LINQUAD: AN INCOMPLETE DEMAND SYSTEM APPROACH TO DEMAND ESTIMATION AND EXACT WELFARE MEASURES

by

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ABSTRACT

The use of an incomplete demand system approach facilitates the exploration of a new functional form for applied demand analysis. LinQuad is a flexible functional form, linear in income and linear and quadratic in prices. A generalized PIGL form of LinQuad is introduced which nests LinQuad and an AIDS-like PIGLOG form. Three levels of theoretical restrictions, symmetry, joint symmetry and concavity, and concavity conditional on symmetry, are not rejected at the 5% and 1% level of significance for the optimal generalized PIGL form and LinQuad, respectively. These empirical models thus meets all integrablity conditions. Two hypothetical welfare applications to the dairy industry illustrate the dangers of using a misspecified PIGLOG model for policy evaluation.

CHAPTER ONE

INTRODUCTION

1.1 Problem Statement

Consumer demand theory tells us that the demand for a good can be explained as a function of prices and income. Applied demand analysis uses econometrics to extend this theory into the empirical realm, using it to quantify these market dynamics. The results can be both practical and powerful. One application, welfare analysis, can take advantage of this understanding of market dynamics to quantify the effects of policy-induced price changes on consumers, producers and even the efficient functioning of the market itself. The power of this kind of application, however, is fundamentally determined by the quality of the underlying demand analysis.

The study of demand analysis is the exploration of how best to use economic theory to describe the empirical functioning of a market. The goal is a mathematical statement that explains how price, income or expenditure and any other desired variables interact to determine the quantity demanded of a good. This functional form provides the underlying structure while the econometrics provides the unique expression of actual data within that functional form. For example, a very basic linear functional form would indicate that the effect of price and income on demand is simply additive. The econometrics provides the weights, or the marginal price and income effects, which identify the equation with the specific data being used. Now, if the true relationship between the price and income is multiplicative, then the model is misspecified. The misspecified linear functional form will do a poor job of explaining the data and practical applications like welfare analysis might be altogether wrong. The problem of specification and, specifically, choice of functional form is thus of central importance to the quality of applied demand estimation.

1.1.1 Choice of Functional Form

Functional form issues tend to be specific to an empirical approach. One particular area of demand analysis that has received a disproportionate amount of attention due to the combination of practical applicability and theoretical challenge is the study of the market interaction between a small group of goods. A narrow approach is widely useful because it allows a subtle understanding of closely related goods, both substitutes and complements. The challenge lies in the translation of general demand theory into this particular kind of scenario. Two particular challenges have bedeviled this area. First, the separability theory used to justify focussing on a subgroup has its problematic aspects. Second, empirical results do not always fully conform to the expectations of theory.

For almost two decades the Almost Ideal Demands System (AIDS), introduced by Deaton and Muellbauer (1980a) has been the standard approach to performing subgroup demand analysis. When it was introduced it offered an alternative to the models in use at the time. It did not, as its name indicates, solve all the problems and work has continued to improve on the AIDS model. For a model to be a useful alternative to the AIDS model it would have to increase the already flexible AIDS form and address issues related to separability.

Even if an alternative could be found, directly comparing functional forms in a quantitative manner can be difficult. The complexity of the underlying assumptions along with the variety of measures of quality can make comparison of results uninformative or arbitrary. It can also be difficult to determine an objective benchmark by which to compare results. Where functional forms are similar in structure and certain results are expected a priori, comparison becomes easier. A particularly strong comparison can be made between two or more functional forms nested within another functional form. In a nested specification, the log of the likelihood function becomes a simple measure of the relative ability of each nested functional form to fit the data. Direct tests can determine which form is closest to the optimal expression of the overarching functional form. Under these circumstances it is, in fact, possible to make a reasonable comparison between two functional forms.

1.1.2 Demand Analysis Application

An important further measure of the success of a functional form is performance in application. The market for dairy products is a particularly good place to compare different functional forms through the welfare measures they produce. The dairy industry is an ideal set of closely related goods (milk, butter, cheese, frozen, and other dairy products) to study as a subset and there is no shortage of policy effects to quantify.

The structure of the dairy sector is a well-documented case of government intervention (Bailey 1997). The federal government has maintained price supports for milk at the wholesale level since WWII. There has also been a system of differential pricing through milk marketing orders which has distorted price relationships within the subgroup. Economic theory tells us that both consumers and dairy farmers have experienced transfers of money as a result of market distortion and that the resulting inefficiency can be measured as deadweight loss. There have been many attempts to quantify the costs and benefits (LaFrance and de Gorter 1985; Ippolito and Masson 1978; Dahlgren 1980; Gould, Cox and Perali 1990; Kaiser, Streeter and Liu 1988; Liu et al. 1991). This work has all been based on demand analysis.

While the dismantling of a large part of the federal dairy program is now in its final stages as a result of the 1996 farm bill, there remain many possible policy decisions that deserve welfare analysis. Removal of government presence is far from complete. A policy decision that has reasserted a government presence in the dairy market is presently in effect in the Northeastern U.S. With the stated goal of supporting producer prices in participating states, the Northeastern Interstate Dairy Compact (NIDC) increased the support price for fluid milk.(Chite 1997) One apparent effect has been an increase in retail milk prices. This kind of policy effect provides a reasonable template for comparing welfare measures from different functional forms.

Price discrimination, the practice of supporting fluid milk production at a higher price than milk for manufactured products, provides a scenario for a very different kind of policy effect. Price discrimination is a policy many feel should be removed. This scenario would have a very different effect on prices than the NIDC. Fluid prices would drop relative to the prices of manufactured dairy products.

The policy-induced price changes that would result in each of these scenarios provide very different welfare measures. Simply put, the first scenario shows the imposition of further policy while the second scenario shows the removal of a policy. Furthermore, the two scenarios are different in the interaction of price changes, as the NIDC support price increase should have no effect on manufacturing precisely as a result of the structure that allows price discrimination. The welfare measures for both of these scenarios can be decomposed into net transfers and deadweight loss. The latter is the most important measure as it reflects the actual gains or losses, net of transfers, that result from a policy decision. Deriving welfare measures for these two scenarios provides realistic and varied results to compare the efficacy of different empirical approaches to demand analysis.

1.2 Research Objectives

In this thesis, a new functional form is proposed that is well suited for applied demand analysis. It is presented in a particularly useful nested specification. The functional form can be tested on its own merits and can be compared directly to an incomplete demand expression of the AIDS model. The functional form allows estimation fully consistent with economic theory. It is a flexible functional form able to fully exploit the time series data utilized here. Its structure and theoretical derivation are particularly well suited for derivation of exact welfare measures.

1.3 Organization of the Study

The thesis is organized as follows. Chapter two discusses recent critiques of the AIDS model and then provides the theoretical argument for an incomplete demand system approach. Chapter three provides derivations for the three models under consideration, LinQuad, the AIDS-like PIGLOG form and the generalized PIGL form which nests them. Chapter three goes on to derive exact welfare measures for these models, discusses the imposition of theoretical restrictions and addresses a number of technical issues important to the estimation procedure. Chapter four reports the results of the estimations and the relevant welfare measures. Chapter five is concluding remarks.

CHAPTER TWO

LITERATURE REVIEW AND THEORY

2.1 Applied Demand Analysis

2.1.1 Complete Demand Systems

Consumer economics is built on the utility function. The utility function is a mathematical statement of how much utility is produced by some combination of goods. A set of axioms that delineates how people order their preferences for different goods makes it possible to represent the relationship between these goods mathematically. It is assumed that utility will be maximized within the limitations of funds and thus the prices play a central role. Utility, in itself, is not a very useful concept as there is no common currency for something so subjective. Combined with a budget constraint, however, it provides a link with which the relationships of much more concrete entities like prices and quantity can be explored. Central to this research is the notion that, without ever defining the units of utility, the quantities demanded of a set of goods can be described as a function of income and the prices of those goods.

The utility function posited by economic theory encompasses all goods. In theory, a set of equations detailing the demand for every good could be derived, directly or indirectly, from this utility function. In practice, however, such complete demand systems are of limited use. Data limitations force the applied demand analyst to use highly aggregate groups of goods for estimation. This aggregation means that only under limited conditions are the underlying preferences theoretically sound. Even more important, from a practical perspective, information on individual commodities is not recoverable.

If the desired scope of analysis includes information on individual commodities, another approach is needed. A subset of goods needs to be separated off, facilitating a full exploration of their structural relationship, without losing the foundation of utility theory. Derivation of demand equations for a subset of goods has been approached one of two ways. One option has been to assume separability of the utility function and derive conditional demand equations. The other option directly specifies a demand system and then tries to reconcile it with economic theory after the fact.

2.1.2 Conditional Demand Systems

Conditional demand systems rely on separability to limit the scope of analysis in a theoretically sound manner. If the preference ordering within a subset of goods can be done independently of quantities of goods in other subsets, that subset is said to be separable (Deaton and Muellbauer 1980b). Of the various forms of separability, the least restrictive is weak separability. Weak separability implies the presence of subutility functions within the overall utility function. This can be written $u = v(x_1, x_2, x_3, x_4) = f[v_a(x_1, x_2), v_b(x_3, x_4)]$ where f is an increasing function and v_a and v_b are subutility functions.

The separable approach to estimating demands for a subgroup of goods can be best understood as a two-step budgeting procedure. Expenditure is allocated to aggregated subgroups first and only then further allocated within each subset. If a family allocates a certain amount of money to groceries and then decides on a certain mix of goods at the store, then a kind of two-step budgeting has occurred. It is less clear that two-step budgeting can explain budgeting for a subgroup like dairy. Regardless of this question, the separable approach produces demand equations derived from the maximization of the subutility function with respect to the subgroup budget allocation. These demands are conditional on the first step of budget allocation.

2.1.2.1 The Almost Ideal Demand System (AIDS)

The Almost Ideal Demand System (AIDS) derived by Deaton and Muellbauer (1980a) is a model that is commonly used under an assumption of separability. The derivation of the AIDS model does not limit it to conditional demand systems. Following a common approach to deriving demand systems, the AIDS model is derived from an expenditure function which represents a well behaved preference ordering.. The expenditure function is the dual expression of the utility function. Rather than maximizing utility subject to a budget constraint, expenditure is minimized given some fixed level of utility.

The AIDS model is a flexible functional form that allows the imposition and testing of theoretical restrictions. It was originally proposed as a complete system using heavily aggregated groups of goods. More recently, under the assumption of separability, it has been used to estimate demands for subsets of goods. The system is derived from the sub-expenditure function related to the subutility function of the separable group of goods. The process of the initial allocation of expenditure among the aggregate subgroubs is usually not explicitly addressed.

For a full derivation of the AIDS model refer to the original Deaton and Muellbauer article. They explain why they chose the expenditure function they start with and how with Shephard's Lemma and a substitution, the demands are derived. The basic process is identical to the process that will be used later in this thesis. In short, an AIDS demand system is of the form,

$$\mathbf{w} = \alpha + B\ln(\mathbf{p}) + \gamma\left[\ln(e) - \alpha'\ln(\mathbf{p}) - .5\ln(\mathbf{p})'\mathbf{B}\ln(\mathbf{p})\right] \qquad (2.1.1)$$

where **w** is a vector of budget shares, ln(**p**) a vector of logged prices, ln(e) logged subgroup expenditure and α and **B** and γ are parameters to be estimated.

The AIDS model is non-linear in its parameters. This makes the econometrics computationally more difficult. This was a serious consideration when computer power was an expensive and limited resource. The popularity of the AIDS model grew, in part, due to an easily estimated linear approximation(L-AIDS) to the original non-linear AIDS specification. Use of a price index makes the model linear in parameters.

$$\mathbf{w} = \alpha + B\ln(\mathbf{p}) + \gamma\left[\ln(e) - \mathbf{P}\right] \qquad (2.1.2)$$

where **P** is some logged price index, traditionally Stone's index, the sum of the expenditure-weighted logged prices. The authors offered evidence that the linear model was a reasonable approximation of the non-linear AIDS specification but warned this might not always be the case. Over the years a number of criticisms have been leveled at

AIDS. They fall into two categories: Problems with the L-AIDS and econometric issues implicit in the separable functional form.

2.1.2.2 Critiques of the AIDS Model

Buse (1998), Moschini (1995) and others have pointed out that there are a number of problems with L-AIDS. Comparisons of the commonly used Stone's index and other indexes indicated a potentially substantial bias depending on the choice of index. Furthermore, Moschini demonstrated that the Stone's index is not invariant to scale. As a result a different index has been proposed that appears to solve the problem.

Correcting problems with the index does not solve a further problem of theoretical consistency. One of the great advantages of a demand system derived from an expenditure function is the luxury of returning to the expenditure function to derive exact welfare measures. The use of the Linear approximation changes the functional form of the AIDS model so that it is no longer strictly speaking integrable to the original expenditure function of the nonlinear AIDS model (Buse 1994). The problematic implications of this may also be solved as LaFrance (1998b) has recently recovered the expenditure function for the L-AIDS model.

The second set of criticisms is aimed at AIDS only as one of the more popular separable specifications commonly in use. There appears to be a bias inherent in welfare measures derived from conditional demand systems. LaFrance (1993) points out that exact welfare measures are based on overall expenditure rather than subgroup expenditure. Calculating welfare measures from a subgroup demand system cannot account for the possible changes in the initial budgeting process to the aggregated subgroups. As a result compensating variation will be biased downwards and equivalent variation biased upwards. In a Monte Carlo experiment intended to validate his results from actual data, LaFrance found that the average conditional estimate of compensating variation was only 75% of the average estimate of the unconditional specification. The only circumstances under which no bias would be present is a fixed coefficients, Leontief relationship between the subutility function for the separable goods and all other goods. This indicates a serious limitation in separable demand systems including the non-linear AIDS specification.

The use of subgroup expenditure as a right-hand side variable is a further problem for separable specifications (LaFrance 1991a). It is common practice to assume exogeneity of expenditure to avoid simultaneity bias. There is growing evidence that expenditure cannot be viewed as exogenous. Using the data set that will be used in this thesis, LaFrance (1992,1993) showed expenditure failed tests of exogeneity. Wu-Hausman tests performed for that research clearly rejected exogeneity in the conditional demand specification while failing to reject exogeneity in prices, income or joint price and income in the alternative incomplete specification. Simultaneous equations bias results from joint determination of quantities demanded and expenditure. As another possible explanation for simultaneity, Lewbel (1996) shows that measurement error results in a correlation between individual commodity expenditure and subgroup expenditure.

2.1.3 Directly Specified Demand Models

A second approach to estimating demands for a subset of goods avoids two step budgeting and conditional demands by directly specifying demand equations for a subset of goods. For example, to avoid the possible simultaneity bias resulting from subgroup expenditure, a direct specification might include income instead as a right hand side variable. An informal separability may still motivate the selection of the subgroup but demands are not the result of subutility function maximization with respect to the subgroup budget allocation. While this approach has the advantage of choice of structure and variables unrestricted by economic theory, the lack of explicit theoretical structure is also a serious disadvantage. A great deal of energy has been expended developing a way of grounding directly specified demand systems in utility theory. The integrability conditions are a set of theoretical restrictions that establish the connection back to a well-behaved expenditure function and, thus, theoretical consistency.

Duality theory provides the theoretical link between preference ordering and a system of demands. Without the ability to make this connection there is no guarantee that a demand system reflects any rational preference ordering. To maintain rational preference ordering, duality theory implies a number of restrictions on the expenditure function. It must be increasing, homogeneous of degree one in prices and concave in prices. A fourth requirement called the adding up condition requires that all expenditure must be accounted for in the demand system. If a directly specified demand system can be integrated to an expenditure function that fits these requirements this implies a reasonable underlying preference ordering. A directly specified subgroup demand system that uses income as a variable fails the adding up condition. The other three integrability conditions, however, remain important for reasonable preference ordering. In empirical demand analysis, these restrictions have proven elusive.

2.2 Incomplete Demand Systems

In recent years, work on a theory of incomplete demand systems has developed to better understand the theoretical implications of directly specified demand systems. It has grown out of the well-established tradition of theoretical work on complete demand system theory (Gorman 1961, 1981; Deaton and Muellbauer 1980a, 1980b; Lewbell 1987, 1990; Muellbauer 1975; Pollak and Wales 1992). The goal has been to make incomplete demand systems fully consistent with duality theory, thus allowing a full exploration of the relationship between incomplete demands and the underlying preference ordering.

2.2.1 Duality theory for Incomplete Demand Systems

Maximization of a utility function subject to a budget constraint results in a complete set of demand functions with certain properties. If a subset of this complete set of demand functions is considered separately from the whole, the properties only change slightly, and in fact become more general in nature. This insight, along with a relaxation of the assumption of uniform functional form, allows the development of an incomplete system duality theory that is solidly grounded in traditional duality theory but takes full advantage of the incomplete approach.

Let $\mathbf{x} = [x_1, \ldots, x_n]'$ be a vector of non-negative consumption levels for the commodities of interest and $\mathbf{p} = [p_1, \ldots, p_n]'$ be the corresponding price vector; let $\mathbf{x}^o = [x^o{}_1, \ldots, x^o{}_n]'$ be the vector of non-negative consumption levels of all other commodities, and $\mathbf{q} = [q_1, \ldots, q_n]'$ be the corresponding price vector; and let income be m.

For a complete demand system, where there are goods of interest, $\mathbf{x}$, and other goods, $\mathbf{x}^o$, maximizing an increasing, quasiconcave utility function, $u(\mathbf{x},\mathbf{x}^o)$, with respect to the budget constraint, $\mathbf{p}'\mathbf{x} + \mathbf{q}'\mathbf{x}^o \leq m$, results in demands for the goods of interest with four properties:

1) Demands are positively valued: $\mathbf{h}^x(\mathbf{p},\mathbf{q},m) \geq 0$.

2) Demands are homogeneous of degree zero in all prices and income, $\mathbf{h}^x(\mathbf{p},\mathbf{q},m) \equiv \mathbf{h}^x(t\mathbf{p},t\mathbf{q},tm)$ for all $t \geq 0$.

3) Subgroup expenditure is strictly less than income: $\mathbf{p}'\mathbf{h}^x(\mathbf{p},\mathbf{q},m) < m$.

4) The n x n Slutsky matrix, $\partial\mathbf{h}^x/\partial\mathbf{p}' + \partial\mathbf{h}^x/\partial m * \mathbf{h}^{x'}$, is symmetric, negative semidefinite.

Constrained maximization also implies the existence of an expenditure function, e(**p**,**q**,u) that is continuous and increasing in prices and utility, linearly homogeneous and concave in prices.

These properties of an incomplete demand system, concerned only with demand for **x**, are identical to the complete set of demands that would include $\mathbf{x}^{o}$, except in property 3. Property three is the adding up condition and in a complete system expenditure on goods is assumed to equal income. The important question about an incomplete system is how much information is lost by forgoing the adding-up condition. If the system is incomplete then there must be at least one other good at a positive consumption level with a positive price to make the complete system adding up condition hold as an equality. If there is only one other good, information about it can be recovered through the adding up condition which, with the addition of the single other good, is now defines a complete system. Clearly if there are two or more other goods, the incomplete system adding up condition remains strictly less than y but the unique demands for the other goods are no longer individually distinguishable through the complete system adding up equality condition. This information is lost in the incomplete system.

Viewing the system from the perspective of the demands, this framework clarifies what information is recoverable regarding the underlying utility function, and thus the preference ordering, when starting from an incomplete system of demand equations. Global integrability, which would imply the existence of an increasing, quasiconcave utility function across all goods, is difficult to establish because of the insufficient information to distinguish other goods. But global integrability is a stronger condition than is necessary to maintain a foundation in duality theory. LaFrance and Hanemann (1989) propose a weak integrability that takes full advantage of all the information present for included goods but remains flexible to the unknown information for other goods.

A theoretical link between complete and incomplete systems is achieved with a composite commodity encompassing all other goods. Expenditure on this composite good is defined as $s = \mathbf{q}'\mathbf{x}^o \equiv m - \mathbf{p}'(\mathbf{h}^x(\mathbf{p},\mathbf{q},m))$. With a properly defined utility function and the price of s normalized to one, duality applies to the incomplete system just as if it were a complete system. The four properties of incomplete demands and this new budget identity are equivalent to the existence of an expenditure function, $e(\mathbf{p},\mathbf{q},u)$, that is increasing and concave in $\mathbf{p}$, linearly homogenous in $\mathbf{p}$ and $\mathbf{q}$, and satisfies the adding up condition

$$\mathbf{p}'\mathbf{h}[\mathbf{p},\mathbf{q},\, e(\mathbf{p},\mathbf{q},u)] + \sigma[\mathbf{p},\mathbf{q},\, e(\mathbf{p},\mathbf{q},u)] \equiv e(\mathbf{p},\mathbf{q},u) \tag{2.2.1}$$

where $\sigma = s$. Implicit in this approach is a relaxation of an assumption that commonly holds in demand system theory. The functional form for the composite commodity of other goods is unknown, relaxing the assumption of uniformity of functional form. This further increases the generality of incomplete demand systems. Assuming a different form for the aggregate demand function for expenditure on other goods allows the conditions for integrability to be maintained where they would fail under uniformity. For example, a complete demand system cannot be linear in all prices and income and satisfy adding up conditions. An incomplete system, linear in income and prices for all goods of interest, must only be strictly less than income. Non-uniformity of demand for other goods increases the flexibility of functional form choice for the desired subset of goods.

2.2.2 Weak Integrability and Restrictions on Preferences

This theoretical approach is applicable in practice provided the four properties of incomplete demands are present. Positive demands and an expenditure that sums to strictly less than income are assumed in an incomplete demand system. Symmetry and concavity can be imposed and are testable hypotheses. Homogeneity, however, must be imposed on the system from the outset.

Prices and income are deflated to achieve the required zero degree homogeneity of the demands. Let the deflator, $\pi(Q)$, be a known, twice continuously differentiable, positive valued, nondecreasing (strictly increasing in some Q_i), linearly homogeneous, concave function of other prices. In the context of food demands, for example, the non-food consumer price index or the price of gold could be used. As long as it fits the above description the deflator does not affect the qualitative results. Demands must be zero degree homogeneous in all prices so an expenditure function linearly homogeneous in all prices can be recovered.

One further property proves useful in the practical application of the dual structure of incomplete demand systems. Symmetry restrictions are derived from the Slutsky matrix

$$\mathbf{S} = \frac{\partial \mathbf{h}^x}{\partial \mathbf{p}'} + \frac{\partial \mathbf{h}^x}{\partial m}\mathbf{h}^{x\prime} \qquad (2.2.2)$$

As the only equality constraint on the demands, much of the information regarding the underlying preference structure is derived from symmetry. Incomplete system demand functions are usually assumed twice differentiable. This allows for the further differentiation of the Slutsky matrix and thus another set of identities. These identities provide additional information regarding potential restrictions on the preference ordering (i.e. the parameters of the demand model).

The demands, $h^x(\mathbf{p},\mathbf{q},m)$, are integrated with respect to $\mathbf{p}$ to recover the quasi-expenditure function $\varepsilon[\mathbf{p},\mathbf{q}, \theta(\mathbf{q},u)]$. The quasi-expenditure function is related to the expenditure function by the identity

$$e(\mathbf{p},\mathbf{q},u) \equiv \varepsilon[\mathbf{p},\mathbf{q}, \theta(\mathbf{q},u)] \qquad (2.2.3)$$

A quasi-expenditure function that is increasing and concave in $\mathbf{p}$ is a necessary, but not sufficient, condition for the existence of the true expenditure function. This quasi-expenditure function exhibits the qualities of weak integrability in that $\theta(\mathbf{q},u)$ is an unknown constant of integration which limits the knowledge of preferences to those of goods included in the incomplete demand system. The global integrability mentioned earlier would require a single expenditure function, $e(\mathbf{p},\mathbf{q},u)$, that held information on all preferences. The quasi-expenditure function recovered by weak integrability applied to deflated demands , $\varepsilon[\mathbf{p},\mathbf{q}, \theta(\mathbf{q},u)]$, defines a class of expenditure functions that are all related to the same set of incomplete demands. As long as $\theta(\mathbf{q},u)$ is zero degree homogeneous in other prices, demands are invariant across this set of expenditure functions. The quasi-expenditure function has insufficient information to recover preferences for other goods but exhausts the potential information implied by duality theory regarding preferences of included goods.

A quasi-utility function and the effective restrictions on preference structure are recovered using the precepts of duality theory applied to these quasi-functions. Setting $\varepsilon[\mathbf{p},\mathbf{q},\theta(\mathbf{q},u)] = m$, and inverting with respect to $\theta(\mathbf{q},u)$ recovers the quasi-indirect utility function, $\varphi(\mathbf{p},\mathbf{q},m)$. This is related to the indirect utility function, $\nu(\mathbf{p},\mathbf{q},m)$, by the identity $\nu(\mathbf{p},\mathbf{q},)\equiv\psi[\mathbf{q}, \varphi(\mathbf{p},\mathbf{q},m)]$ where ψ once again encapsulates all information in $\mathbf{q}$ and u. The quasi-utility function is expressed as $\omega(\mathbf{x},s,\mathbf{q})$ and is recovered by recognizing the utility function as the solution to the minimization of the indirect utility function with respect to prices and income subject to the budget constraint. The result, $\omega(\mathbf{x},s,\mathbf{q})$, is increasing and quasi-concave in $(\mathbf{x},s)$. Thus, the properties of incomplete system demands are equivalent to weak integrability and allow the recovery of the quasi-utility function. With this duality theory of incomplete demand systems made explicit, the effect of symmetry restrictions on conditional preferences can be established.

LaFrance (1985,1986,1990) has applied this framework to the most common functional forms used in directly specified demand systems. His results expose the weaknesses of most of the forms that are used. When integration is carried out on a system of the simplest of the integrable demand functions, those linear in prices and income, some surprising results are found. Despite being integrable, very restrictive conditions are necessary to recover the underlying preference maps. Income effects must either be all zero, all nonzero with the same sign, or some combination of these two cases. The limitation of zero income effects is apparent. Non-zero income effects of the same sign reveal conditional preferences with a fixed coefficients, Leontief structure from a translated origin. A mix of these two restrictions adds the further limitation that demands with zero income effects are also completely independent of all prices. Constant elasticity and Semilog demand models are found to have similarly unrealistic restrictions on the preference structure.

CHAPTER 3

ANALYTICAL FRAMEWORK

3.1 Derivation of LinQuad

3.1.1 The Basic Model

While applying weak integrability to commonly used functional forms exposed overly restrictive constraints on preferences, the application of duality theory to incomplete demand systems also opened new territory. Integration of demand functions that are linear in prices and income revealed the structure of the class of deflated expenditure functions that produce all demands linear in prices and income. All such demands are generated from the quasi-expenditure function

$$\varepsilon(\mathbf{p},\mathbf{q},\mathbf{z},\theta) = \mathbf{p}'\alpha + \mathbf{p}'\mathbf{A}_z\mathbf{z} + \delta(\mathbf{z}) + \theta(\mathbf{q},u,\mathbf{z})e^{\gamma' p} \qquad (3.1.1)$$

where $\mathbf{p}$ is now the vector of deflated prices, $[p_1/\pi(Q), \ldots, p_i/\pi(Q)]$, $\mathbf{z}$ is a set of demographic shifters, relevant other prices or lagged demand, $\delta(\mathbf{z})$ is an arbitrary real valued function of all variables in $\mathbf{z}$, $\theta(\mathbf{q},u,\mathbf{z})$ is the constant of integration and α, $\mathbf{A}_z$ and $\mathbf{B}$ are the parameters to be estimated. This quasi-expenditure could easily be made more flexible with the addition of a quadratic term in prices (LaFrance 1990),

$$\varepsilon(\mathbf{p},\mathbf{q},\mathbf{z},\theta) = \mathbf{p}'\alpha + \mathbf{p}'\mathbf{A}_z\mathbf{z} + .5\mathbf{p}'\mathbf{B}\mathbf{p} + \delta(\mathbf{z}) + \theta(\mathbf{q},u,\mathbf{z})e^{\gamma' p}. \qquad (3.1.2)$$

The above quasi-expenditure function creates a new class of quasi-expenditure functions which produce demands with more desirable qualities. Applying Shepherd's lemma generates demands of the form,

$$\mathbf{x} = \alpha + \mathbf{A}_z\mathbf{z} + B\mathbf{p} + \gamma[\theta(\mathbf{q},u,\mathbf{z})e^{\gamma'p}]. \quad (3.1.3)$$

Solving the original LinQuad expenditure function (3.1.2) for $\theta(\mathbf{q},u,\mathbf{z})e^{\gamma'p}$, and replacing expenditure with m for income, gives a final demand specification of

$$\mathbf{x} = \alpha + \mathbf{A}_z\mathbf{z} + B\mathbf{p} + \gamma[m - \alpha'\mathbf{p} - \mathbf{p}'A_z\mathbf{z} - .5\mathbf{p}'\mathbf{B}\mathbf{p} - \delta(\mathbf{z})]. \quad (3.1.4)$$

This is the original LinQuad model (LaFrance 1990,1998a). The quadratic term in prices increases the flexibility in Slutsky symmetry removing the restrictions that constrain the preference ordering of the linear system. The LinQuad expenditure function in (3.1.2) is now a second order Taylor series approximation to any arbitrary expenditure function. The Slutsky substitution matrix is

$$\mathbf{S} = B + [m - \alpha'\mathbf{p} - \mathbf{p}'A_z\mathbf{z} - .5\mathbf{p}'\mathbf{B}\mathbf{p} - \delta(\mathbf{z})]\gamma\gamma'. \quad (3.1.5)$$

Symmetry of the Slutsky matrix is determined by **B**. **B** is not necessarily symmetric so symmetry is a testable hypothesis. The matrix of price effects,

$$\frac{\partial \mathbf{x}}{\partial \mathbf{p}'} = B + \gamma(\alpha - A_z\mathbf{z} - \mathbf{p}'\mathbf{B}) \quad (3.1.6)$$

is similarly not necessarily symmetric. Thus, another problem identified in some functional forms, homothetic conditional preferences, is avoided. Finally, there are no restrictions on individual income coefficients.

This incremental derivation of LinQuad proves to be a fundamentally strong result. Shephard's lemma establishes that a quasi-expenditure function of the original LinQuad form (3.1.2) is sufficient for demands linear in income and linear and quadratic in prices. It can also be shown that demands of this form are a necessary outcome of the

LinQuad expenditure function. If demands linear in income and linear and quadratic in prices are written in a general form they appear as

$$x_i = \alpha_i(q) + \sum_{j=1}^{n} \beta_{ij} p_j + \tfrac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \Omega_{ijk} p_j p_k + \gamma_i m, \tag{3.1.7}$$

where, without loss in generality, $\Omega_{ijk} = \Omega_{ikj}$ for all $i, j, k = 1, K, n$, and we let $\gamma_1 \neq 0$. Generating the Slutsky substitution matrix and imposing symmetry onto this generalized form of the demands produces demands identical to LinQuad. Using the LinQuad quasi-expenditure function is the only way to derive demands linear in deflated income and linear and quadratic in deflated prices and consistent with weak integrability (LaFrance (1998a).

3.1.2 The PIGLOG Model

The derivation of demands from the LinQuad quasi-expenditure function does not rely on linearity in prices and income. A logarithmic version of the LinQuad expenditure function has the same structure but is a function of the natural logs of prices and income. If this logarithmic expenditure function is differentiated with respect to logged prices, demands of the form

$$\mathbf{w} = \alpha + \mathbf{A}_z \mathbf{z} + B \ln(\mathbf{p}) + \gamma[\ln(m) - \alpha' \ln(\mathbf{p}) - \ln(\mathbf{p})' A_z \mathbf{z} - .5 \ln(\mathbf{p})' \mathbf{B} \ln(\mathbf{p}) - \delta(\mathbf{z})] \tag{3.1.8}$$

result, where **w** is a vector of budget shares, which falls out of the necessary chain differentiation. With the exception of the extra terms included in the vector **z**, these demands are identical to those of the AIDS model reported earlier. The important innovation is the inclusion of the natural log of income as a right-hand side variable in the place of subgroup expenditure. Following Deaton and Muellbauer we call this the Price Independent Generalized Logarithmic (PIGLOG) specification of the LinQuad quasi-expenditure function. AIDS is a member of the PIGLOG class of preferences derived from a logged expenditure function.

The similarity of the PIGLOG specification to AIDS is a striking result. The PIGLOG specification maintains the basic form of the AIDS model and includes logged prices and quadratic logged prices. Having income as a right hand side variable in place of subgroup expenditure avoids one of the problematic aspects of the AIDS model. The result is an AIDS-like model that avoids the simultaneity issues that accompany the use of subgroup expenditure. Furthermore, the inclusion of logged income comes at no theoretical cost. In fact, the problems associated with welfare measures in separable demand systems might make this PIGLOG version of the LinQuad quasi-expenditure model preferable. For the purpose of this research we view the PIGLOG model as a reasonable, if not improved, stand-in for the AIDS model to use as an alternative specification to LinQuad.

3.1.3 The Generalized PIGL Model

The comparison of LinQuad to the PIGLOG model is facilitated by a transformed version of the LinQuad quasi-expenditure function that nests both models. Another commonly used transformation of an expenditure function, which results in demands with increased flexibility in expenditure, is called the Price-Independent Generalized Linear (PIGL) form of the expenditure function. The expenditure function is raised to the power of some constant, κ. The PIGL expenditure function is usually specified as $\varepsilon^{\kappa}/\kappa$. The resulting demands allow income to vary with respect to the parameter κ. This idea of transforming variables is extended here to include prices.

The Box-Cox transformation is a commonly used tool in economic analysis. It allows flexibility between linear and logarithmic forms of a variable. Here, the Box-Cox transformation is applied to both deflated income (expenditure) and deflated prices to create a transformed and normalized form of the deflated LinQuad expenditure function. Because it is a natural extension of the PIGL form we call it the generalized PIGL form. The deflated expenditure function becomes

$$m(\kappa) = \frac{(\varepsilon(\mathbf{p},\mathbf{q},\mathbf{z},\theta)^{\kappa} - 1)}{\kappa}. \tag{3.1.9}$$

As κ varies between 0 and 1 the expenditure term varies between ln(ε) and ε-1. Each deflated price is similarly transformed using the parameter λ. Let $p_i(\lambda) = (p_i^{\lambda}-1)/\lambda$, i= 1, . . ,n; and $\mathbf{p}(\lambda) = [p_1(\lambda) \ldots p_n(\lambda)]'$. In the following derivation, where variables are not specifically indicated as transformed, they are the original untransformed variable. **e** and **w** are still vectors of expenditure on, and budget share of, goods, respectively. The **p** and **w** vectors are also reworked as diagonal matrices, **P** and **W**, respectively.

The Box-Cox transformed, normalized expenditure function becomes

$$m(\kappa) = \mathbf{p}(\lambda)'\alpha + \mathbf{p}(\lambda)'\mathbf{A}_z\mathbf{z} + .5\mathbf{p}(\lambda)'\mathbf{B}\mathbf{p}(\lambda) + \delta(\mathbf{z}) + \theta(\mathbf{q},u,\mathbf{z})e^{\gamma' p(\lambda)}. \tag{3.1.10}$$

Differentiating with the chain rule reveals $\mathbf{h}^x(\mathbf{p},\mathbf{q},y)=\partial m/\partial \mathbf{p}=\mathbf{x}$ combined with the extra terms from the chain-rule differentiation of the Box-Cox transformation.

$$\frac{\partial m(\kappa)}{\partial m}\frac{\partial \mathbf{p}}{\partial \mathbf{p}(\lambda)}\frac{\partial m}{\partial \mathbf{p}} = \mathbf{P}^{1-\lambda}\frac{\partial m}{\partial \mathbf{p}}m^{k-1} = \alpha + \mathbf{A}_q\mathbf{q} + \mathbf{B}\mathbf{p}(\lambda) + \gamma'\theta\exp\{\gamma'\mathbf{p}(\lambda)\} \tag{3.1.11}$$

With the vector of deflated expenditures on included goods as the left-hand side variable, the transformed PIGL demands are

$$\mathbf{e} = \mathbf{P}\frac{\partial m}{\partial \mathbf{p}} = m^{1-\kappa}\mathbf{P}^{\lambda}\{\alpha + \mathbf{A}_z\mathbf{z} + \mathbf{B}\mathbf{p}(\lambda) + \gamma[m(\kappa) - \alpha'\mathbf{p}(\lambda) - (\mathbf{A}_z\mathbf{z})'\mathbf{p}(\lambda) - .5\mathbf{p}(\lambda)'\mathbf{B}\mathbf{p}(\lambda) - \delta(\mathbf{z})]\} \tag{3.1.12}$$

Note that when $\kappa = \lambda = 0$, we have the PIGLOG form of the incomplete demand system,

$$\mathbf{e} = m\{\alpha + \mathbf{A}_z\mathbf{z} + \mathbf{B}\ln(\mathbf{p}) + \gamma[\ln(m) - \alpha'\ln(\mathbf{p}) - (\mathbf{A}_z\mathbf{z})'\ln(\mathbf{p}) - .5\ln(\mathbf{p})'\mathbf{B}\ln(\mathbf{p}) - \delta(\mathbf{z})]\} \tag{3.1.13}$$

When $\kappa = \lambda = 1$, the Box-Cox transformations for income and prices are equal to m-1 and p_i-1, respectively. Carrying this extra negative one through and grouping appropriately, the demands take the form,

$$\mathbf{e} = \mathbf{P}\left\{\begin{matrix}(\alpha - \mathbf{B}\tau) + \mathbf{A}_z\mathbf{z} + \mathbf{B}\mathbf{p} + \\ \gamma[m - (\alpha - .5\mathbf{B}'\tau - .5\mathbf{B}\tau)'\mathbf{p} - (\mathbf{A}_z\mathbf{z})'\mathbf{p} - .5\mathbf{p}'\mathbf{B}\mathbf{p} - 1 - .5\tau'\mathbf{B}\tau + \alpha'\tau + (\mathbf{A}_z\mathbf{z})'\tau)]\end{matrix}\right\} \tag{3.1.14}$$

with τ=[1 . . .1]', a column vector of ones. This can be rewritten as

$$\mathbf{e} = \mathbf{P}\{\tilde{\mathrm{a}} + \mathbf{A}_z\mathbf{z} + \mathbf{B}\mathbf{p} + \gamma[m - \tilde{\mathrm{a}}'\mathbf{p} - (\mathbf{A}_z\mathbf{z})'\mathbf{p} - .5\mathbf{p}'\mathbf{B}\mathbf{p} - \delta(\mathbf{z})] \tag{3.1.15}$$

where $\tilde{\alpha} = \alpha - \mathbf{B}\tau$ under symmetry and $\tilde{\alpha} = \alpha - .5\mathbf{B}'\tau - .5\mathbf{B}\tau$ in the unrestricted model and $\delta(\mathbf{z}) = 1 + .5\tau'\mathbf{B}\tau - \alpha'\tau - (\mathbf{A}_z\mathbf{z})'\tau$ is a specific function of the other parameters and the demographic variables. This reparameterization of the model shows that the generalized PIGL with κ = λ = 1 is the same model as the original LinQuad demand model. Slight adjustments take place at all levels of the theoretical restrictions as a result of the added demographic terms in δ(**z**). In the unrestricted model, where **B** is not necessarily symmetric, the reparameterization of $\tilde{\alpha}$ implies an averaging of the off-diagonal terms, $.5(B_{ij} + B_{ji})$. The overall effect of this will be small as long as the γ_i terms are small. The generalized PIGL form remains a consistent specification of the normalized and transformed expenditure function across the Box-Cox transformation of the price and income variables.

The term, δ(**z**), which was left unidentified in earlier specifications like the PIGLOG model but is explicitly defined as a function of parameters and variables in the LinQuad version of the generalized PIGL model is an interesting parameter even outside this transformed specification. This parameter is an arbitrary real-valued function of **z** held in common by demand systems derived from the expenditure function. It is constant with respect to **p** so enters into the demands only when the expenditure function is solved for $\theta(\mathbf{q}, u, \mathbf{z})e^{\gamma' p}$ and substituted into the demands. For the generalized PIGL form, it would amount to a summation of δ(**z**) parameters on the demographic variables included in **z**. (including a constant) that enters the parenthetical income component of the model.

Since they enter the model only through the income component held in common by all of the equations these parameters could have the intuitive interpretation of system-wide demographic and income (from the constant) effects. Empirical difficulty estimating these parameters in the AIDS model has led researchers to set them equal to zero, a priori. Estimating the original LinQuad demand system, a similar approach had to be taken with all $\delta(\mathbf{z})$ parameters.

In LaFrance (1998a), where the LinQuad expenditure function is shown to be both necessary and sufficient for demands linear in income and linear and quadratic in prices, there is an important corollary finding regarding $\delta(\mathbf{z})$. A critical reparameterization in that work indicates that for the PIGLOG form of the LinQuad expenditure function, δ_0, the intercept component of $\delta(\mathbf{z})$, is empirically unidentified. This justifies the common practice of setting the parameter to zero. The parameters associated with the demographic variables also appear to be empirically unidentified. Use of the generalized PIGL form of LinQuad forces a consistent treatment of $\delta(\mathbf{z})$ throughout the Box-Cox transformation of variables from logarithmic to linear. For the PIGLOG form of the generalized PIGL, all $\delta(\mathbf{z})$ parameters are set to zero for the purpose of estimation. As κ and λ increase from zero to one, the elements of $\delta(\mathbf{z})$ takes on specific values as a result of the reparamaterization that uniquely identify this term as a function of all of the other parameters.

3.2 Welfare Measures for LinQuad

One of the most useful properties of the LinQuad quasi-expenditure function is its complete characterization of the included goods with regards to prices and income. This result from the duality theory of incomplete demand systems allows exact welfare measures to be obtained from the quasi-indirect utility function (LaFrance 1991b). This is easily done and superior to depending on consumer surplus as an estimate of welfare measures. Consumer surplus is not an exact measure of welfare. Provided it has a unique solution, it is only an approximation bounded by equivalent variation (*ev*) and compensating variation (*cv*). It is only equal to *ev* and *cv* when income effects on demands are zero. With the ease of using direct welfare under the circumstances, there is no reason to work with consumer surplus.

For the generalized PIGL form of LinQuad, *ev* and *cv* are derived as follows. Remember the quasi indirect utility function $\varphi(\mathbf{p},\mathbf{q},\mathbf{z},m)$ is derived by setting the quasi-expenditure function, $\varepsilon(\mathbf{p},\mathbf{q},\mathbf{z},\theta)$, equal to m and inverting with respect to θ. $\theta(\mathbf{q},\mathbf{z},u)$ is the monotonic transformation of u containing the information regarding other goods that is not available in the incomplete demand system. $\theta(\mathbf{q},\mathbf{z},u)$ allows the definition of a set of expenditure functions that are sufficient to define the preference map of the subset of goods in which we are interested. Inverting produces the function $\theta = \varphi(\mathbf{p},\mathbf{q},\mathbf{z},m)$. This is related to the indirect utility function, $\nu(\mathbf{p},\mathbf{q},\mathbf{z},m)$ by

$$\nu(\mathbf{p},\mathbf{q},\mathbf{z},m)\equiv\psi[\mathbf{q},\mathbf{z},\varphi(\mathbf{p},\mathbf{q},\mathbf{z},m)] \qquad (3.2.1)$$

where $\psi[\mathbf{q},\theta] = u$, is the inverse of $\theta(\mathbf{q},\mathbf{z},u)$ with respect to u. (3.2.1) clearly shows that the quasi-indirect utility function contains all the information available regarding prices, **p**, of goods included in the demand system and income, *m*. Thus, all compensated changes in prices are fully contained in $\varphi(\mathbf{p},\mathbf{q},\mathbf{z},m)$. This is a particularly important result in light of the biased welfare measures produced by separable demand specifications. Because all information relating to income is found in the quasi-indirect utility function, the proper welfare measures as they relates to income, as opposed to subgroup expenditure, can be derived.

Equivalent variation, is the change in income that would produce the equivalent change in utility as the price change,

$$\varphi(\mathbf{p}^0,\mathbf{q},\mathbf{z},m+ev) = \varphi(\mathbf{p}^1,\mathbf{q},\mathbf{z},m). \qquad (3.2.2)$$

Compensating variation, the compensating change in income needed to reestablish original utility levels after a price change has taken place, can be defined by

$$\varphi(\mathbf{p}^0,\mathbf{q},\mathbf{z},m) \equiv \varphi(\mathbf{p}^1,\mathbf{q},\mathbf{z},m-cv). \qquad (3.2.3)$$

The generalized PIGL transforms income through the Box-Cox transformation. Welfare measures have the same units as income so must be inside the Box-Cox transformation. It is argued that *ev* is a better measure for policy analysis because it allows comparison starting from the same baseline of $\mathbf{p}^0$. The derivation of the two measures is almost identical so only *ev* will be derived.

To derive *ev* for the generalized PIGL form of LinQuad functional form, the normalized and transformed expenditure function must be inverted with respect to θ after being set equal to income, *m*.

$$\varphi(\mathbf{p},\mathbf{q},\mathbf{z},m) = [m(\kappa) - \mathbf{p}(\lambda)'\alpha - \mathbf{p}(\lambda)'\mathbf{A}_z\mathbf{z} - .5\mathbf{p}(\lambda)'\mathbf{B}\mathbf{p}(\lambda) - \delta(\mathbf{z})]e^{\gamma' p(\lambda)} \tag{3.2.4}$$

The *ev* identity becomes

$$\left\{\left[\frac{(m+ev)^{\kappa}-1}{\kappa}\right] - \alpha'\mathbf{p}(\lambda)_0 - \mathbf{p}(\lambda)_0'\mathbf{A}_z\mathbf{z} - .5\mathbf{p}(\lambda)_0'\mathbf{B}\mathbf{p}(\lambda)_0 - \delta(\mathbf{z})\right\}e^{-\gamma' p_0} =$$
$$\left\{\left[\frac{(m)^{\kappa}-1}{\kappa}\right] - \alpha'\mathbf{p}(\lambda)_1 - \mathbf{p}(\lambda)_1'\mathbf{A}_z\mathbf{z} - .5\mathbf{p}(\lambda)_1'\mathbf{B}\mathbf{p}(\lambda)_1 - \delta(\mathbf{z})\right\}e^{-\gamma' p_1} \tag{3.2.5}$$

where $\mathbf{p}(\lambda)$ is still a vector of Box-Cox transformed prices whether before or after the price change. After isolating *ev*, the welfare measure is expressed

$$ev = \left[\begin{array}{l}\kappa\left[m(\kappa) - \alpha'\mathbf{p}(\lambda)_1 - \mathbf{p}(\lambda)_1'\mathbf{A}_q q - .5\mathbf{p}(\lambda)_1'\mathbf{B}\mathbf{p}(\lambda)_1 - \delta(\mathbf{z})\right]e^{-\gamma'(p_0-p_1)} \\ \qquad - \kappa\left[-\alpha'\mathbf{p}(\lambda)_0 - \mathbf{p}(\lambda)_0'\mathbf{A}_q q - .5\mathbf{p}(\lambda)_0'\mathbf{B}\mathbf{p}(\lambda)_0 - \delta(\mathbf{z})\right] - 1\end{array}\right]^{\frac{1}{\kappa}} - m \tag{3.2.6}$$

By deriving exact welfare measures with transformed income and prices, comparisons can be made between LinQuad, PIGLOG and the optimal generalized PIGL model, which should fall between the others with respect to (κ,λ). The most important aspect of *ev* to compare across the functional forms is deadweight loss. Deadweight loss equals the remainder of *ev* after net transfers are removed. This is the measurement of greatest importance to policy analysis as it indicates the specific monetary losses of policy induced inefficiencies.

3.3 The Econometric Model

3.3.1 Choice of Left-Hand Side Variable

For the sake of econometric estimation, an error term is appended to each demand equation. A system of five equations of the form

$$e_i = m^{1-\kappa} p_i^{\lambda} \left\{ \alpha_i + \sum_j \beta_{ij} p(\lambda)_j + \gamma_i \left[m(\kappa) - \sum_i \alpha_i p(\lambda)_i - .5 \sum_i \sum_j \beta_{ij} p(\lambda)_i p(\lambda)_j \right] \right\} + u_i \qquad (3.3.1)$$

is estimated, where $u_{it} = [u_{1t}, \ldots, u_{5t}]$ is assumed to be distributed $N(0, \Sigma)$. The Box-Cox transformed PIGL specification as well as the PIGL and Linear versions of LinQuad can all be estimated with quantity, expenditure or budget share as the left-hand side variable. In the incomplete specification, budget shares have the disadvantage of being extremely small. Estimating with quantity as a left-hand side variable, on the other hand, implies a heteroscedastic Σ-matrix.

The heteroscedasticity is evident if we consider the adding-up condition for the incomplete specification,

$$m = s + \sum_i p_i x_i, \qquad (3.3.2)$$

where m is income and s is expenditure on all other goods. For an estimated system of demands, error terms should be included,

$$m = f(s) + u_s + \sum_i p_i x_i + \sum_i p_i u_i \,, \tag{3.3.3}$$

where f(s) is estimated expenditure on other goods and u_s is the error term for expenditure on other goods. Adding up implies that the price-weighted, mean levels of the estimated demands along with expenditure on other goods add up to income. This leaves the price-weighted errors and expenditure residual from f(s) adding to zero,

$$0 = u_s + \sum_i p_i u_i \tag{3.3.4}$$

Clearly, the adding up condition makes this complete system singular necessitating the dropping of an equation for the purposes of estimation. More importantly, the adding up condition implies

$$u_s = -\sum_i p_i u_i = -\mathbf{u}'\mathbf{p} \tag{3.3.5}$$

Taking the variance of both sides,

$$\mathrm{Var}(u_s) = \mathbf{p}'\Sigma\mathbf{p}, \tag{3.3.6}$$

where Σ is the variance-covariance matrix for the demands. Suppose that the variance of u_s is assumed constant. With **p** varying through the sample, the assumption of a constant Σ is impossible unless it is zero. This implies heteroscedasticity in Σ.

This result is an argument for the use of deflated expenditure as the left-hand side variable. This is achieved by simply multiplying both sides of each equation by its corresponding price. Returning to the budget identity, it is apparent how this approach avoids this source of heteroscedasticity. Now the adding up condition appears

$$m \equiv \sum_i e_i + s \tag{3.3.7}$$

where e_i is deflated expenditure. If error terms were included they would no longer be price-weighted, thus,

$$\text{Var}[u_s] = \tau'\Sigma\tau, \tag{3.3.8}$$

where τ is again a vector of ones. This solution provides a logical rationale for choice of left-hand side variable.

3.3.2 Demographic Variables and Other Prices

The generalized PIGL form in all of its specifications allows the inclusion of demographic variables and specific other prices. These variables serve two purposes. First, in time series estimation, trends in population composition, both with regards to age and ethnicity, have an effect on the observed demand behavior. Inclusion of variables relating to age and ethnicity allow for the explicit accounting of variation due to these factors. The vector **z** is a vector of these variables. The matrix $\mathbf{A_z}$ is the matrix of coefficients that captures the effects. In this estimation, population mean, variance and skew have been included to capture the effects of such demographic phenomena as the post WW-II baby boom, increased longevity and decreasing birthrates. Ethnicity variables that indicate percentage of population in various ethnic groups track changes in the ethnic composition of the country. Explicitly accounting for these variables improves the fit of the model and lessens potential bias resulting from omitted variables.

Second, other prices can also be included in the vector **z** and estimated as shift parameters. This is where complements and substitutes not included in the subgroup being estimated can be included. This allows other goods to be included in the estimation without making the overall size of the system untenable. In this research, for the sake of simplicity, no other prices were included.

3.3.3 The Problem of Serial Correlation

In all time series econometrics, the potential problem of serial correlation must be addressed. Because the purpose of this thesis is comparing functional forms, serial correlation is addressed in two different ways. First, a reasonable effort is made to account for serial correlation through model specification. Second, remaining serial correlation is measured to compare across the different forms of the generalized PIGL model.

To account for some of the serial correlation, lagged quantity demanded is included on the right-hand side of the demand equations in vector $\mathbf{q}$ along with the demographic variables. This approach allows the model to explicitly account for variation due to lagged demand. The coefficients on these lagged variables are invariably highly significant indicating the necessity of their inclusion.

Beyond this basic incorporation of lagged demand, it is more important to determine how the different models under consideration perform with respect to serial correlation. A convenient way to do this in a system of demands is with a Prais-Winston auto-correlation coefficient. This is a single coefficient for all of the equations estimated in a system of equations, based on the demand equation residuals of the form:

$$u_i^P = \delta(\sqrt{1-r^2})u_i + (1-\delta)\{u_i - r[u_i(-1)]\} \tag{3.3.9}$$

where δ =1 in the first year of the sample and zero elsewhere, r is the coefficient being estimated, u_i is the residual from equation i of the demand system, $u_i(-1)$ is that same residual lagged one period and u_i^P is the resulting Prais-Winston-corrected residual. The Prais-Winston procedure is similar to the Cochrane-Orcutt procedure in that it measures first-degree autocorrelation, or the degree to which unexplained right-hand side variation is determined by the left-hand side variable lagged one period. The advantage of the Prais-Winston procedure is the explicit inclusion of the first sample year in the estimation. The coefficient varies between zero and one, with numbers closer to zero indicating less serial correlation.

These coefficients are not tests of serial correlation but give an indication of the magnitude of the auto-correlation coefficient that would arise if the whole demand system were estimated under the Prais-Winston procedure. A major concern in demand estimation of this kind is what happens as theoretical restrictions are imposed. Theoretical restrictions generally increase autocorrelation problems. The Prais-Winston autocorrelation coefficient allows us to compare the effect of imposed theoretical restrictions on the extent of the presence of autocorrelation within each functional form at the same time as we compare across the forms.

3.3.4 Choice of Estimation Technique

Systems of equations like AIDS and LinQuad are usually estimated using either maximum-likelihood (ML) or iterative seemingly unrelated regressions techniques (ITSUR). With the AIDS model the motivation is clear. Dropping one of the subgroup demand equations is necessary to avoid a singular variance-covariance matrix. These two estimators provide results invariant to which equation is dropped. This motivation is not relevant for incomplete demand systems. An incomplete demand system encompasses all goods but the demand equation for all non-subgroup goods is always disregarded. Thus, in the system to be estimated, there is not a singular variance-covariance matrix.

Seemingly unrelated regressions (SUR) technique, with only one iteration on the variance-covariance matrix, produces consistent, efficient and asymptotically normal parameter estimates while avoiding potential difficulties inherent in ITSUR and ML methods. Both Deaton and Muellbauer (1980a) and LaFrance (1997) point out that ITSUR and ML can overfit an equation in systems of equations like LinQuad with a large number of parameters shared across equations.

For this thesis, parameters will be estimated with SUR estimation techniques in the statistical package TSP (Hall 1996). One graph will be created using ITSUR because it is convenient to compare likelihood functions across specifications and the visual representation is extremely helpful in organizing the results. The difference in point estimates and standard errors between SUR and iterative-SUR are quite small. Other than the graph, all results will be derived using SUR techniques.

3.3.5 Lagrange Multiplier Based F-Test

There are three commonly used tests for joint restrictions on an econometric model: the Wald, likelihood ratio (LR) and Lagrange multiplier (LM). In large demand models such as this, all three are known to over-reject a true null (Laitinen 1978; Meisner 1970; and Bera, Byron, and Jarque 1981). Of these asymptotic χ^2 tests, the Wald test is the most likely to reject a true null while LM is least likely to reject. LaFrance (1997) has developed an approximate F-test based on the LM test that partially corrects this tendency to over-reject.

In SUR estimation, first round residuals are used to estimate a variance–covariance matrix for the system. If the estimate of Σ is ζ, then the least squares criterion for the second round of estimation is to minimize:

$$s(\zeta) = \sum_{t=1}^{T} u'_{it}\, \zeta^{-1} u_{it} \tag{3.3.10}$$

where u_{it} is the equation residual, i indicates the equation and t the sample year. The F-test uses the least square criterions from both restricted and unrestricted models. Using a "^" to indicate the unrestricted least square criterion and Σ matrix, gives us $\hat{s}(\hat{\Sigma})$. Using "~" to indicate the restricted least square criterion and Σ matrix gives us is $\tilde{s}(\tilde{\Sigma})$. Where the F-test is unusual is in its use of a mixed sum of squares, using residuals from the unrestricted model but the Σ matrix from the restricted model. This mixed sum of squares is represented: $\hat{s}(\tilde{\Sigma})$. The F-statistic is

$$F(G, NT-K) = \frac{\left(\tilde{s}(\tilde{\Sigma}) - \hat{s}(\tilde{\Sigma})\right)/G}{\hat{s}(\hat{\Sigma})/(NT-K)} \tag{3.3.11}$$

where G is the number of restrictions, N the number of equations and K the number of parameters. Use of the restricted Σ matrix in the numerator guarantees a positive test statistic. The numerator converges in distribution to a $\chi^2(G)/G$ random variable. Under normality of the residuals, the denominator converges in distribution to a $\chi^2(NT-K)/(NT-K)$. This F-statistic provides a test of restrictions that is better suited to the kind of model being estimated with the number of restrictions being imposed.

3.3.6 Imposition of Symmetry Restrictions

One set of theoretical restrictions commonly tested in applied demand analysis is the symmetry restrictions. Most functional forms have some way of restricting the estimated model such that symmetry of the off-diagonal elements of the Slutsky matrix will be maintained. Hypothesis testing establishes whether the restricted model is statistically similar to the original, unrestricted model. Since symmetry is one of the basic theoretical requirements of demand systems, not rejecting symmetry restrictions strengthens the model's explanatory validity. Many models do not make it over even this hurdle.

Imposition of symmetry on the generalized PIGL form of LinQuad is not difficult. Further differentiation of the Box-Cox transformed demands (3.1.12) reveals the Slutsky matrix for the generalized PIGL model, in a convenient form,

$$m^{\kappa-1}\mathbf{P}^{1-\lambda}\frac{\partial^2 m}{\partial\mathbf{p}\partial\mathbf{p}'}\mathbf{P}^{1-\lambda} = \mathbf{B} + \left[m(\kappa) - \alpha'\mathbf{p}(\lambda) - \mathbf{A}_z{}'\mathbf{p}(\lambda) - .5\mathbf{p}(\lambda)'\mathbf{B}\mathbf{p}(\lambda) - \delta_q\right]\gamma\gamma'$$
$$+ m^{\kappa}\mathbf{P}^{-\lambda}\left[(1-\kappa)\mathbf{w}\mathbf{w}' - (1-\lambda)\mathbf{W}\right]\mathbf{P}^{-\lambda}$$

(3.3.12)

where $\frac{\partial^2 m}{\partial\mathbf{p}\partial\mathbf{p}'} = \mathbf{S}$ is the Slutsky matrix. It is unnecessary to isolate **S** because, for all κ and λ, $m^{\kappa-1}$ and $\mathbf{P}^{1-\lambda}$ are positive and only scale the Slutsky matrix while having no effect on either symmetry or concavity considerations.

Close consideration of the right-hand side of (3.3.12) reveals that symmetry of the Slutsky matrix is completely determined by the symmetry of **B**. Symmetry restrictions are simply $B_{ij} = B_{ji}$. In this five equation system this amounts to ten linear restrictions on the parameters. Estimating the restricted system is straightforward.

3.3.7 Concavity Restrictions

The last restriction to impose on the demand system is quasi-concavity. This is done by imposing negative semidefiniteness on the Slutsky matrix. Quasi-concavity is the final property of integrable demand equations and allows the theoretically consistent return to the normalized and transformed expenditure function using the estimated parameters of the demand functions. The exact welfare measures that make the generalized PIGL model so attractive are derived from this expenditure function. This is the practical crux of integrability.

Negative semidefiniteness of the Slutsky matrix can be imposed on the generalized PIGL form of LinQuad for all sample points. Further consideration of the right-hand side of the Slutsky matrix equation (3.3.12) reveals that the two components other than **B** are quite small. The income parameters (γ) are small, as are the budget shares (**w**). In fact, for $\kappa = \lambda = 1$, the last component drops out completely. If **B** is restricted to being sufficiently negative definite for all sample points, then negative semidefiniteness of the Slutsky matrix is guaranteed. The positive semidefinite influence of the second component, with m(κ) in it, is largely determined by the size of this income variable. The last year in the sample, 1995 has the largest income and therefore produces the largest influence. Imposing negative semidefiniteness using 1995 levels assures quasi-concavity of the demand system for all sample points.

The imposition of quasi-concavity is achieved with a Choleski factorization of the scaled Slutsky matrix,

$$m^{\kappa-1}\mathbf{P}^{1-\lambda}\frac{\partial^2 m}{\partial \mathbf{p}\partial \mathbf{p}'}\mathbf{P}^{1-\lambda} = -\mathbf{LL}' \qquad (3.3.13)$$

where **L** is a lower triangular 5x5 matrix, **-LL'** is symmetric and negative semidefinite. It has 15 unknown parameters, the same number of unknown parameters as a symmetric **B** matrix. Substituting this negative semidefinite representation of the scaled **S** back into the Slutsky equation we get,

$$-\mathbf{LL}' = \mathbf{B} + \left[m(\kappa) - \alpha'\mathbf{p}(\lambda) - \mathbf{A}_z'\mathbf{p}(\lambda) - .5\mathbf{p}(\lambda)'\mathbf{B}\mathbf{p}(\lambda) - \delta_q\right]\gamma\gamma' \\ + m^{\kappa}\mathbf{P}^{-\lambda}\left[(1-\kappa)\mathbf{ww}' - (1-\lambda)\mathbf{W}\right]\mathbf{P}^{-\lambda} \quad (3.3.14)$$

Evaluated at any particular sample point, the last component of (3.3.13) is a constant and independent of **B**. The income index term,

$$\left[m(\kappa) - \mathrm{a}'\mathbf{p}(\lambda) - \mathbf{A}_z'\mathbf{p}(\lambda) - .5\mathbf{p}(\lambda)'\mathbf{B}\mathbf{p}(\lambda) - \delta_q\right] = \tilde{m}(\kappa), \quad (3.3.15)$$

however, varies with **B.** Evaluating $\tilde{m}(\kappa)$ at 1995 levels of prices and income along with all the parameter estimates from the symmetry restricted model we get,

$$\mathbf{B} = -\mathbf{LL}' - \tilde{m}(\kappa)\gamma\gamma' - m^{\kappa}\mathbf{P}^{-\lambda}\left[(1-\kappa)\mathbf{ww}' - (1-\lambda)\mathbf{W}\right]\mathbf{P}^{-\lambda}. \quad (3.3.16)$$

B is thereby reparameterized in terms that force quasi-concavity onto the model as a whole. $\tilde{m}(\kappa)$ depends on **B** so it is necessary to iterate on $\tilde{m}(\kappa)$, updating the parameter values that define it and then re-estimating the parameter values, until it converges. This iterative process generally took fewer than five iterations in this study. Parameters estimated in this fashion are known to be consistent but not efficient because the interaction between $\tilde{m}(\kappa)$ and the other parameters is ignored at each iteration of the estimation procedure. Consistent estimates of the covariance matrix for the parameters are obtained by evaluating the empirical information matrix at the final consistent, symmetry and concavity restricted point estimates of the structural parameters.

The Slutsky matrix for the LinQuad has three negative eigen values. The PIGLOG version only has one negative value. The 3 negative eigen values indicate negative semidefiniteness in only three dimensions for LinQuad, one dimension for the

PIGLOG form. This determines the number of parameter restrictions on **L** necessary to estimate the concave version of the model. It also determines the rank of the Slutsky matrix. Restricting l_{44}, l_{45}, and l_{55} to zero limits LL' to a factorization in three dimensions. Where there are positive eigen values there will be no information that can enrich a negative semidefinite representation of the matrix. With out these restrictions TSP will try to identify these last two dimensions indefinitely. An LM based F-test of the further restrictions of l_{33}, l_{34} and l_{35} has a p-value of .9845 indicating that while these parameters can be identified there is nothing to be gained statistically by their inclusion. This makes intuitive sense. The third negative eigen value for the LinQuad model is close to zero and of a magnitude much smaller than the other two negative eigen values. Thus, the LinQuad model is estimated with 6 additional restrictions, l_{33}, l_{34}, l_{35}, l_{44}, l_{45}, and l_{55}, over and above the ten symmetry restrictions already imposed.

The PIGLOG model must be estimated as a rank one Slutsky matrix. This means only l_{11}, l_{12}, l_{13}, l_{14} and l_{15} are identified. The10 additional zero restrictions on l_{22} and the rest, below and to the right, are necessary for the concave version. That the Slutsky matrix for the PIGLOG model is rank one in its negative semidefinite representation is an early sign of the limited explanatory power of this model.

3.3.8 Data

The data set for this research consists of annual time series observations from 1919 to 1995. Per capita consumption of the five dairy categories (milk, butter, cheese, frozen and other), and their average retail prices were constructed from several USDA and Bureau of Labor Statistics (BLS) sources. To get a consistent series of disaggregate retail prices per pound a baseline was established with detailed price estimates from 1967 combined with their corresponding quantity observations. Then, using either disaggregate consumer price indices or average retail food prices, the full series of retail prices was constructed for each category. The nonfood consumer price index is used as a general deflator for all prices and income.

Demographic variables included in this research come from the Bureau of Census. They include the first three moments (mean, variance and skewness) of the empirical age distribution for the U.S. population. Ethnicity variables track the proportion of the population that is black or neither white nor black. The variable for white population is dropped to avoid a singularity. Finally, income is per capita disposable income, and also comes from the Bureau of Census. All data used and data sources are included in appendix A.

CHAPTER FOUR

ESTIMATION RESULTS

4.1 Demand System Results

There are a variety of results of interest in this research. For the estimation of the demand system, there are two different kinds of restrictions being imposed on the model. The generalized PIGL model varies from LinQuad to the PIGLOG specification as (κ,λ) vary from (1,1) to (0,0). Restrictions on κ and λ allow nested tests of these two specifications. Meanwhile, the imposition of symmetry, concavity and joint symmetry and concavity[1] are testable hypotheses for the LinQuad, PIGLOG and optimal generalized PIGL model. Finally, there is added information on the serial correlation characteristics of each specification.

There is a convenient way of organizing the results with this functional form. In the Box-Cox transformation, κ and λ determine the degree of curvature in the variable. In the generalized PIGL form, the amount of curvature preferred by the prices is independent of the curvature preferred by the income variable. A single restriction, that $\kappa = \lambda$, forces the model to determine the optimal degree of curvature for both prices and income, jointly. This is a testable hypothesis for all three levels of the theoretical restrictions. Results are in Table 4.1. $\kappa = \lambda$ is clearly not rejected by the data. A p-value

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[1] Symmetry is a necessary condition for the imposition of concavity. When concavity restrictions are discussed, symmetry is thus assumed. An important distinction exists, however, when restrictions are tested. Joint imposition of symmetry and concavity is a different test than testing the imposition of concavity conditional on symmetry.

Table 4.1 Generalized PIGL Form: Test for $\kappa = \lambda$

		F-Statistic	P-Value
Unrestricted	F(1,318)	0.003363	0.953789
Symmetric	F(1,328)	0.003579	0.952331
Concave	F(1,334)	0.917864	0.338729

as low as .05 would be acceptable.

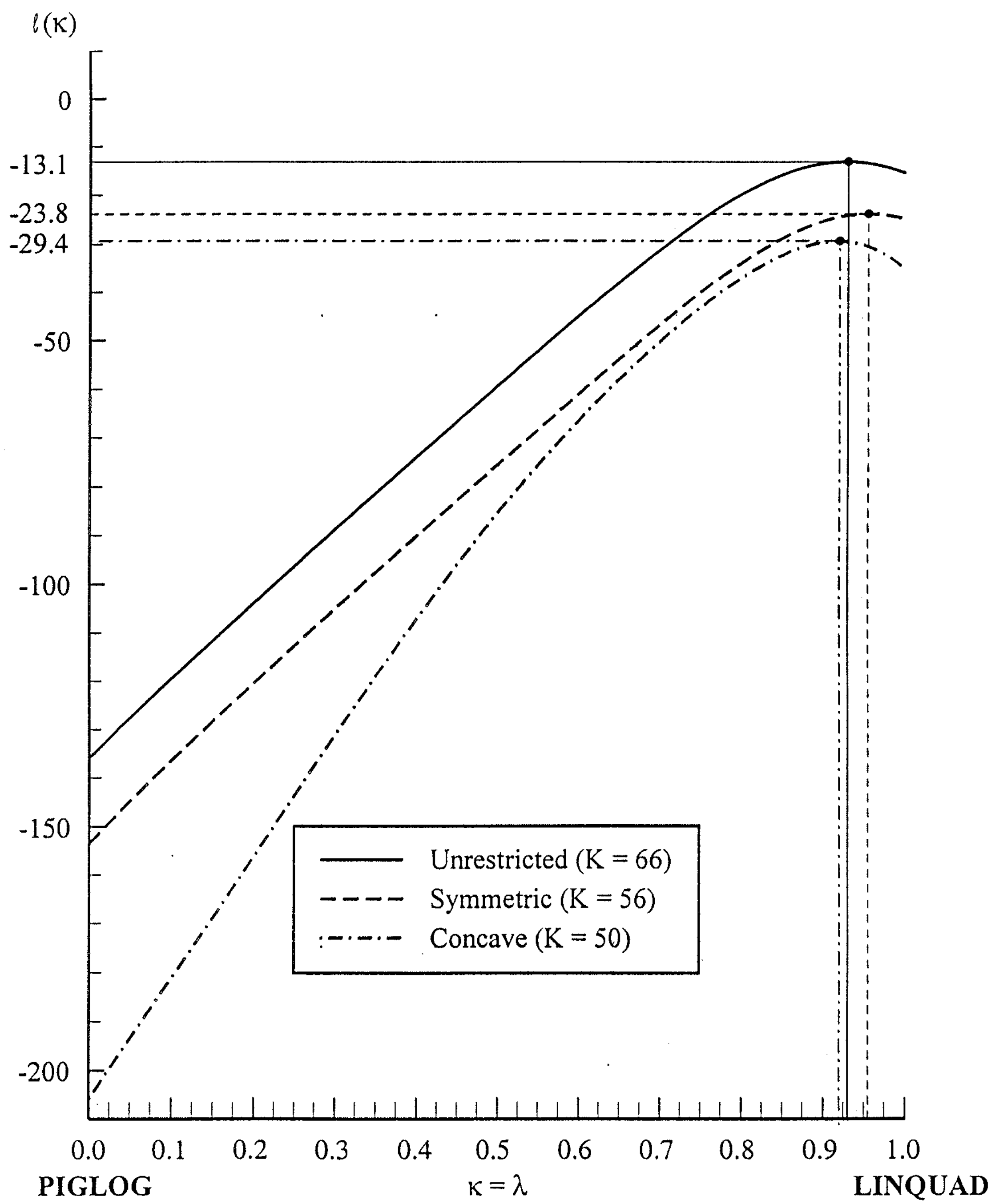
The simplification allowed by setting $\kappa = \lambda$ facilitates a general introduction to the results of the generalized PIGL functional form. Specifically, setting $\kappa = \lambda$ allows a two-dimensional graph of the log-likelihoods of all three levels of theoretical restrictions as $\kappa = \lambda$ varies from 0 to 1. This visual representation, Figure 4.1, helps to organize the remaining statistical results.

Figure 4.1 is based on iterative SUR results because of the ease of comparing the log-likelihood function values. The chart is bounded by $\kappa = \lambda = 0$ on the left, the AIDS-like PIGLOG form, and $\kappa = \lambda = 1$ on the right, the LinQuad form. The optimal values for $\kappa = \lambda$ are indicated by the vertical lines corresponding with the apexes of the parabolas.

There are two different sets of test results to report. First we will look at each of the three forms carefully so as to fully understand the characteristics of each one. This will involve reviewing the Prais-Winston autocorrelation coefficients for all of the theoretical restrictions within each specification and then testing those restrictions. With regards to the theoretical restrictions, the null hypothesis is that the restricted model is still the same model as the unrestricted model. The visual representation of these tests is

Figure 4.1, The Generalized PIGL Demand Model,

Concentrated Log-Likelihood Function



a comparison of the vertical distance between the lines representing different levels of restriction at $\kappa = \lambda = 0$, $\kappa = \lambda = 1$ and optimal $\kappa = \lambda$. As is indicated by the graph, the different model specifications have varying degrees of success with the theoretical restrictions.

The second set of tests compares across the different specifications. It is possible to use the nested generalized PIGL form to directly compare all three versions of the LinQuad and PIGLOG specifications to the results with optimal κ and λ and optimal $\kappa = \lambda$. If a LR test was used this would involve comparing the optimal log-likelihood at the apex of each parabola to the log-likelihoods at $\kappa = \lambda = 0$ and $\kappa = \lambda = 1$. There is the question of which form has the highest log likelihood, and whether either the PIGLOG specification or LinQuad is statistically indistinguishable from the optimal model.

4.1.1 Generalized PIGL Form

The optimal generalized PIGL form with $\kappa = \lambda$ is represented by the apex of each of the three parabolas in Figure 4.1. A three dimensional map would allow us to view three versions of the generalized PIGL where κ and λ are unrestricted.

Prais-Winston coefficients for the generalized PIGL model unrestricted with respect to κ and λ are in Table 4.2, while coefficients for the model with $\kappa = \lambda$ restrictions are in Table 4.3. The Prais-Winston coefficients are small and stay small

Table 4.2 Prais-Winston Coefficients, Optimal PIGL Model, κ and λ Unrestricted

	Coefficient	Standard Error	T-Statistic
Unrestricted	0.102737	0.051523	1.99399
Symmetry-Restricted	0.111731	0.052331	2.13507
Concavity-Restricted	0.096388	0.052165	1.84777

Table 4.3 Prais-Winston Coefficients, Optimal PIGL Model, $\kappa = \lambda$

	Coefficient	Standard error	T-Statistic
Unrestricted	0.058707	0.053861	1.08997
Symmetry-Restricted	0.094596	0.052311	1.80833
Concavity-Restricted	0.100112	0.052105	1.92133

with the imposition of theoretical restrictions. With κ and λ free to estimate separately, the coefficient actually decreases with the imposition of concavity restrictions. What constitutes a small and reasonable coefficient is not the focus of this research. T-statistics indicate that all three coefficients in the $\kappa = \lambda$ model are not statistically different from zero. This gives some indication that in this generalized PIGL model that autocorrelation is largely accounted for by the inclusion of lagged demand as a right-hand side variable. Most importantly, these autocorrelation coefficients provide a benchmark with which to compare the PIGLOG and LinQuad forms of the generalized PIGL model.

4.1.1.1 Tests on Theoretical Restrictions

The generalized PIGL form also provides a benchmark for comparing the effect of theoretical restrictions on the performance of the models in a more general sense. With these models, theoretical restrictions are testable hypotheses. The null hypothesis is that the restricted model is the same model as the unrestricted model. The restrictions lessen the flexibility of the model to fit the data by requiring the model to perform in a manner that is theoretically consistent. What the unrestricted model offers in flexibility to fit the data the restricted models counter with theoretical consistency. If the models are found to be statistically indistinguishable the implication is that theoretical assumptions are consistent with the data as expressed through the functional form.

Imposition of symmetry is a frequently tested restriction and under many circumstances is not rejected. If a restriction like symmetry is rejected it is common to lay the blame on the functional form rather than question basic economic theory. This is why the effect of theoretical restrictions on a functional form becomes an important point of comparison between functional forms.

Table 4.4 Tests on Theoretical Restrictions, Optimal PIGL Form, κ and λ Unrestricted

		F-Statistic	P-Value
Unrestricted v. Symmetry	F(10,318)	1.714394	0.076465
Unrestricted v. Concave	F(16,318)	1.526865	0.088431
Symmetry v. Concave	F(6,328)	1.397798	0.214799

Table 4.5 Tests on Theoretical Restrictions, Optimal PIGL Form, κ = λ

		F-Statistic	P-Value
Unrestricted v. Symmetry	F(10,319)	1.76224	0.066641
Unrestricted v. Concave	F(16,319)	1.60206	0.066542
Symmetry v. Concave	F(6,329)	1.54477	0.162862

The results from the LM-based F tests derived from non-iterated SUR results for both forms of the generalized PIGL form are in Table 4.4 and Table 4.5. As indicated, the tests reflect the imposition of symmetry on the unrestricted model, the joint imposition of symmetry and concavity on the unrestricted model and the imposition of concavity on the symmetry restricted model. P-values indicate that not a single test can be rejected at the 5% level of significance. Interestingly, the additional imposition of concavity on the unrestricted model lessens the likelihood of rejection in the first model where κ and λ are unrestricted and has almost no effect in the second model. Imposition of concavity on the symmetry-restricted model is not rejected at the 21% and 16% level of significance, respectively.

These are extremely strong results. Non-rejection of concavity restrictions imposed over all sample points is almost unheard of in the economics literature. This functional form allows for the estimation of demand parameters under strict theoretically consistent constraints. This is both an exciting result with regards to demand estimation and also a strong argument for the use of exact welfare measures that require this theoretical consistency.

4.1.2 PIGLOG Form Results

The primary usefulness of the generalized PIGL functional form is that it nests both the PIGLOG and LinQuad specifications. This allows for direct comparison of these two specifications to the optimal generalized PIGL model results just reported. At this point the differences are not being tested statistically but statistical results within each specification can be compared to get an indication of relative performance. Returning to Figure 4.1, these tests are vertically oriented within each specification, at zero, one and the vertical lines corresponding with the optimal generalized PIGL model results. The relationships within each model can be tested statistically and the results compared across the three different specifications. Statistical tests of relationships across the three specifications will be discussed in a later section.

The Prais-Winston autocorrelation coefficient for the PIGLOG specification indicates a dramatic increase in the presence of autocorrelation. At all three levels of restrictions, the coefficient is large and highly significant. Though the magnitude of the

Table 4.6 Prais-Winston Coefficients, PIGLOG Form ($\kappa = \lambda = 0$)

	Coefficient	Standard Error	T-Statistic
Unrestricted	0.620098	0.053228	11.6498
Symmetry-Restricted	0.555373	0.054244	10.2385
Concavity-Restricted	0.382269	0.032167	11.884

coefficient falls with the imposition of restrictions it still indicates a serious problem with autocorrelation. The smallest coefficient of the PIGLOG specification, the concavity restricted version, has an autocorrelation coefficient more than 3 times larger than the largest coefficient of both generalized PIGL models. Coefficients of this size could not be ignored in a demand system estimation process.

4.1.2.1 Tests of Theoretical Restrictions

Tests on the imposition of theoretical restrictions on the PIGLOG specification do not exhibit the positive results found in the optimal generalized PIGL model. In the

Table 4.7 Tests of Theoretical Restrictions, PIGLOG Form ($\kappa = \lambda = 0$)

		F-Statistic	P-Value
Unrestricted v. Symmetry	F(10,320)	2.593881	0.004891
Unrestricted v. Concave	F(20,320)	4.138038	2.14E-08
Symmetry v. Concave	F(10,330)	7.423184	1.2E-10

results in Table 4.7, symmetry restrictions are narrowly rejected at the 1% significance level. The addition of concavity restrictions, either jointly with symmetry restrictions or conditional on symmetry restrictions, drops P-values effectively to zero. Concavity can be imposed on the PIGLOG specification but it is no longer the same model. This indicates a weakness in this functional specification.

4.1.3 LinQuad Results

As the visual map (Figure 4.1) indicates, the results for LinQuad should be closer to the results generated for the optimal PIGL specification. In fact, LinQuad results do have most of the desirable characteristics of the optimal generalized PIGL results.

Table 4.8 Prais-Winston Coefficients, LinQuad From ($\kappa = \lambda = 1$)

	Coefficient	Standard Error	T-Statistic
Unrestricted	0.106286	0.052414	2.02782
Symmetry-Restricted	0.108786	0.0524	2.07609
Concavity-Restricted	0.12509	0.051537	2.42718

The Prais-Winston coefficents for the three levels of restrictions in LinQuad are reported in Table 4.8. The magnitude of these coefficients is quite small. While larger than the Prais-Winston coefficients from both optimal generalized PIGL models in all but one case, and similarly more significant, the differences are modest. The differences are particularly small when compared to the Prais-Winston coefficients for the PIGLOG specification. The best autocorrelation coefficient from the PIGLOG specification, from the concavity restricted version, is still more than three times the size of the largest Prais-Winston coefficient from Linquad. With regards to autocorrelation, LinQuad clearly out performs the PIGLOG specification.

4.1.3.1 Tests of Theoretical Restrictions

The results from tests of the imposition of theoretical restrictions on LinQuad fall between the optimal generalized PIGL models and the PIGLOG specification. As the visual map indicates, the results are much closer to those of the optimal generalized PIGL specification. The results are in Table 4.9.

Table 4.9 Tests of Theoretical Restrictions, LinQuad Form ($\kappa = \lambda = 1$)

		F-Statistic	P-Value
Unrestricted v. Symmetry	F(10,320)	1.547145	0.121616
Unrestricted v. Concave	F(16,320)	1.899994	0.019797
Symmetry v. Concave	F(6,330)	2.755366	0.012578

Imposition of symmetry restrictions is not rejected at the 12% significance level. Joint concavity and symmetry restrictions and concavity restrictions conditional on symmetry are not rejected at the 1% significance level. While these results are not as strong as for the generalized PIGL model, they are far superior to the PIGLOG specification, and represent a substantial degree of success with regards to the estimation of a theoretically consistent model.

4.1.4 Testing the Optimal PIGL Versus the PIGLOG Form

The statistical tests that take full advantage of the nested relationship of the three functional forms focus on the parameters, κ and λ. The PIGLOG and LinQuad forms result from testable restrictions on κ and λ. Starting from the fully generalized PIGL form, where κ and λ are both free to estimate, both PIGLOG and LinQuad forms result from two parameter restrictions limiting $\kappa = \lambda = 0$ and $\kappa = \lambda = 1$, respectively. If already restricted by $\kappa = \lambda$, as in the visual representation above, then both PIGLOG and LinQuad forms result from a single further restriction. Tests of these restrictions are designed to determine whether either the PIGLOG or LinQuad forms can be considered statistically indistinguishable from either of the generalized PIGL forms.

Using the LM-based F-test, the PIGLOG form, when $\kappa = \lambda = 0$, is soundly rejected at all three levels of theoretical restrictions. Results for the test are in Table 4.10. The P-values for every test are effectively zero. There are a variety of single parameter Z-test that can be performed on these restrictions. All Z-tests on restrictions at all three levels of theoretical restrictions return P-values effectively equal to zero. Which Z-tests are performed can be seen in the LinQuad section where results are more interesting.

Table 4.10 Optimal PIGL v. PIGLOG(AIDS)

	κ and λ Unrestricted v. κ = λ =0			κ = λ v. κ = λ =0		
		F-Statistic	P-Value		F-Statistic	P-Value
Unrestricted	F(2,318)	52.20877	2.47E-20	F(1,319)	103.7337	2.8E-21
Symmetric	F(2,328)	18.81856	1.83E-08	F(1,329)	22.46509	3.19E-06
Concave	F(6,334)	23.99873	1.33E-23	F(5,335)	28.82241	2.6E-24

4.1.5 Testing Optimal PIGL Versus the LinQuad Form

The same statistical tests, used above with the PIGLOG form, bear out the proximity of Linquad to the generalized PIGL. Results are in the Table 4.11. As the visual representation indicates, the optimal generalized PIGL ($\kappa = \lambda$) with symmetry restrictions imposed, is the closest to the LinQuad form. A P-value of .4864 shows a substantial failure to reject the null hypothesis that LinQuad with symmetry restrictions is the same model as the relevant PIGL model. However, with both concavity and symmetry imposed, the null hypothesis that LinQuad is the same as the optimal generalized PIGL form is soundly rejected.

Table 4.11 Optimal PIGL v. LinQuad, Test of κ = λ =1 Restriction Using LM F-Test

	κ and λ Unrestricted v. κ = λ =1			κ = λ v. κ = λ =1		
		F-Statistic	P-Value		F-Statistic	P-Value
Unrestricted	f(2,318)	1.971776	0.140908	F(1,319)	3.951766	0.047677
Symmetric	F(2,328)	0.442137	0.643044	F(1,329)	0.485563	0.486405
Concave	F(2,334)	5.698428	0.003685	F(1,335)	10.28397	0.001471

The results above are reiterated by the variety of single parameter z-tests that can be done testing whether κ, λ or κ = λ=κλ equals one. The tests are essentially T-tests but since the model is non-linear, not to mention the large number of degrees of freedom, we must appeal to asymptotic theory and use the standard normal distribution.

Table 4.12 LinQuad v. Optimal PIGL, Tests of κ = 1 and λ =1

		Z-Statistic	P-Value
Unrestricted	κ=1	-2.89882	0.003746
	λ=1	-0.68083	0.495982
Symmetry	κ=1	-1.97389	0.048394
	λ=1	0.231633	0.816823
Concavity	κ=1	-2.74643	0.006025
	λ=1	-2.38025	0.017301

The results in Table 4.12 show the results of tests when κ and λ are estimated separately. P-values reflect results from the earlier F-tests but decompose the effects into the two variables. λ, the variable that transforms the prices, has a lower probability of rejection than κ in every case. For the Concavity restricted model, λ=1 no longer has the high P-value found at the other levels of restriction. λ=1 is not rejected at 1% significance, but the change explains why when considered jointly with κ, LinQuad is soundly rejected in the concavity restricted model.

When the model is estimated with κ = λ=κλ, similar tests can be performed on the single parameter (κλ) that transforms both prices and income. These results mirror the results of the F-tests in Table 4.11 except that there is a much higher tendency to reject the null. A Z-test of this sort is the square-root of a Wald test of a single restriction. The Wald test's tendency to over-reject the null was the motivation behind the LM-based F-test discussed above and therefore we should interpret these results rather cautiously.

Table 4.13 LinQuad v. Optimal PIGL Tests of $\kappa = \lambda = \kappa\lambda = 1$

		Z-Statistic	P-Value
Unrestricted	$\kappa\lambda=1$	-2.88304	0.003939
Symmetry	$\kappa\lambda=1$	-2.01354	0.044058
Concavity	$\kappa\lambda=1$	-3.05356	0.002262

One final set of results puts both the PIGLOG and LinQuad results in perspective. Using the above single-parameter Z-statistics, 95% confidence intervals for κ, λ or $\kappa\lambda$ can be constructed. The results, once again, leave little doubt as to the superiority of LinQuad over the PIGLOG specification. Another interesting result that is evident throughout all of the tests but is particularly clear in this set of confidence intervals is the relationship between the joint $\kappa\lambda$ parameter and the separate κ and λ parameters. κ clearly dominates in the joint $\kappa\lambda$ parameter.

Table 4.14 95% Confidence Intervals in Optimal Generalized PIGL Model for κ, λ, and $\kappa\lambda$

Unrestricted	κ	0.858541	to	0.985659
	λ	0.808450	to	1.087224
	$\kappa\lambda$	0.858603	to	0.983883
Symmetry	κ	0.884394	to	1.004002
	λ	0.879854	to	1.149206
	$\kappa\lambda$	0.885214	to	0.998454
Concavity	κ	0.860702	to	0.976518
	λ	0.775747	to	0.964675
	$\kappa\lambda$	0.853468	to	0.962996

4.1.6 Price Elasticities for All Three Models

Income compensated price elasticities are easily calculated from the Slutsky matrix. The Slutsky matrix combines the price effects with the income effects in one measure. They can be made into elasticites by transforming each element by its corresponding ratio of price and demand. The elasticites in Tables 4.14, 4.15 and 4.16 are calculated from the Slutsky matrix of the concavity restricted model. These examples give another indication of how different from the optimal PIGL and LinQuad forms are the results generated by the PIGLOG form.

Table 4.15 Optimal PIGL Income-Compensated Price Elasticities

	Milk	Butter	Cheese	Frozen	Other
Milk	-0.27031	-0.05032	-0.0649	0.031623	0.045341
Butter	-0.61597	-0.2067	0.02556	0.22919	0.28365
Cheese	-0.05782	0.00175	-0.03851	-0.0147	-0.01489
Frozen	0.14913	0.088704	-0.07959	-0.12156	-0.14446
Other	0.43288	0.22181	-0.16442	-0.29195	-0.34948

Table 4.16 LinQuad Form Income-Compensated Price Elasticities

	Milk	Butter	Cheese	Frozen	Other
Milk	-0.25813	-0.04096	-0.0298	0.024379	0.035813
Butter	-0.50055	-0.14475	0.021841	0.15198	0.2274
Cheese	-0.02551	0.00153	-0.00975	-0.00653	-0.00995
Frozen	0.11523	0.058789	-0.03608	-0.07582	-0.11394
Other	0.34212	0.17778	-0.11107	-0.23029	-0.34612

Table 4.17 PIGLOG (AIDS) Form Income-Compensated Price Elasticities

	Milk	Butter	Cheese	Frozen	Other
Milk	-0.00126	-0.00134	-0.00908	0.000818	0.003792
Butter	-0.01643	-0.01747	-0.11799	0.010623	0.049262
Cheese	-0.00777	-0.00826	-0.05581	0.005025	0.023301
Frozen	0.003864	0.004109	0.027751	-0.0025	-0.01159
Other	0.03622	0.038512	0.2601	-0.02342	-0.1086

4.2 Exact Welfare Measures

The focus of this study has been functional form and the estimation of demand. The demand equations estimated are not embedded in a larger system of equations designed to model the industry and reflect long term economic effects of a price change. Furthermore, the dairy data being utilized here is annual data. As a result, the welfare measures generated will reflect a one-time mean annual price change. Two theoretical price changes are proposed that provide reasonable scenarios for changes that could result from policy changes.

The Northeast Interstate Dairy Compact, now in place in the Northeastern U.S., provides an example of a simple policy that has a clear effect on retail dairy prices. The NIDC increased the base rate paid to Northeastern U.S. producers for fluid milk. The immediate effect of this increase was a reported \$.20 to \$.25 per gallon rise in the price of milk at the retail level (IDFA 1998). In terms of 1995 per pound dairy prices this is an increase of between 7 and 8%. This price increase appeared to persist at approximately the same level for many months.

Another measure that can be calculated in rough terms is the effect of completely removing price discrimination from the dairy industry. As a result of federal milk marketing orders, a premium is paid for fluid milk relative to milk for manufactured products. This is another distortion of the dairy industry that has occupied economists over the years. Present policy calls for the simplification of the system but not its removal. LaFrance (1992,1993) estimated the change in prices that would take place if federal support prices for fluid and manufacturing milk were equalized, removing price discrimination in the dairy industry. Applying these changes to 1995 prices gives another theoretical price regime grounded in the reality of the dairy industry.

The *ev* measures are money metrics of the per capita loss or gain as a result of the policy decision in pre-policy dollars. The population of New England is roughly 13 million. New York, which has been considering joining the compact, would add 18 million. The total cost of the NIDC-related price increases to consumers is not a trivial amount even for the low range estimates.

Table 4.18 Per Capita *ev* for Fluid Milk Price Increase, Per Capita $1995

Increase	Optimal PIGL	LinQuad	PIGLOG (AIDS)
5%	-4.44813 (0.029532)	-4.43687 (0.02823)	-4.40956 (0.088171)
10%	-8.83685 (0.064185)	-8.81481 (0.060785)	-8.81531 (0.236143)
15%	-13.1681 (0.105702)	-13.1338 (0.099223)	-13.209 (0.450488)

Standard Errors in Parentheses

The *ev* results for the NIDC scenario appear quite similar across the range of increase in the price of fluid milk. These numbers multiplied by whatever population measure is used, represent the total cost to consumers of the corresponding increase. It can be argued that the transfers inherent in price changes are not necessarily problematic. The purpose of NIDC is to support dairy producers, after all, and it came about through the political process. However, *ev* is a combination of net transfers and deadweight loss. When net transfers are removed from the *ev* measure to get the more important measure of deadweight loss, the differences between functional forms become less subtle.

Table 4.19 Per Capita Deadweight Loss for Fluid Milk Price Increase, Per Capita $1995

Increase	Optimal PIGL	LinQuad	PIGLOG (AIDS)
5%	-1.40619 (0.205869)	-1.23967 (0.187532)	-0.0435 (1.836718)
10%	-2.82616 (0.410575)	-2.53628 (0.381516)	-0.28392 (3.511324)
15%	-4.25996 (0.616155)	-3.88983 (0.582125)	-0.69534 (5.04185)

Standard Errors in Parentheses

Deadweight loss estimates for the PIGLOG version are more than 97% smaller than the generalized PIGL model measure for a 5% increase in fluid milk price. For the same price increase LinQuad underestimates deadweight loss by less than 12%. Furthermore, the standard error for the PIGLOG form is large compared to the *ev* point estimate. The policy ramifications are non-trivial. For total deadweight loss in New England, the optimal PIGL model and LinQuad indicate $23.91 and 21.07 million lost respectively. The PIGLOG form indicates a loss of .74 million.

The scenario of removal of price discrimination provides an even more dramatic example of the potential difference in welfare measures across these three models. In this scenario, fluid milk price drops by 14%, while butter, cheese and other dairy products increase by 19%, 4% and 5.5% respectively. The price of frozen dairy remains unchanged. The removal of price discrimination should have positive implications for both transfers and deadweight loss. Consumers will benefit more from the decrease of fluid milk prices than they will suffer from the increases in most other dairy products

Table 4.20 Effect of Removal of Price Discrimination, Per Capita $1995

	Equivalent Variation		Deadweight Loss	
PIGL	6.596522	(-0.096593)	2.961649	(0.646167)
LinQuad	6.595972	(0.092208)	2.587817	(0.571495)
PIGLOG	5.418765	(0.466145)	-2.272864	(4.337326)

Standard Errors in Parentheses

The overall *ev* estimate for the PIGLOG model is no longer so close to the generalized PIGL estimate. It underestimates *ev* by over 15%. While this is problematic, the deadweight loss estimates are stunning. While LinQuad underestimates deadweight loss by 13%, the PIGLOG version has the wrong sign. Contrary to economic theory, the PIGLOG model indicates that removing price controls would have a substantial, negative effect on market efficiency. The difference between the different models is also substantial. If price discrimination affects 200 million Americans (not all states participate in milk marketing orders), the difference between the *ev* measures for the optimal PIGL and PIGLOG models top one billion dollars. These results indicate that the PIGLOG model may have serious drawbacks for use in estimating welfare measures if it is seriously misspecified.

CHAPTER FIVE

SUMMARY AND CONCLUSIONS

Applied demand estimation is a many-facetted problem. When choosing a functional form, or developing a new functional form, a number of different issues must be taken into consideration. The functional forms proposed in this thesis provide reasonable solutions to some of the problems that trouble other extant functional forms.

The incomplete demand system approach provides a theoretically sound way of limiting demand estimation to small groups of goods while avoiding the shortfalls of separable specifications. Within this theoretical structure it is possible to find functional forms that are practical models to estimate that reflect sound preference ordering. LinQuad is such a functional form. LinQuad is linear in income and linear and quadratic in prices. It avoids the problem of endogeneity of subgroup expenditure while providing flexibility with regards to price and income effects. The LinQuad expenditure function is the only quasi-expenditure that can produce demands with these characteristic and it provides a theoretically sound way of recovering exact welfare measures.

An interest in testing the different specifications possible from the LinQuad expenditure function led to a normalized and transformed expenditure function that produces what we called the generalized PIGL form of LinQuad. This model nested both the linear form, LinQuad and the AIDS-like PIGLOG form in the overarching form. The PIGLOG form is identical to the AIDS model except in its use of income rather than subgroup expenditure as a right-hand side variable. While maintaining the exact structure of the AIDS model the PIGLOG form avoids the simultaneity issues. An argument can be made that this is a partial solution to one of the weaknesses of AIDS.

As an AIDS stand-in, the PIGLOG model did not fair well in tests comparing it with LinQuad and the optimal generalized PIGL forms. The imposition of symmetry and concavity restrictions were soundly rejected for the PIGLOG model. Autocorrelation appeared to be a substantial problem. Similarity with the optimal generalized PIGL was rejected for all three levels of theoretical restrictions. Finally, the *ev* measures for the PIGLOG model bore no relation to the estimates from the optimal generalized PIGL model, even producing a deadweight loss measure completely inconsistent with accepted economic theory.

LinQuad, on the other hand, faired quite well relative to the optimal generalized PIGL model. All restrictions were not rejected at at least the 1% level of significance. Autocorrelation appeared to be a minor problem. Only the concavity-restricted model was rejected in tests directly comparing LinQuad to the generalized PIGL model. Finally, the *ev* measures for LinQuad were consistently close to values from the generalized PIGL model and in both examples, slightly underestimated the "true" measures.

These results clearly establish the superiority of LinQuad over the PIGLOG model. They also raise questions as to the effectiveness of the AIDS model for modeling demands and estimating welfare measures. LinQuad is as easy to estimate as the nonlinear AIDS model and avoids all of the problems of separable demand models. Welfare measures are much easier to derive from LinQuad than an AIDS specification.

In fact, in light of the results reported here, the only decision is whether the improved results for the generalized PIGL form are worth the increased model complexity beyond the relatively simple, linear LinQuad form. The generalized PIGL model is only slightly more complicated to estimate and will always give at least as good results as LinQuad. The added flexibility of allowing the data to choose the degree of curvature could prove even more useful on other data sets. The results of this research indicate that one of these two models offers the best choice of functional form for applied demand estimation.

Appendix A Data Used in Estimation

Dairy Product Prices and Nonfood Consumer Price Index

Year	Milk	Butter	Cheese	Frozen	Other	Nonfood CPI
1919	0.071	0.641	0.395	0.138	0.136	0.524
1920	0.077	0.662	0.385	0.188	0.13	0.606
1921	0.067	0.489	0.316	0.143	0.118	0.542
1922	0.06	0.452	0.305	0.133	0.096	0.508
1923	0.064	0.527	0.347	0.139	0.104	0.517
1924	0.062	0.493	0.336	0.146	0.098	0.518
1925	0.064	0.521	0.348	0.152	0.098	0.531
1926	0.064	0.506	0.349	0.158	0.099	0.536
1927	0.065	0.532	0.358	0.141	0.099	0.526
1928	0.065	0.538	0.378	0.138	0.096	0.519
1929	0.066	0.525	0.366	0.131	0.094	0.545
1930	0.065	0.438	0.339	0.13	0.087	0.539
1931	0.058	0.339	0.276	0.118	0.078	0.515
1932	0.049	0.262	0.226	0.091	0.065	0.479
1933	0.048	0.262	0.221	0.095	0.062	0.448
1934	0.052	0.298	0.232	0.095	0.064	0.448
1935	0.054	0.34	0.25	0.098	0.067	0.449
1936	0.055	0.374	0.262	0.101	0.073	0.454
1937	0.058	0.384	0.273	0.102	0.073	0.47
1938	0.058	0.328	0.25	0.096	0.069	0.475
1939	0.056	0.307	0.235	0.096	0.066	0.472
1940	0.059	0.342	0.242	0.095	0.067	0.473
1941	0.063	0.391	0.281	0.095	0.075	0.487
1942	0.069	0.45	0.326	0.116	0.085	0.521
1943	0.072	0.496	0.352	0.13	0.096	0.536
1944	0.072	0.471	0.358	0.139	0.095	0.557
1945	0.072	0.477	0.354	0.143	0.096	0.569
1946	0.082	0.674	0.476	0.159	0.11	0.594
1947	0.092	0.764	0.56	0.187	0.125	0.649
1948	0.101	0.823	0.623	0.186	0.141	0.696
1949	0.098	0.689	0.571	0.194	0.126	0.703
1950	0.096	0.692	0.567	0.193	0.122	0.711
1951	0.106	0.777	0.645	0.192	0.138	0.757

1952	0.111	0.812	0.664	0.193	0.143	0.775
1953	0.11	0.754	0.662	0.192	0.14	0.79
1954	0.108	0.691	0.631	0.188	0.133	0.795
1955	0.108	0.676	0.631	0.185	0.131	0.797
1956	0.111	0.692	0.634	0.185	0.135	0.811
1957	0.115	0.713	0.639	0.189	0.14	0.838
1958	0.117	0.712	0.64	0.19	0.145	0.857
1959	0.118	0.723	0.64	0.191	0.146	0.873
1960	0.121	0.72	0.664	0.189	0.152	0.888
1961	0.122	0.735	0.706	0.189	0.153	0.897
1962	0.121	0.724	0.703	0.188	0.15	0.908
1963	0.12	0.723	0.706	0.186	0.148	0.92
1964	0.121	0.73	0.726	0.183	0.148	0.932
1965	0.12	0.742	0.746	0.179	0.152	0.945
1966	0.128	0.808	0.835	0.183	0.159	0.967
1967	0.133	0.83	0.872	0.188	0.169	1
1968	0.138	0.837	0.89	0.188	0.172	1.044
1969	0.142	0.847	0.939	0.189	0.178	1.101
1970	0.148	0.867	1.008	0.197	0.189	1.167
1971	0.152	0.878	1.055	0.2	0.2	1.221
1972	0.155	0.874	1.087	0.2	0.203	1.258
1973	0.169	0.919	1.208	0.213	0.227	1.307
1974	0.203	0.947	1.464	0.251	0.291	1.437
1975	0.203	1.031	1.533	0.285	0.311	1.571
1976	0.214	1.271	1.732	0.298	0.346	1.675
1977	0.216	1.343	1.784	0.314	0.367	1.784
1978	0.228	1.489	1.87	0.334	0.384	1.912
1979	0.255	1.684	2.098	0.371	0.42	2.13
1980	0.277	1.892	2.318	0.421	0.465	2.44
1981	0.293	2.038	2.509	0.465	0.503	2.706
1982	0.294	2.084	2.572	0.47	0.518	2.884
1983	0.296	2.108	2.615	0.479	0.533	2.983
1984	0.299	2.159	2.639	0.494	0.546	3.113
1985	0.303	2.182	2.688	0.511	0.568	3.233
1986	0.302	2.19	2.701	0.519	0.572	3.286
1987	0.307	2.177	2.764	0.533	0.585	3.401
1988	0.314	2.165	2.849	0.545	0.589	3.542
1989	0.339	2.14	3.068	0.572	0.6	3.703
1990	0.376	1.999	3.423	0.61	0.605	3.9
1991	0.363	1.942	3.464	0.639	0.605	4.074
1992	0.377	1.942	3.534	0.652	0.605	4.215

1993	0.377	1.662	3.306	0.627	0.569	4.343
1994	0.391	1.602	3.316	0.652	0.579	4.46
1995	0.391	1.612	3.352	0.665	0.591	4.582

First Four Moments of the U.S. Population Age Distribution
and Per Capita Disposable Income, $/yr.

Year	Average Age	Age Variance / 10	Age Skewness / 100	Age Kurtosis /10,000	Per Capita Disposable Income
1919	28.256	38.246	46.55	37.923	631
1920	28.338	38.348	46.247	37.928	654
1921	28.388	38.489	46.333	38.118	508
1922	28.451	38.692	46.595	38.399	541
1923	28.548	38.843	46.752	38.613	616
1924	28.644	38.993	46.894	38.832	610
1925	28.764	39.177	46.887	39.009	636
1926	28.917	39.302	46.701	39.066	651
1927	29.067	39.42	46.389	39.084	645
1928	29.244	39.533	45.923	39.066	653
1929	29.438	39.607	45.398	39.002	683
1930	29.679	39.699	44.931	38.983	605
1931	29.9	39.92	44.633	39.152	516
1932	30.135	40.106	44.283	39.267	390
1933	30.378	40.267	43.843	39.334	362
1934	30.63	40.406	43.289	39.357	414
1935	30.86	40.564	42.742	39.42	459
1936	31.088	40.715	42.153	39.484	518
1937	31.308	40.881	41.539	39.591	552
1938	31.498	41.107	40.949	39.813	504
1939	31.671	41.345	40.388	40.078	537
1940	31.847	41.533	39.863	40.281	568
1941	32.008	41.794	39.248	40.55	689
1942	32.141	42.161	38.773	40.996	863
1943	32.189	42.582	38.493	41.565	972
1944	32.282	42.915	38.058	41.99	1052
1945	32.406	43.373	37.625	42.551	1066
1946	32.524	43.761	37.32	43.015	1124
1947	32.43	44.415	37.757	43.99	1171

1948	32.409	44.862	38.185	44.607	1283
1949	32.397	45.382	38.493	45.322	1260
1950	32.356	45.933	39.146	46.161	1368
1951	32.314	46.529	39.61	47.003	1475
1952	32.304	46.866	40.443	47.419	1528
1953	32.275	47.289	41.227	47.994	1599
1954	32.236	47.748	42.155	48.644	1604
1955	32.19	48.209	43.043	49.293	1687
1956	32.138	48.577	43.915	49.813	1769
1957	32.076	48.942	44.948	50.372	1833
1958	32.026	49.212	45.997	50.814	1865
1959	31.985	49.514	47.072	51.266	1971
1960	31.954	49.774	48.057	51.65	2008
1961	31.897	50.004	49.23	52.024	2062
1962	31.911	49.981	49.959	52.039	2151
1963	31.912	49.964	50.691	52.039	2225
1964	31.93	49.965	51.377	52.052	2384
1965	31.968	49.922	51.925	51.973	2541
1966	32.046	49.781	52.21	51.712	2715
1967	32.152	49.625	52.285	51.409	2877
1968	32.28	49.421	52.074	51.028	3096
1969	32.389	49.251	52.092	50.738	3297
1970	32.488	49.181	52.213	50.641	3545
1971	32.608	49.097	52.035	50.552	3805
1972	32.727	49.012	51.858	50.462	4074
1973	32.906	48.878	51.403	50.273	4552
1974	33.095	48.725	51.007	50.057	4928
1975	33.287	48.627	50.798	49.925	5367
1976	33.506	48.475	50.207	49.636	5837
1977	33.693	48.425	49.614	49.554	6362
1978	33.86	48.402	48.995	49.54	7097
1979	34.044	48.319	48.22	49.468	7861
1980	34.149	48.343	47.665	49.571	8665
1981	34.246	48.337	47.191	49.669	9566
1982	34.393	48.355	46.087	49.707	10108
1983	34.528	48.393	45.172	49.79	10764
1984	34.665	48.348	44.224	49.741	11887
1985	34.79	48.305	43.425	49.717	12587
1986	34.925	48.246	42.577	49.68	13244
1987	35.051	48.258	41.752	49.717	13849
1988	35.143	48.282	40.919	49.734	14857

1989	35.239	48.374	39.869	49.853		15742
1990	35.307	48.47	39.206	49.995		16670
1991	35.367	48.588	38.302	50.141		17191
1992	35.479	48.727	36.94	50.153		18062
1993	35.58	48.832	35.825	50.151		18552
1994	35.685	48.847	34.635	50.006		19253
1995	35.809	48.83	33.412	49.797		20063

U.S. Per Capita Consumption of Dairy Products, 1919-95
(Pound per Person per Year)

Year	Milk	Butter	Cheese	Frozen	Other
1919	314.9	15.2	4.8	8.4	11.9
1920	326.3	14.9	4.6	9.2	10.5
1921	324	16.3	4.6	9.2	11.4
1922	319.5	17.1	4.8	9.8	12.6
1923	304.7	17.8	5.1	10.6	13.8
1924	314.9	17.8	5.4	10.4	13.8
1925	314.9	18.1	5.6	11.2	14.2
1926	313.8	18.3	5.6	11	15.1
1927	311.5	18.3	5.6	11.4	15.2
1928	311.5	17.6	5.6	11.4	16.1
1929	313.8	17.6	5.9	12.2	18.1
1930	311.5	17.6	5.9	11.3	17.9
1931	310.4	18.3	5.7	10.2	17.4
1932	314.9	18.5	5.7	8	17.8
1933	313.8	18.2	5.8	7.8	17.4
1934	300.1	18.6	6.1	8.9	18.9
1935	302.4	17.6	6.6	9.9	20.4
1936	304.7	16.8	6.8	11.3	20.7
1937	304.7	16.8	7	12.4	21.9
1938	302.4	16.6	7.5	12.2	22.8
1939	305.8	17.4	7.9	13.2	23.4
1940	304.7	17	7.9	13.7	25.2
1941	307	16.1	7.9	15.8	25.6
1942	331.9	15.9	8.4	18.1	26
1943	359.2	11.8	7	17.2	27.2
1944	371.7	11.9	7.1	18.5	24.1
1945	380.8	10.9	9.3	20.7	29
1946	364.9	10.5	9.2	25.4	32.7

1947	343.3	11.2	9.2	22.4	31.2
1948	328.5	10	9.4	21	29.4
1949	326.3	10.5	10	20.5	27.8
1950	335.7	10.7	10.8	21.1	29.8
1951	333.2	9.6	10.5	21.1	28
1952	332.5	8.6	11	21.6	28
1953	328.1	8.5	11.1	22.5	27.2
1954	323.3	8.9	11.7	22.5	27
1955	328.1	9	11.8	23.5	27.2
1956	330.3	8.7	12.4	23.8	26.5
1957	324.7	8.3	12.2	24.1	26.3
1958	319.2	8.3	12.7	24.2	25.8
1959	312.5	7.9	12.7	25.8	26.3
1960	301.7	7.5	13	25.7	25.5
1961	295.3	7.4	13.2	25.8	25.4
1962	292.6	7.3	13.8	26.4	24.5
1963	293.1	6.9	13.8	27	23
1964	293.1	6.9	14.1	27.6	23.2
1965	292.2	6.4	14.3	28.1	22.6
1966	290.8	5.7	14.4	28.1	22.5
1967	282.7	5.5	14.6	27.8	21.2
1968	282.2	5.7	15.2	28.7	20.9
1969	279.8	5.4	15.7	28.7	20.1
1970	277	5.3	16.7	28.5	19.2
1971	274.6	5.1	17.5	28.2	18.9
1972	275.6	4.9	18.6	28	17.6
1973	271.5	4.8	18.9	28	17.5
1974	262.3	4.5	19.3	27.7	15.4
1975	266.7	4.7	19	28.6	14.4
1976	264.1	4.3	20.4	27.5	14.7
1977	259.9	4.3	20.8	27.5	14.3
1978	257.3	4.4	21.7	27.3	13.5
1979	253.2	4.5	21.7	26.5	13.8
1980	249.6	4.5	22.1	26.4	13.2
1981	245.4	4.3	22.7	26.5	12.7
1982	241.9	4.3	24.4	26.4	12.6
1983	242.3	4.9	24.8	27.1	12.9
1984	243.3	4.9	25.8	27.2	13.7
1985	245.1	4.9	26.5	27.9	13.8
1986	244.7	4.6	27.2	27.9	14.8
1987	242.7	4.7	28	28.2	14.8

1988	238.7	4.5	27.6	27.7	14.7
1989	240.5	4.4	27.4	28.8	14.2
1990	237.3	4.4	28.1	28.9	15.3
1991	237	4.2	28.3	29.3	15
1992	234.8	4.2	29.4	29.3	15.8
1993	230.8	4.5	29.7	29	14.5
1994	229.8	4.8	30.3	29.9	17.5
1995	226.7	4.5	30.9	28.2	15.7

U.S Population Ethnic Makeup

Year	White	Black	Neither White nor Black
1919	89.62	9.98	0.4
1920	89.7	9.9	0.4
1921	89.71	9.88	0.41
1922	89.72	9.86	0.42
1923	89.73	9.84	0.43
1924	89.74	9.82	0.44
1925	89.75	9.8	0.45
1926	89.76	9.78	0.46
1927	89.77	9.76	0.47
1928	89.78	9.74	0.48
1929	89.79	9.72	0.49
1930	89.8	9.7	0.5
1931	89.8	9.71	0.49
1932	89.8	9.72	0.48
1933	89.8	9.73	0.47
1934	89.8	9.74	0.46
1935	89.8	9.75	0.45
1936	89.8	9.76	0.44
1937	89.8	9.77	0.43
1938	89.8	9.78	0.42
1939	89.8	9.79	0.41
1940	89.8	9.8	0.4
1941	89.76	9.82	0.42
1942	89.72	9.84	0.44
1943	89.68	9.86	0.46

1944	89.64	9.88	0.48
1945	89.6	9.9	0.5
1946	89.56	9.92	0.52
1947	89.52	9.94	0.54
1948	89.48	9.96	0.56
1949	89.44	9.98	0.58
1950	89.4	10	0.6
1951	89.32	10.05	0.63
1952	89.24	10.1	0.66
1953	89.16	10.15	0.69
1954	89.08	10.2	0.72
1955	89	10.25	0.75
1956	88.92	10.3	0.78
1957	88.84	10.35	0.81
1958	88.76	10.4	0.84
1959	88.68	10.45	0.87
1960	88.6	10.5	0.9
1961	88.5	10.56	0.94
1962	88.4	10.62	0.98
1963	88.3	10.68	1.02
1964	88.2	10.74	1.06
1965	88.1	10.8	1.1
1966	88	10.86	1.14
1967	87.9	10.92	1.18
1968	87.8	10.98	1.22
1969	87.7	11.04	1.26
1970	87.6	11.1	1.3
1971	87.43	11.17	1.4
1972	87.26	11.24	1.5
1973	87.09	11.31	1.6
1974	86.92	11.38	1.7
1975	86.75	11.45	1.8
1976	86.58	11.52	1.9
1977	86.41	11.59	2
1978	86.24	11.66	2.1
1979	86.07	11.73	2.2
1980	85.9	11.8	2.3
1981	85.7	11.85	2.45
1982	85.5	11.9	2.6
1983	85.3	11.95	2.75
1984	85.1	12	2.9

1985	84.9	12.05	3.05
1986	84.7	12.1	3.2
1987	84.5	12.15	3.35
1988	84.3	12.2	3.5
1989	84.1	12.25	3.65
1990	83.9	12.3	3.8
1991	83.6	12.4	4
1992	83.4	12.45	4.15
1993	83.2	12.5	4.3
1994	83	12.55	4.45
1995	82.8	12.6	4.6

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