}}$

μ and σ are the expected value and the standard deviation, respectively, of the random

---

[1] See Jondeau and Rockinger (2003)

variable y. λ is a parameter which controls the skewness, and is bounded such that $-1< \lambda <1$. k controls the thickness of the tails and the height, and is constrained by $k>0$. $\Gamma(a) = \int_0^{+\infty} z^{a-1}e^{z}dz$ is the gamma function, and *sign* is the sign function.

When $\lambda=0$ and $k=1$, the SGED becomes the Laplace distribution, when $\lambda=0$ and $k=2$, the normal distribution, and when $\lambda=0$ and $k=\infty$, the uniform. The 3rd and 4th centered moments are as follows.[2]

(2.7) $m_3 = E(y-\mu)^3 = (A_3 - 3\delta - \delta^3)\sigma^3$

and

(2.8) $m_4 = E(y-\mu)^4 = (A_4 - 4A_3\delta + 6\delta^2 + 3\delta^4)\sigma^4$

where

(2.9) $A_3 = 4\lambda(1+\lambda^2)\Gamma\left(\frac{4}{k}\right)\Gamma\left(\frac{1}{k}\right)^{-1}\theta^3$

(2.10) $A_4 = (1 + 10\lambda^2 + 5\lambda^4)\Gamma\left(\frac{5}{k}\right)\Gamma\left(\frac{1}{k}\right)^{-1}\theta^4$

The skewness and kurtosis are

(2.11) $SK = \frac{m_3}{\sigma^3} = (A_3 - 3\delta - \delta^3)$

(2.12) $KU = \frac{m_4}{\sigma^4} = (A_4 - 4A_3\delta + 6\delta^2 + 3\delta^4)$

Although skewness and kurtosis depend on three parameters (σ, λ, and k), λ is closely associated with skewness while k is closely associated with kurtosis. Figures 2.1 and 2.2 show how varying λ and k affect the distribution. When $k<1$ then the distribution has a higher peak than the normal distribution, is more leptokurtic, as well as having thicker tails. As k grows to be greater than 2, the tails die off more rapidly and the middle portion

[2] For derivations of the moments of the SGED as well as the skewness and kurtosis see Theodossiou (2001).

of the distribution becomes more uniform. Figure 2.3 shows the effect of λ on the skewness for different values of k. There is a positive relationship between λ and skewness. This relationship is stronger when the value of k is smaller. As the absolute value of λ approaches 1, the change in skewness becomes smaller. Larger values of k also limit the maximum value of the skewness. Although k is the parameter most closely associated with kurtosis, the kurtosis does change as λ changes. Figure 2.4 shows how the kurtosis is related to λ for different values of k. The kurtosis is at it's minimun when $\lambda=0$. As λ departs from 0 the kurtosis increases. For larger values of k, the kurtosis is not as affected by changes in λ.The SGED is a very flexible distribution, which can allow for a wide range of skewness and kurtosis values.

Figure 2.1: The effect of λ for the SGED

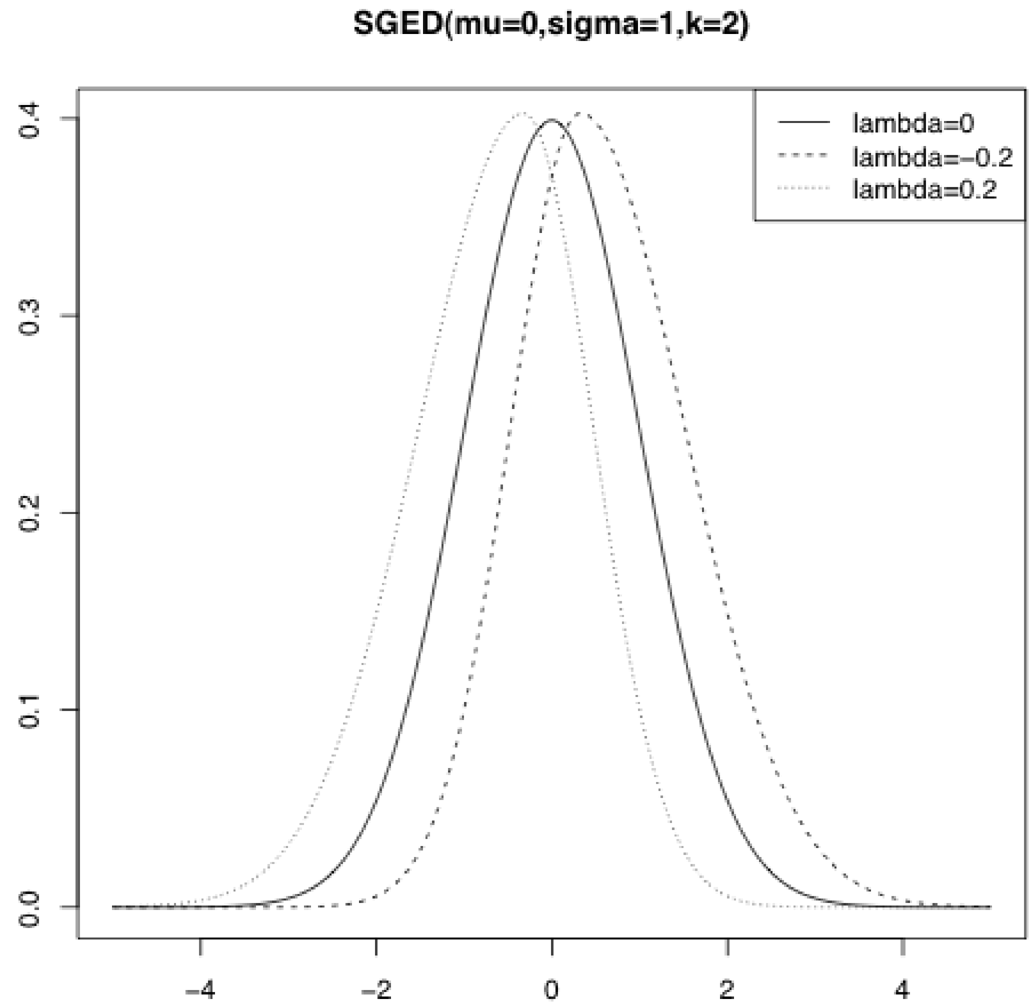


Figure 2.2: The effect of k for the SGED

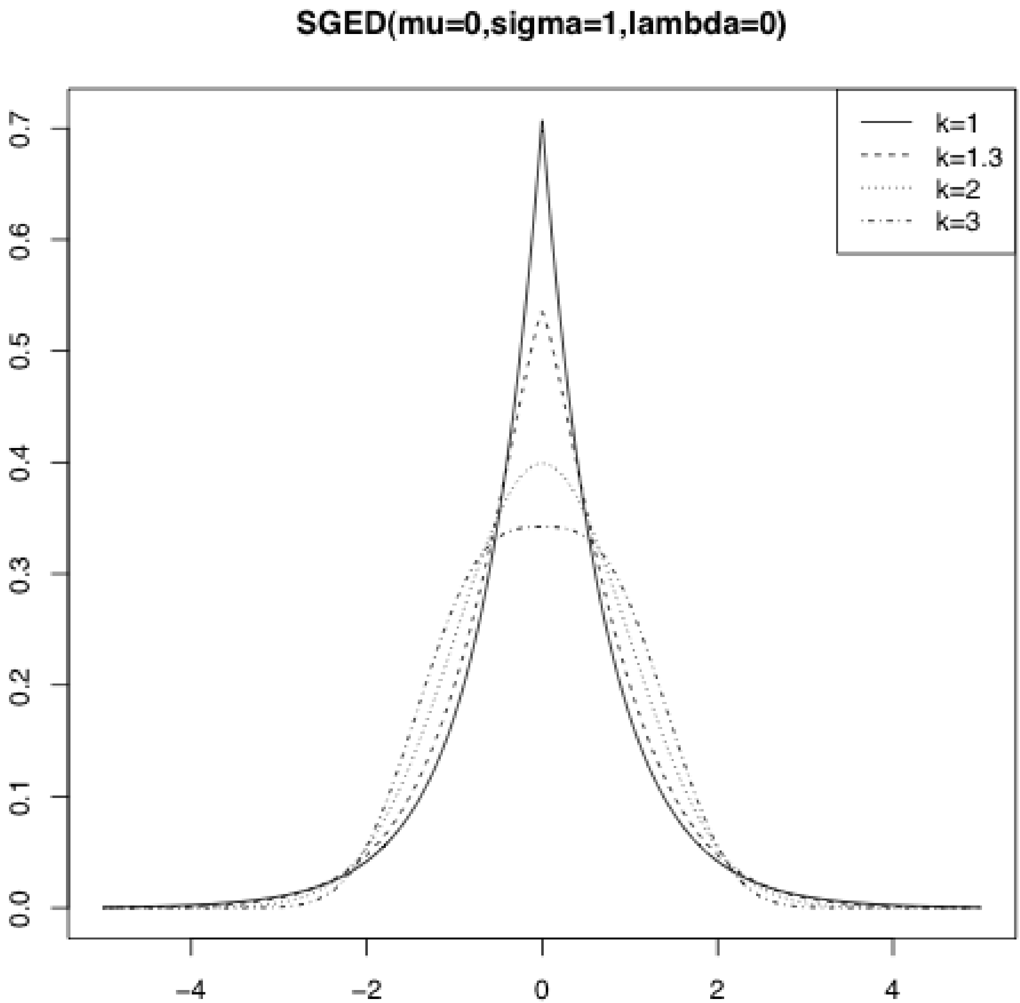


Figure 2.3: The effect of λ on skewness for the SGED

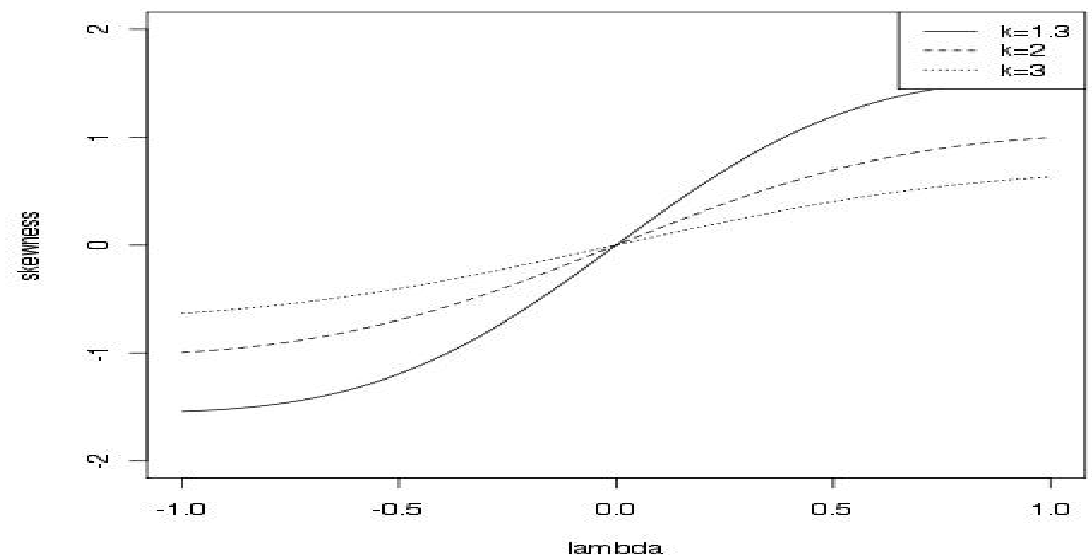
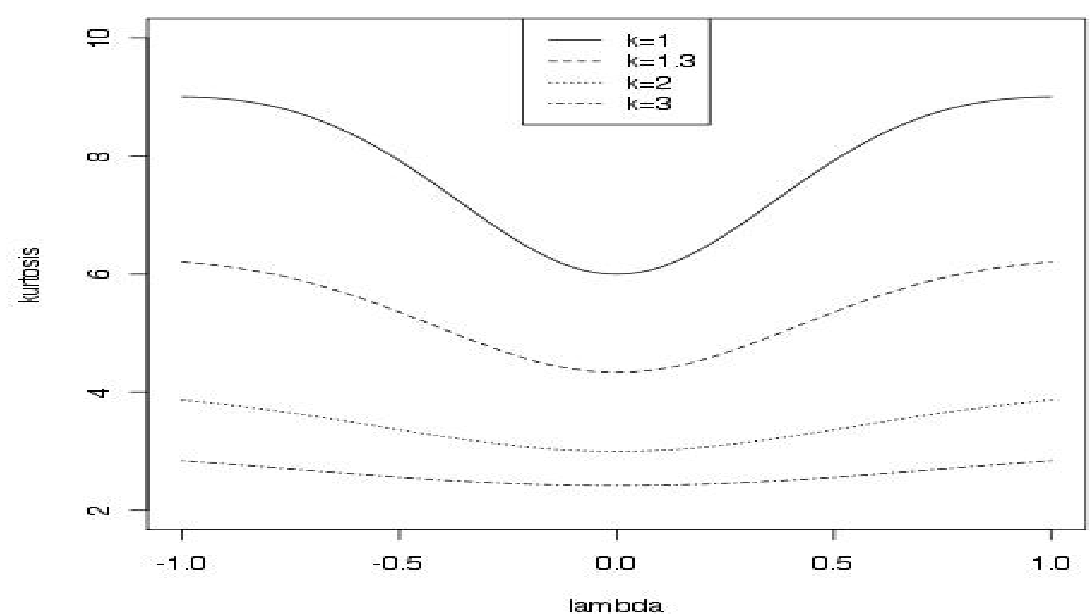


Figure 2.4: The effect of λ on kurtosis for the SGED



CHAPTER 3

DATA SOURCE AND METHODOLOGY

The data used for this study consist of daily observations of the closing price for the S&P 500 from January 3rd 1950 through February 4th 2011 (source: www.finance.yahoo.com) (T = 15,372), as well as daily observations of the spot price of crude oil in Cushing, Oklahoma from January 2nd 1986 through March 29th 2011 (source: www.eia.doe.gov) (T= 6,366). The returns were calculated to be the difference of the daily logarithms multiplied by 100. $r_t = 100\times(\ln P_t - \ln P_{t-1})$ where r_t and P_t represent the returns at time t, and the index price at time t, respectively. Dickey-Fuller unit root tests indicated that returns for S&P 500 and oil prices are stationary.

Table 3.1 gives descriptive statistics for the daily returns of both the S&P 500 and the oil spot price. The means are 0.028 and 0.022 respectively with standard errors of 0.008 and 0.033. Both sets of data display negative skewness along with leptokurtosis. Jarque-Bera test statistics soundly reject the null hypothesis that the returns are normally distributed at the 1% level for each set of data. Finally Ljung-Box Q^2 statistics indicate that there is a linear dependence on returns squared along with ARCH effects. These results indicate that a GARCH model with a SGED distribution should be able to model the returns effectively.

Table 3.1: Descriptive Statistics of Daily Returns for S&P 500 and oil prices

Data	Mean	Std. error	Skewness	Kurtosis	Jarque-Bera	Q(20)	Q^2(20)
S&P500	0.028	0.008	-1.060	29.130	546,620.8***	111.27***	3431.63***
Oil	0.022	0.033	-0.770	14.570	56,977.57***	67.69***	731.00***

Notes: 1. *** denotes statistical significance at the 1% level
2. Jarque-Bera is a test for normality.
3. Q() and Q^2() are Ljung-Box Q tests for serial correlation in returns and returns squared.

Figure 3.1: Distribution of S&P 500 from January 3rd 1950 to February 4th 2011

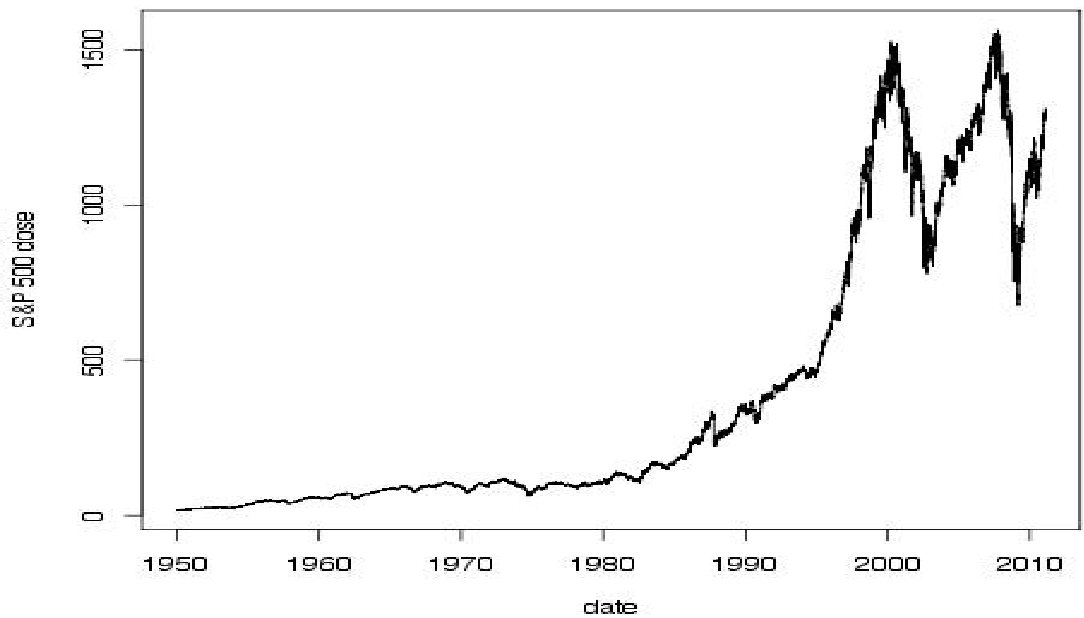


Figure 3.2: Daily S&P 500 returns from January 3rd 1950 to February 4th 2011

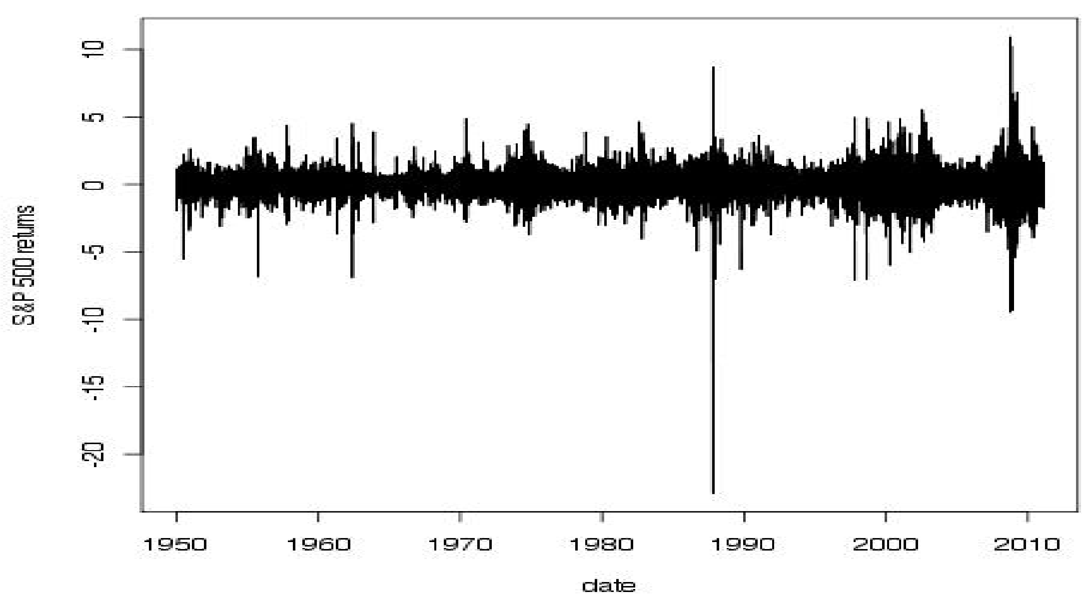


Figure 3.3: Distribution of oil spot prices from January 2nd 1986 to March 29th 2011

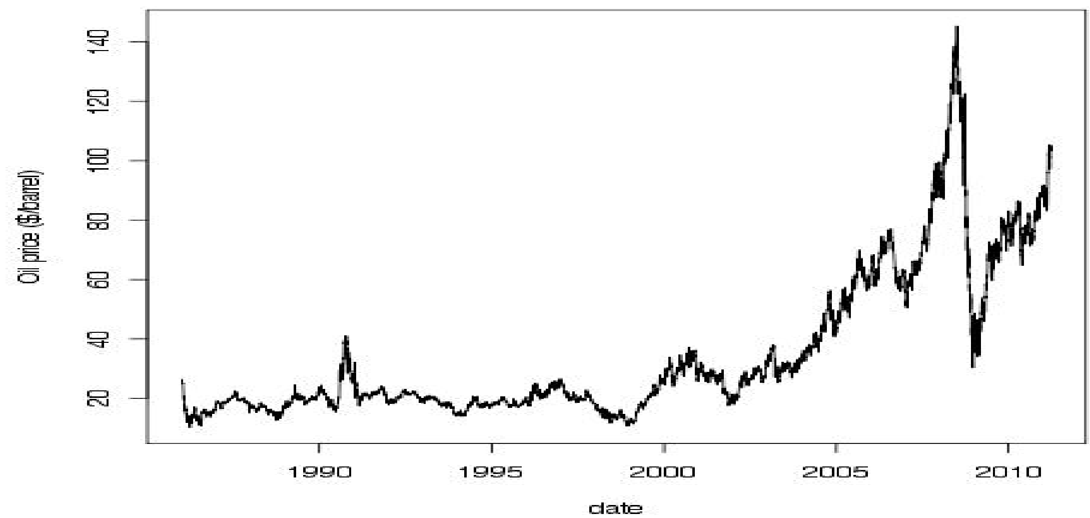
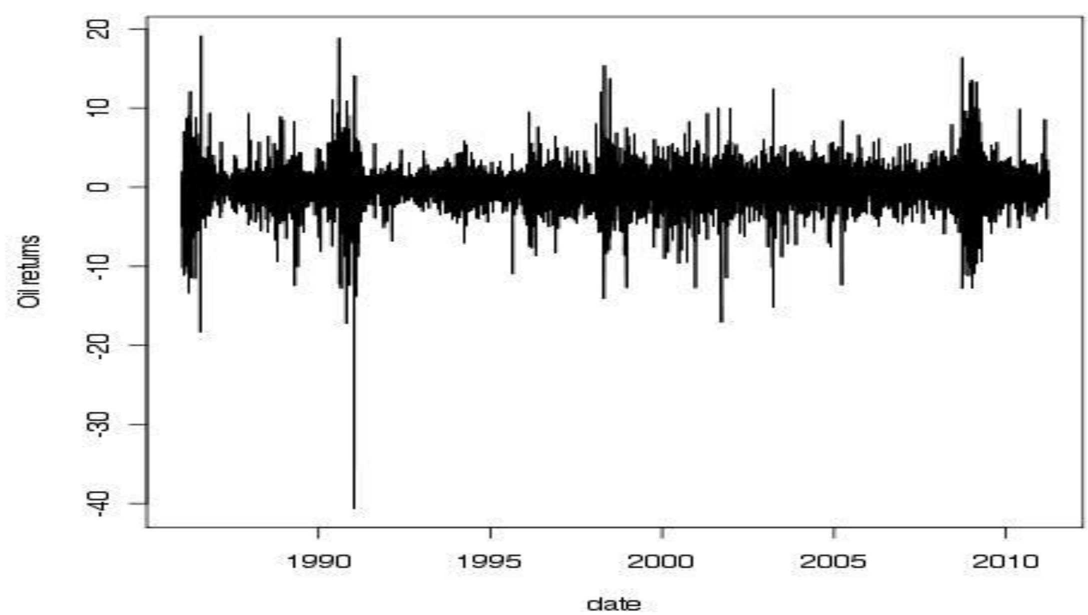


Figure 3.4: Daily oil returns from January 2nd 1986 to March 29th 2011



The first model estimated for this study was a GARCH(1,1) model with a SGED distribution. This model is as follows.

(3.1) $r_t = a_0 + a_1 r_{t-1} + \varepsilon_t, \;\; \varepsilon_t \sim SGED(0, \sigma, \lambda, k)$

(3.2) $\sigma_t^2 = e^{b_0} + e^{b_1}\varepsilon_{t-1}^2 + e^{b_0}\sigma_{t-1}^2$

a_0 and a_1 are constant parameters, ε is distributed SGED. By taking the exponent of b_0, b_1, and b_2 we are assured the parameters will be non-negative. Also $e^{b_1} + e^{b_2}$ is restricted to be less than 1. This model allows variance to be time varying, while keeping skewness and kurtosis constant.

The next step in the model specification was to explicitly model the skewness parameter λ to be time varying. This was done through a GARCH like framework as follows.

(3.3) $\lambda_t = c_0 + c_1 \varepsilon_{t-1}^{3*} + c_2 \lambda_{t-1}$

where

(3.4) $\varepsilon_{t-1}^{3*} = \frac{\exp(\varepsilon_{t-1}^3) - 1}{\exp(\varepsilon_{t-1}^3) + 1}$

c_0, c_1, and c_2 are constant parameters with $c_1 + c_2 < 1$. The cubed errors were transformed in order to magnify the parameter c_1 while maintaining values of λ_t in the range to which it is restricted.[3] If this were not done then the value of c_1 must be extremely small so that λ will remain between -1 and 1 for large values of ε^3.

The final addition to the model was including a dummy variable (weekend_t), which takes values of 1 when the day of the week is a Monday, or when there was no closing price the previous day. (i.e. if there was no close price reported for Wednesday, the returns for Thursday would have a value of $\text{weekend}_t = 1$.) This variable was added to

[3] The restriction for λ in the SGED distribution is $-1 < \lambda < 1$

the mean, variance, and skewness parameter equations.

(3.5) $\mu_t = a_0 + a_1 r_{t-1} + a_2 weekend_t$

(3.6)[4] $\sigma_t^2 = e^{b_0} + e^{b_1} + e^{b_2}\sigma_{t-1}^2 + b_3 weekend_t$

(3.7) $\lambda_t = c_0 + c_1 \varepsilon_{t-1}^{3*} + c_2 \lambda_{t-1} + c_3 weekend_t$

The same three models were estimated for both S&P 500 data and oil spot price data. All models were estimated by maximum likelihood estimation. The log-likelihood of the SGED is

(3.8) $L(k, \lambda, \mu, \sigma) = Tln(C) - Tln(\sigma) - \sum \frac{|y_t - \mu + \delta\sigma|^k}{(1 + sign(y_t - \mu + \delta\sigma)\lambda)^k \theta^k \sigma^k}$

where C, θ, and δ are as defined in 2.2-2.4, and T is the sample size.

The Nelder-Mead method of maximum likelihood was used to obtain estimates. Starting values for model 1 were those that would assume errors to be distributed normal with constant variance. The maximum likelihood estimates for the more restricted models were then used as the starting values for the less restricted models. Parameter estimates will be discussed in the next section.

[4] Note that the parameter b_3 is not restricted to be positive; however, estimated values for σ_t^2 remain non-negative for both models.

CHAPTER 4

EMPIRICAL ANALYSIS

Tables 4.1 and 4.2 show the maximum likelihood estimates for the three models described in chapter 3 for S&P 500 data and oil data respectively.

Table 4.1: Maximum likelihood estimates for daily S&P 500 returns

	Model 1		Model 2		Model 3	
Parameter	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
a_0	0.134***	0.011	0.145***	0.013	0.164***	0.015
a_1	0.083***	0.008	0.105***	0.015	0.103***	0.011
a_2					-0.094***	0.015
b_0	-5.049***	0.159	-5.041***	0.159	-8.208**	3.797
b_1	-2.528***	0.065	-2.521***	0.065	-2.495***	0.065
b_2	-0.090***	0.006	-0.091***	0.006	-0.094***	0.006
b_3					0.029***	0.007
k	1.375***	0.021	1.375***	0.021	1.382***	0.021
c_0			0.013	0.008	0.009**	0.004
c_1			0.023**	0.011	0.017**	0.008
c_2			0.793***	0.143	0.875***	0.083
c_3					-0.006	0.007
λ	0.050***	0.006				
Log-Likelihood T=15,372	-17,918.64		-17,908.28		-17,879.69	

Note: * ,** and *** denote statistical significance at the 10%, 5% and 1% levels respectively

Table 4.2: Maximum likelihood estimates for daily oil returns

	Model 1		Model 2		Model 3	
Parameter	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
a_0	0.167***	0.061	0.160***	0.062	0.210***	0.063
a_1	-0.017	0.012	-0.061***	0.023	-0.054**	0.025
a_2					-0.234**	0.117
b_0	-2.699***	0.225	-2.737***	0.219	-7.607	6.204
b_1	-2.565***	0.119	-2.615***	0.110	-2.531***	0.116
b_2	-0.090***	0.010	-0.086***	0.009	-0.097***	0.011
b_3					0.381***	0.086
k	1.330***	0.030	1.332***	0.030	1.334***	0.031
c_0			0.023*	0.013	0.035**	0.015
c_1			-0.024**	0.011	-0.019	0.012
c_2			-0.148	0.172	-0.339**	0.134
c_3					-0.035**	0.018
λ	0.021**	0.011				
Log-Likelihood T=6,366	-13,995.16		-13,992.54		-13,985.50	

Notes: * ,** and *** denote statistical significance at the 10%, 5% and 1% levels respectively

Maximum likelihood estimates of k are significantly less than 2 in all model specifications for both sets of data indicating the returns to be leptokurtic. GARCH effects are highly significant in all model specifications as well. The estimates of λ without time varying skewness are 0.050 for the S&P 500 and 0.021 for the oil data. Both indicate slight positive skewness in the returns and are significant at the 1% level for S&P 500 and the 5% level for oil. When time varying skewness is introduced, the S&P 500 dataset indicates strong overall significance based on likelihood ratio tests (20.72), but for oil returns, the significance is not as great (5.24). Only the parameter associated with the modified lagged error cubed is significant at the 5% level. For S&P 500 both the c_1 and c_2 parameters in the skewness equation are significant at at least the 5% level. The final addition of a weekend dummy variable proves significant at the 1% level in the variance equation for both models. The weekend variable is significant at the 5% level in both the mean and skewness equations for the oil model, whereas it is highly significant in the mean equation but not significant at all for the skewness for the S&P 500 returns. Jointly the addition of the weekend variable to the mean, variance, and skewness equations is significant for both sets of data based on likelihood ratio tests (57.18 for S&P 500 and 14.08 for oil). The signs of the parameters are also consistent between the data sets. Returns on Mondays or days after holidays seem to be less than compared with all other days. Also they display greater variance along with more negative skewness.

Table 4.3 shows values for Ljung-Box test statistics for raw returns as well as standardized residuals for each model for different lag lengths. For S&P 500 data, models 2 and 3 have the least autocorrelation, while all models have no ARCH effects in the standardized residuals. All models for oil returns are free from ARCH effects, however only model 1 is free of autocorrelation in the standardized residuals. Breusch-Pagan tests for autocorrelation and ARCH effects presented in table 4.4 also show the same results.

Table 4.3: Ljung-Box statistics

Data/model	Q(5)	Q(20)	Q(40)	$Q^2(5)$	$Q^2(20)$	$Q^2(40)$
SP returns	57.26***	111.27***	185.36***	1,836.61***	3,431.63***	4,622.50***
SP model 1	22.32***	39.46***	65.88***	8.69	20.73	38.26
SP model 2	13.23*	30.50*	57.00**	9.05	21.45	39.15
SP model 3	13.72**	29.02*	53.52*	8.61	19.88	36.07
Oil returns	34.30***	67.69***	116.93***	369.94***	731.00***	1,102.16***
Oil model 1	5.86	20.04	48.96	7.08	22.69	36.78
Oil model 2	22.61***	37.52**	67.10***	8.85	24.32	37.96
Oil model 3	16.61***	31.87**	62.62**	7.11	23.51	37.71

Notes: * ,** and *** denote statistical significance at the 10%, 5% and 1% levels respectively

Table 4.4: Breusch-Pagan test statistics

	Breusch-Pagan test for autocorrelation			Breusch-Pagan test for ARCH effects		
Model	Lag =5	Lag = 20	Lag = 40	Lag =5	Lag = 20	Lag = 40
SP returns	44.19***	87.25***	139.37***	929.14***	1,065.11***	1,127.43***
SP model 1	16.99***	29.46*	48.72	6.79	14.73	28.32*
SP model 2	9.06	22.66	41.92	6.79	15.86	29.46
SP model 3	10.19*	21.53	39.65	6.79	14.73	27.19*
Oil returns	38.58***	77.80***	125.88***	265.65***	341.55***	405.43***
Oil model 1	6.33	21.50	50.60	6.33	20.87	36.05
Oil model 2	22.77***	37.95***	67.04***	8.22	22.77	36.69
Oil model 3	16.45***	32.89**	63.25**	6.33	22.14	36.69

Notes: * ,** and *** denote statistical significance at the 10%, 5% and 1% levels respectively

Figures 4.1 and 4.2 graph the estimated SGED models with the kernel densities and a fitted normal distribution.[5] In both cases the SGED does a good job modeling the excess kurtosis along with the tail thickness demonstrated by the returns.

[5] The unconditional mean and variance along with the estimates of k and λ for “model 1” are used to generate the SGED graph. Normal distribution is fitted using sample mean and std. dev.

Figure 4.1: SGED vs. kernel density for S&P 500 returns

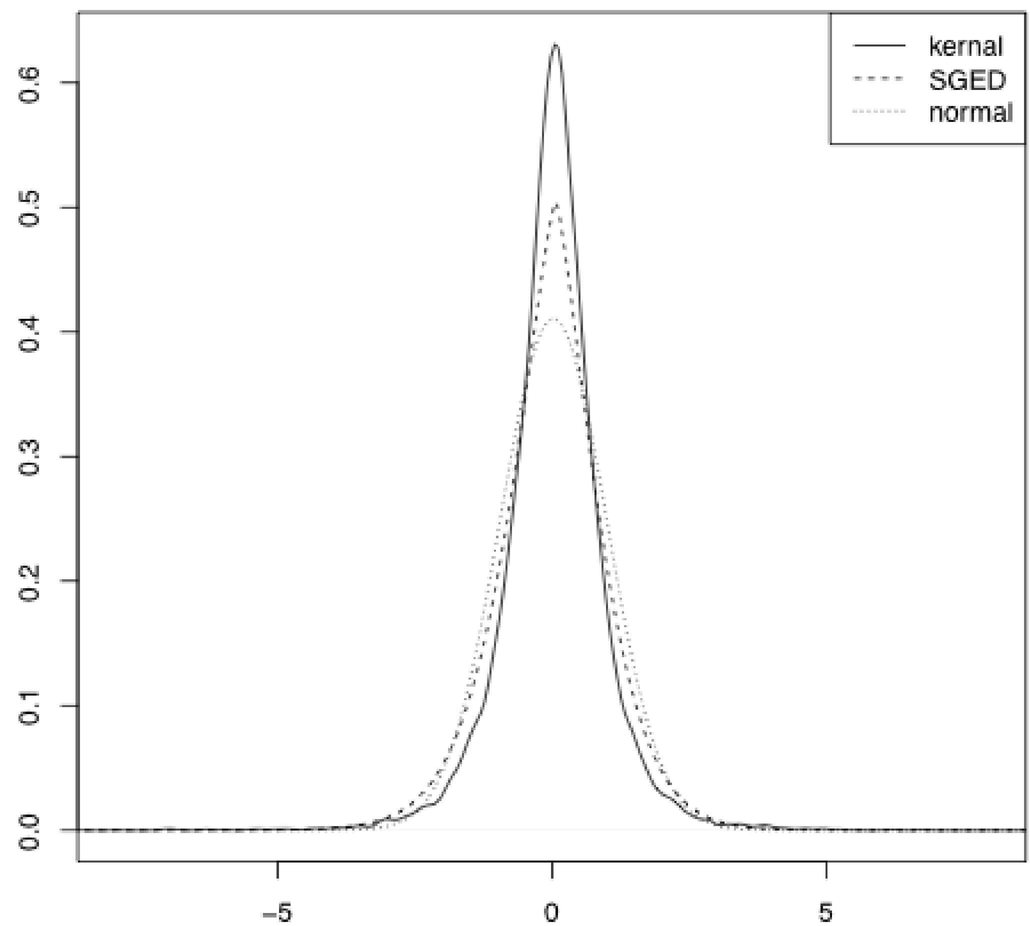
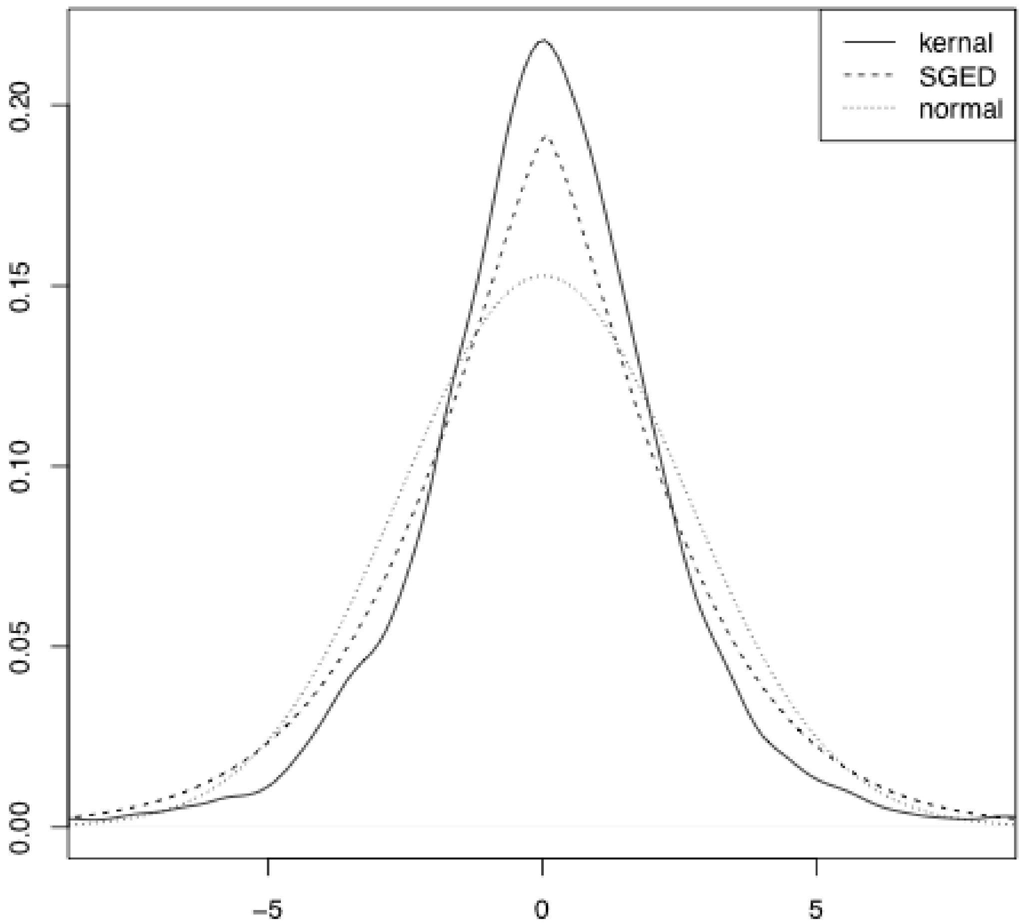


Figure 4.2: SGED vs. kernel density for oil returns



Figures 4.3 and 4.4 show the conditional variance over time, figures 4.5 and 4.6 show the conditional skewness over time, and figures 4.7 and 4.8 show the conditional kurtosis over time. Also figures 4.5-4.8 show the constant skewness and kurtosis from model 1. The conditional variance of the S&P 500 returns is relatively unchanging over the first half of the sample and has a few very large spikes later on. The conditional variance of the oil returns is more varied throughout the entire sample as well as having larger values than those of the S&P 500. The conditional skewness of the S&P 500 returns is mostly positive with only a small portion of the values being negative. The conditional skewness of the oil returns is much less stable and has a higher percentage of negative values compared to the S&P 500 returns. Compared to the conditional kurtosis of the S&P 500 returns, the conditional kurtosis of the oil returns is generally greater, however the range for kurtosis values is smaller for oil returns.

Figure 4.3: Conditional variance of S&P 500 returns over time

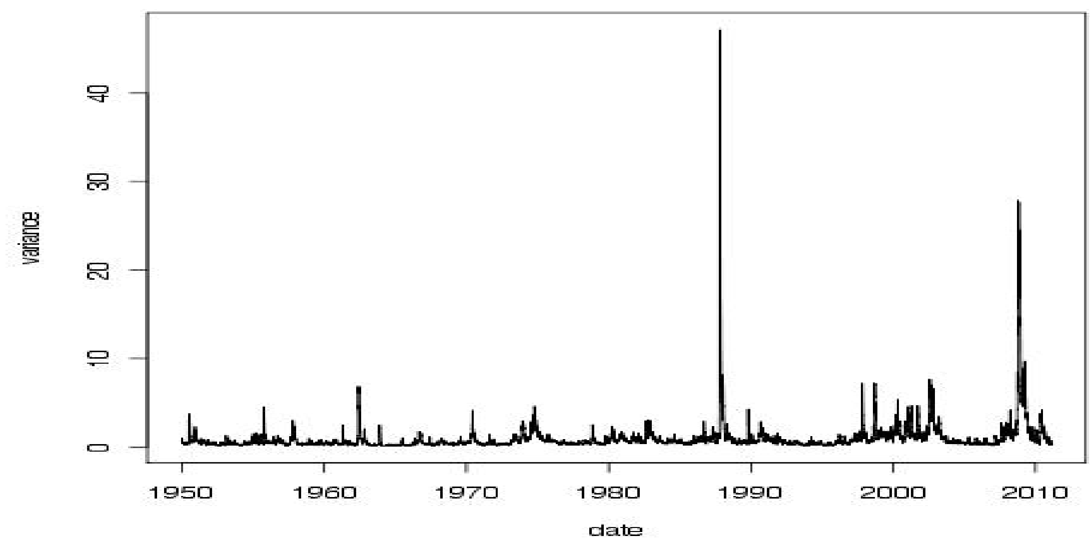


Figure 4.4: Conditional variance of oil returns over time

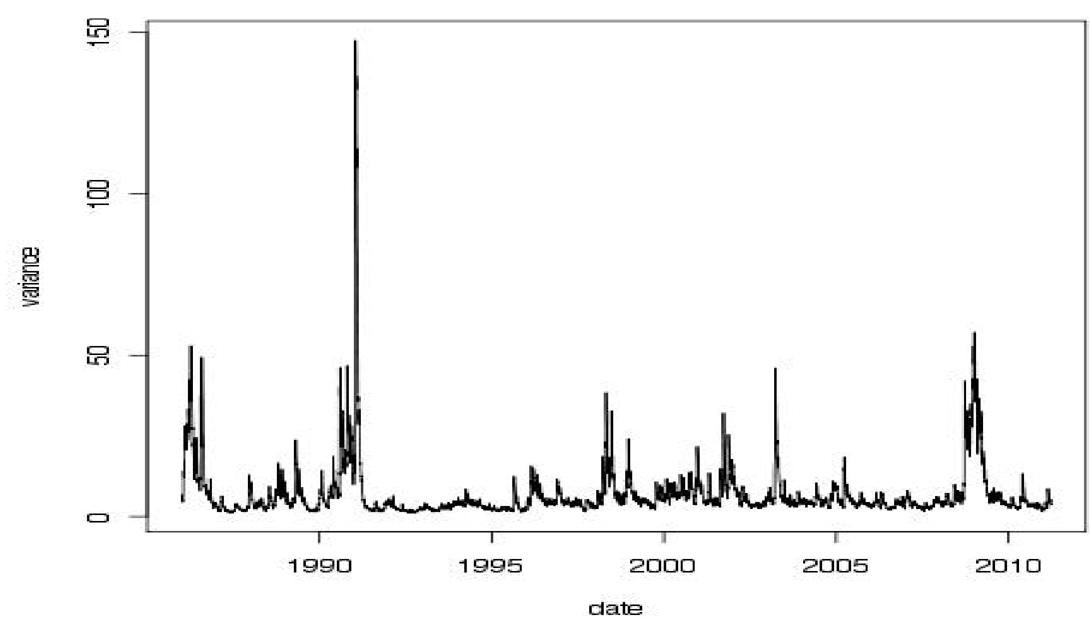


Figure 4.5: Conditional skewness of S&P 500 returns over time

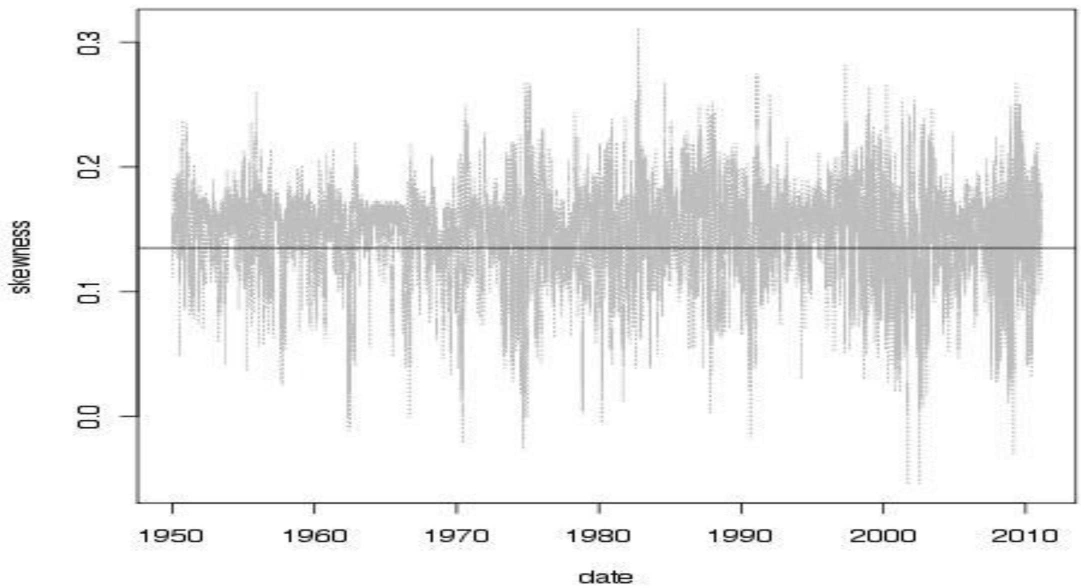


Figure 4.6: Conditional skewness of oil returns over time

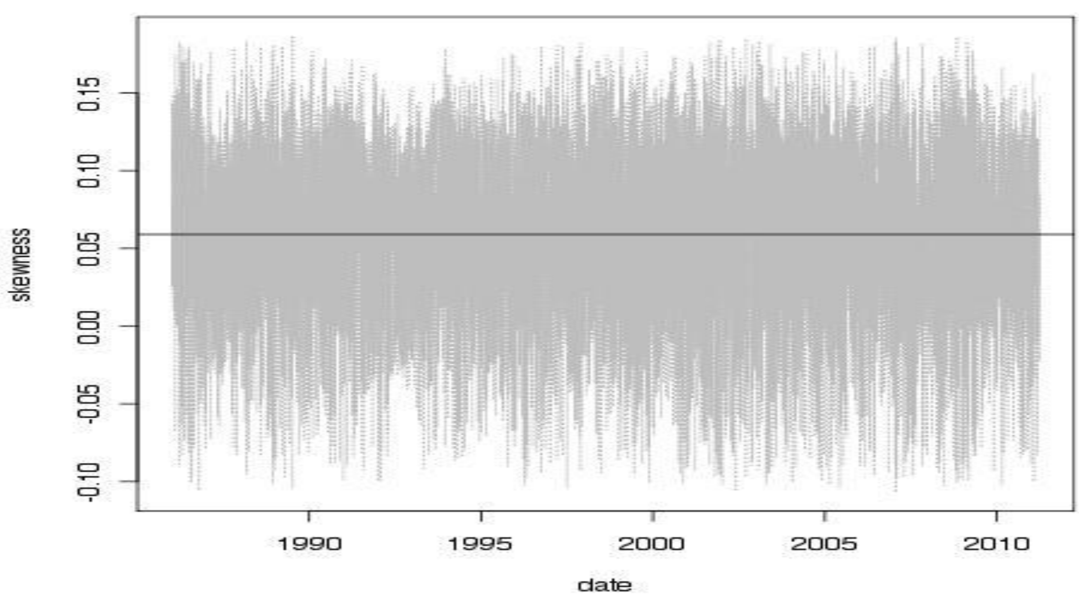


Figure 4.7: Conditional kurtosis of S&P 500 returns over time

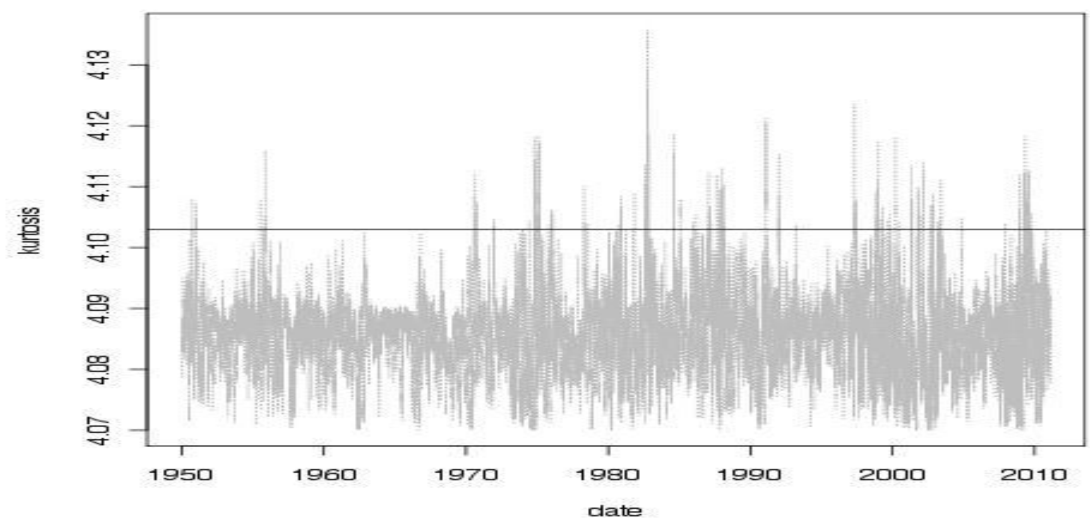
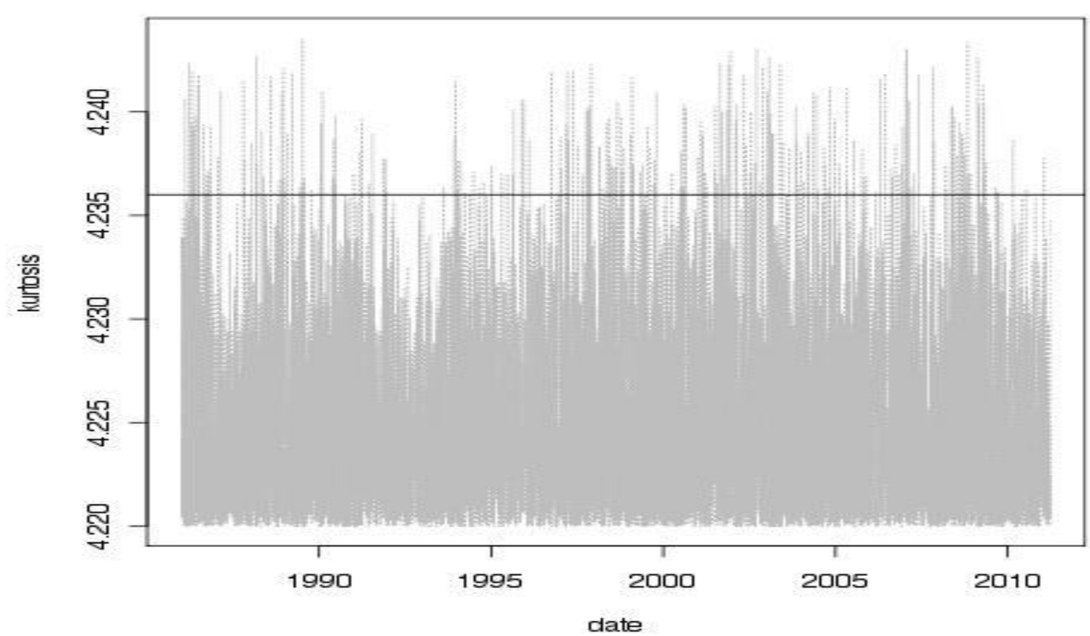


Figure 4.8: Conditional kurtosis of oil returns over time



CHAPTER 5

CONCLUSION

Financial returns have been shown to be distributed non-normal, having thicker tails and are leptokurtic, as well as being asymmetric. This study made use of the skewed generalized error distribution to model the daily returns of the S&P 500 and spot price of oil. Jarque-Bera tests for both sets of data concluded that the data were, in fact, distributed non-normal. Estimates of the parameters for the SGED indicate that both data sets are indeed leptokurtic as well as positively skewed.

In addition to the GARCH model for time varying volatility, skewness was also allowed to vary over time. Based on likelihood ratio tests, we conclude that skewness is indeed time varying for both S&P 500 and oil data.

Finally we introduced a dummy variable for days after weekends or holidays into the mean, variance, and skewness equations. The addition of this variable proved to be significant to the overall model for both data sets, however in the individual equations the results varied. In both S&P 500 and oil returns we found that returns were less in days following weekends and holidays than they were for all other days. Days after weekends and holidays also had more variance compared to other days. There was however, no significant difference in the skewness between days after weekends/holidays and all other days.

Future developments would include out-of-sample predictions, with applications to Value-at-Risk calculations. Also higher order moments, such as kurtosis, could be modeled to be time varying as well. Finally a multivariate SGED would be of interest for jointly modeling returns.

References

Bali, Turan G. “Modeling the Dynamics of Interest Rate Volatility with Skew Fat-tailed Distributions.” *Annals of Operations Research.* 1 (2007): 151-178.

Bollerslev, Tim. “A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return.” *The Review of Economics and Statistics.* 69 (August 1987): 542-547.

Bollerslev, Tim, Chou, Ray Y., and Kroner, Kenneth F. “ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence.” *Journal of Econometrics.* 52 (1992): 5-59

Bond, Shaun A., and Patel, Kanak. “The Conditional Distribution of Real Estate Returns: Relating time variation in higher moments to downside risk measurement.” *Journal of Real Estate and Economics.* 26 (2003): 319-339.

Brannas, Kurt, and Nordman, Niklas. “Conditional Skewness Modelling for Stock Returns.” *Applied Economics Letters.* 10 (2003): 725-728.

Dark, Jonathan G. “Estimation of Time Varying Skewness and Kurtosis with an Application to Value at Risk.” *Studies in Nonlinear Dynamics & Econometrics.* 14 (2010): Article 3.

Engle, Robert F. “Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation.” *Econometrica.* 50 (July 1982): 987-1007.

Fama, Eugene F. “The Behavior of Stock Market Prices.” *Journal of Business.* 38 (January 1965): 34-105

Hansen, Bruce E. “Autoregressive Condition Density Estimation.” *International Economic Review.* 35 (August 1994): 705-730

Harvey, Campbell R., and Siddique, Akhtar. “Autoregressive Conditional Skewness.” *Journal of Financial and Quantitative Analysis.* 34 (December 1999): 465-487.

Jensen, Morten B., and Lunde, Asger. “The NIG-S&ARCH model: a fat-tailed, stochastic, and autoregressive conditional heterskedastic volatility model.” *Econometrics Journal.* 4 (December 2001): 319-342.

Jondeau, Eric, and Rockinger, Michael. “Conditional Volatility, Skewness, and Kurtosis: Existence, Persistence, and Comovements.” *Journal of Economic Dynamics and Control.* 27 (2003): 1699-1737.

Lambert, Philippe, and Laurent, Sebastien. “Modelling skewness dynamics is series of financial data using skewed location-scale distributions.” (2002). Discussion Paper. Institut de Statistique, Louvain-la-Neuve.

Lee, Cheng F., Chen, Gong-meng, and Rui, Oliver M. “Stock Returns and Volitility on China’s Stock Markets.” *The Journal of Financial Research* 24 (2001): 523-543.

Lee, Yen-Hsien, and Pai, Tung-Yueh. “REIT Volatility Prediction for the Skew-GED Distribution of The GARCH Model.” *Expert Systems with Applications.* 37 (July 2010).: 4737-4741.

Mandelbrot, B. “The Variation of Cartain Speculative Prices.” *Journal of Business* 36 (October 1963): 394-419.

Mittnik, Stefan, and Paolella, Marc S. “Conditional Density and Value-at-Risk Prediction of Asian Currency Exchange Rates.” *Journal of Forecasting.* 19 (2000): 313-333.

Nelson, Daniel B. “Conditional Heteroskedasticity in Asset Returns: A New Approach.” *Econometrica* 59 (March 1991): 347-370.

Premaratne, Gamini, and Bera, Anil K. “Modeling Asymmetry and Excess Kurtosis in Stock Return Data.” (November 2000). Illinois Research & Reference Working Paper No. 00-123.

Sadorsky, Perry. “Modeling and Forecasting Petroleum Futures Volatility.” *Energy Economics.* 28 (July 2006): 467-488.

Taylor, Stephen J. “Modeling Stochastic Volatility: A Review and Comparative Study.” *Mathematical Finance*. 4 (April 1994): 183-204.

Theodossiou, P. “Skewed Generalized Error Distribution of Financial Assests and Option Pricing.” Working Paper, School of Business, Rutgers University, New Jersey. 2001.

Wilhelmsson, Anders. “Value at Risk with time varying variance, skewness and kurtosis – the NIG-ACD model.” *The Econometrics Journal.* 12 (2009): 82-104.