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Abstract. This paper conceptually and empirically compares alternative strategies for incorporating weak complementary into continuous demand system models. The repackaging, integrating back, and discontinuity approaches are evaluated in terms of their behavioral implications and potential usefulness for applied research. The conceptual review suggests that the repackaging approach offers the most flexibility and tractability. The empirical comparison suggests that qualitatively similar policy inference arises from previously employed repackaging approaches. These estimates can be similar to use-related welfare estimates derived from non-weakly complementary models using a decomposition approach suggested by Herriges, Kling, and Phaneuf (2004), although the latter are sensitive to arbitrary assumptions about how to decompose use and nonuse values.

JEL Classification: D120, Q260, C240, C110

1. Introduction

In applied demand analysis, neoclassical consumer theory provides guidance for structuring relationships among quantities, prices, and income. The theory is, however, noticeably silent with respect to how the quality attributes of goods should enter these relationships. As a result the analyst has considerable discretion when introducing goods' quality attributes into consumer demand models. Because a significant and growing number of measurement questions arise in the context of quality change, this reality challenges the researcher to develop preference and demand specifications that defensibly incorporate goods' quality attributes.

When developing these specifications, the analyst can sometimes statistically discriminate between alternative hypotheses about how quality enters preference and demand relationships. Otherwise, intuition is the analyst's only guide. One untestable but intuitive restriction on how quality attributes enter these relationships that Mäler (1974) and Bradford and Hildebrandt (1977) proposed three decades ago is weak complementarity. When a good and its quality attributes are weak complements, the individual only values marginal improvements in the good's quality attributes if she consumes it. This restriction implies that all value derived from changes in a good's quality attributes arises through consumption. Whether implicit or explicit, this restriction has been incorporated into preferences for virtually every valuation exercise that relies exclusively on revealed preference data.

This paper conceptually and empirically compares alternative strategies for incorporating weak complementarity into continuous demand system models. Three different strategies – the repackaging approach (Fisher and Shell (1968)), the integrating back approach (Larson (1991)), and the discontinuity approach (Bockstael, Hanemann, and Strand (1986)) – are evaluated in terms of their behavioral implications and potential empirical usefulness. One of the main implications from a conceptual comparison of the approaches is that only the repackaging approach is likely to offer applied researchers much guidance and flexibility when developing weakly complementary demand specifications. An empirical comparison across three weakly complementary specifications developed within the repackaging framework is conducted with a beach recreation data set and estimated within the Bayesian statistical framework. The main empirical finding is that welfare estimates for the loss of beach width are qualitatively similar across the alternative repackaging specifications considered. These estimates can be similar to use-related welfare estimates derived from non-weakly complementary models using a decomposition approach suggested by Herriges, Kling, and Phaneuf (2004) (hereafter HKP), although the latter are sensitive to arbitrary judgments about how to decompose use and nonuse values.

The remainder of the paper is structured as follows. For perspective, section 2 reviews how economists have proposed introducing goods' quality attributes explicitly into continuous demand system models with a special emphasis on the role of weak complementarity. Section 3 then critically reviews the repackaging, integrating back, and discontinuity approaches to developing weakly complementary demand models. Section 4 summarizes the specifications to be compared in the empirical application, and Section 5 briefly summarizes the 1997 mid-Atlantic beach recreation data used in the comparison. Section 6 follows with the parameter estimates for the alternative models, and section 7 discusses the welfare scenario, estimation strategy, and empirical results. Section 8 concludes with a discussion of the implications of the paper's findings for future research.

2. Introducing Quality into Demand Systems & the Role of Weak Complementary: A Review

As discussed in Hanemann (1982), economists have historically exploited one of two generic frameworks for explicitly incorporating the quality attributes of heterogeneous goods into demand system models. The first approach was pioneered by Houthakker (1952-3) and Theil (1952-3) and assumes that goods can be grouped into categories or classes based on their similar functions and characteristics. Within each class, the individual is assumed to consume at most one good, and the similar goods collectively form a perfect continuum of alternatives over the relevant support of all quality attributes. Consumer preferences in this setup can be represented by a direct utility function, $U(\boldsymbol{x},\boldsymbol{Q},z)$, where $\boldsymbol{x}$ corresponds to an N dimensional vector of consumption quantities for the different classes of goods, $\boldsymbol{Q}=[\boldsymbol{q}_1,\ldots,\boldsymbol{q}_N]$ is a matrix of quality attributes for all N good classes, and z is an essential Hicksian composite good. The price of consuming a particular type of good in the jth class is $p_j(\boldsymbol{q}_j)$, where $p_j(\bullet)$ is a smooth, continuous "hedonic" price function.[1] Together these assumptions imply that the consumer's problem can be succinctly stated as:

[1] Alternatively, Theil (1956) interprets $p_j(\boldsymbol{q}_j)$ as a quality adjusted price index for the jth commodity group, which in turn implies that x_j is a quality adjusted quantity index.

$$(1) \qquad \max_{x,Q}\{U(\boldsymbol{x},\boldsymbol{Q},z)\} \quad \text{s.t. } \sum_j p_j(\boldsymbol{q}_j)x_j + z = y,\ \boldsymbol{x} \geq 0,\ \&\ \underline{\boldsymbol{q}}_j \leq \boldsymbol{q}_j \leq \overline{\boldsymbol{q}}_j, \forall j,$$

where $\underline{\boldsymbol{q}}_j$ and $\overline{\boldsymbol{q}}_j$ are the upper and lower bounds of the support for *j*th good's quality attributes. Equation (1) can be interpreted as a multivariate generalization of Rosen's (1974) discrete-continuous formulation of the consumer's problem when $\boldsymbol{x}$ is a scalar equal to one. Two distinguishing features of this representation of the consumer's problem are worth emphasizing. Both the quantities and quality attributes of goods enter as endogenous arguments of the individual's preference ordering that imply first order conditions that implicitly define the consumer's optimal consumption bundle. In addition, the budget constraint may be highly nonlinear in quality attributes, and thus some structure must be placed on the hedonic price functions for unique solutions to exist.[2]

A second strategy for explicitly incorporating quality attributes into demand system models may be preferable when the continuity assumptions that underlie Houthakker and Theil's approach are inappropriate. As Lancaster (1966) and Mäler (1974) argue, the quality attributes of a finite set of goods can be thought of as exogenous fixed factors or rationed goods. Thus quality attributes in this formulation are nothing more than additional parameters that enter the individual's preference ordering. A good's price may also depend on its quality attributes, but because quality attributes are exogenous to the individual, the dependence need not be made explicit. The consumer's problem in this setting can be stated as:

$$(2) \qquad \max_{x}\{U(\boldsymbol{x},\boldsymbol{Q},z)\} \quad \text{s.t. } \boldsymbol{p}^{\mathsf{T}}\boldsymbol{x} + z = y, \boldsymbol{x} \geq 0,$$

where $\boldsymbol{x}$ contains all quality differentiated goods. Notice in (2) that only $\boldsymbol{x}$ is endogenous and the budget constraint is linear. In some sense, this structure is the natural extension to the traditional

[2] Rosen (1974) discusses how market forces will result in the hedonic price functions embodying this additional structure.

formulation of the consumer's problem where the utility function's dependence on the quality attributes of goods is implicit. Because the rationed goods approach does not rely on continuity assumptions that may be implausible in many applications, it is often the preferred approach for incorporating quality attributes explicitly except in the extreme but not altogether uncommon situations considered by Rosen. Thus in the remainder of the paper the discussion is couched exclusively in terms of the rationed goods approach.

When quality is introduced explicitly into consumer demand models, a natural question arises: does neoclassical consumer theory suggest any structure for how quality enters preference and demand functions? In general, the answer is no. The restrictions on preference and demand relationships implied by traditional consumer theory represent the minimal set of assumptions necessary to guarantee a solution to the consumer's constrained optimization problem. So long as preferences satisfy these conditions and the consumer's affordable budget set is convex, a unique solution to the consumer's problem is guaranteed regardless of how goods' quality attributes enter preferences. Thus although consumer theory has much to say about the relationships among quantities, prices, and income in preference and demand functions, it has nothing to say about how quality attributes enter these relationships. This reality is in some sense liberating to the analyst, but it also place a significant burden on her to develop defensible empirical specifications that explicitly incorporate goods' quality attributes.

When developing these specifications, intuition is often the analyst's only guide, although statistical criteria can sometimes be used to discriminate among competing hypotheses on how quality attributes enter demand and preference functions. Weak complementarity, which is the focus of this paper, represents an intuitive but untestable restriction. In essence, weak complementarity implies that all value derived from the quality attributes of a good arise exclusively through the good's use. Two conditions must be satisfied for the preference restriction to hold: 1) the good must be nonessential; and 2) if the good is not consumed, the individual does not benefit from marginal improvements in its quality attributes, i.e., $\partial U(\boldsymbol{x},\boldsymbol{Q},z)/\partial \boldsymbol{q}_j = 0$ if $x_j = 0, \forall j$. Weak complementarity is not a testable restriction because the analyst cannot distinguish between a utility function $U(\boldsymbol{x},\boldsymbol{Q},z)$ that satisfies conditions 1) and 2) and a monotonic transformation of $U(\boldsymbol{x},\boldsymbol{Q},z)$, say $u(U(\boldsymbol{x},\boldsymbol{Q},z),\boldsymbol{Q})$, that does not with just revealed preference data for $\boldsymbol{x}$.[3,4]

Following Smith and Banzhaf (2004), weak complementarity's implications for the structure of preferences are represented graphically in Figure 1 in the context of a simple two good (x,z) model. The horizontal axis measures the quantity of x consumed while the vertical axis measures the consumption level of z. Three indifference curves are drawn that correspond to the same level of utility (i.e., $U(\boldsymbol{q}^0) = U(\boldsymbol{q}^1) = U(\boldsymbol{q}^2)$) but different levels of quality ($\boldsymbol{q}^0 > \boldsymbol{q}^1 > \boldsymbol{q}^2$) where utility is assumed to be strictly increasing in quality. The first condition of weak complementarity requires that these indifference curves must intersect the vertical axis, and the second that they intersect the vertical axis at the same point (in figure 1, point A). Assuming preferences are continuous, they also exhibit what Smith and Banzhaf refer to as the "fanning" property – as x increases, the distance between the indifference also increases.

In the context of a single quality differentiated good, Mäler (1974) showed how weak complementarity and Hicksian demand functions can be used to construct theoretically consistent welfare measures. A key concept in his derivation is the Hicksian "choke" price, or

[3] This point should not be interpreted as suggesting that weak complementarity implies preference functions are sensitive to all monotonic transformations. Although weak complementarity does rule out monotonic transformations of $U(\boldsymbol{x},\boldsymbol{Q},z)$ that change the marginal rates of substitution among $\boldsymbol{x}$, $\boldsymbol{Q}$, and z, it does not rule out those that do not.

[4] However, if both revealed and stated preference data are present, it may be possible to test whether weak complementarity is a valid assumption.

the minimal price that drives the consumer's Hicksian demand for x to zero. Hicksian choke prices will in general depend on $\boldsymbol{q}$ and $\bar{U}$, the relevant utility level. Graphically in Figure 1, they correspond to the slopes of the indifference curves $U(\boldsymbol{q}^0)$, $U(\boldsymbol{q}^1)$, and $U(\boldsymbol{q}^2)$ evaluated at $x = 0$. Mäler demonstrated that weak complementarity implies that the difference between the compensating variations associated with a price change from the observed to the Hicksian choke price evaluated separately at $\boldsymbol{q}^0$ and $\boldsymbol{q}^1$ represents the Hicksian consumer surplus, $CS^H(\boldsymbol{q}^0,\boldsymbol{q}^1)$, arising from the quality change. This can be seen below in equation (3):

$$
\begin{aligned}
CS^H(\boldsymbol{q}^0,\boldsymbol{q}^1) &= E(\bar{p},\boldsymbol{q}^0,\bar{U}^0) - E(\bar{p},\boldsymbol{q}^1,\bar{U}^0) \\
&= E(\bar{p},\boldsymbol{q}^0,\bar{U}^0) - E(\bar{p},\boldsymbol{q}^1,\bar{U}^0) + E(\hat{p}(\boldsymbol{q}^1,\bar{U}^0),\boldsymbol{q}^1,\bar{U}^0) - E(\hat{p}(\boldsymbol{q}^0,\bar{U}^0),\boldsymbol{q}^0,\bar{U}^0) \\
&= \left[E(\hat{p}(\boldsymbol{q}^1,\bar{U}^0),\boldsymbol{q}^1,\bar{U}^0) - E(\bar{p},\boldsymbol{q}^1,\bar{U}^0)\right] - \left[E(\hat{p}(\boldsymbol{q}^0,\bar{U}^0),\boldsymbol{q}^0,\bar{U}^0) - E(\bar{p},\boldsymbol{q}^0,\bar{U}^0)\right], \\
&= \int_{\bar{p}}^{\hat{p}(\boldsymbol{q}^1,\bar{U}^0)} x^h(p,\boldsymbol{q}^1,\bar{U}^0)dp - \int_{\bar{p}}^{\hat{p}(\boldsymbol{q}^0,\bar{U}^0)} x^h(p,\boldsymbol{q}^0,\bar{U}^0)dp \\
&= CV(\bar{p},\hat{p}(\boldsymbol{q}^1,\bar{U}^0),\boldsymbol{q}^1,\bar{U}^0) - CV(\bar{p},\hat{p}(\boldsymbol{q}^0,\bar{U}^0),\boldsymbol{q}^0,\bar{U}^0)
\end{aligned} \tag{3}
$$

where $\bar{p}$ is the observed market price, $\hat{p}(\bullet,\bar{U}^0)$ is the Hicksian choke price, and $E(\bullet,\bullet,\bar{U}^0)$ is the expenditure function. The first line of equation (3) is simply the definition of the Hicksian consumer surplus associated with the quality change from $\boldsymbol{q}^0$ and $\boldsymbol{q}^1$, the second adds in the expression $E(\hat{p}(\boldsymbol{q}^1,\bar{U}^0),\boldsymbol{q}^1,\bar{U}^0) - E(\hat{p}(\boldsymbol{q}^0,\bar{U}^0),\boldsymbol{q}^0,\bar{U}^0)$ which equals zero if weak complementarity holds, the third line simply reorganizes terms, and the fourth and fifth exploit Shephard's lemma and the definitions of Hicksian demand functions, $x^h(\bullet)$, and compensating variation, $CV(\bullet)$, respectively.

One can also graphically illustrate Mäler's result in Figure 1. Imagine that relative prices are such that initially the individual's optimal consumption bundle corresponds to the tangency between line 1 and $U(\boldsymbol{q}^0)$. If the price of z is normalized to one, the expenditures necessary to purchase this bundle correspond to the distance between the origin and point B. The compensating variation associated with the price change from baseline prices $\overline{p}$ to the Hicksian choke price $\hat{p}(\boldsymbol{q}^0,\bar{U}^0)$ (i.e., the slope of $U(\boldsymbol{q}^0)$ evaluated at $x = 0$), corresponds to the distance between points B and A. Weak complementarity implies that a decrease in quality from $\boldsymbol{q}^0$ to $\boldsymbol{q}^1$, although it lowers the choke price from $\hat{p}(\boldsymbol{q}^0,\bar{U}^0)$ to $\hat{p}(\boldsymbol{q}^1,\bar{U}^0)$, does not alter the minimum expenditure necessary to achieve $\bar{U}^0$, i.e., the distance between the origins to point A. A decrease in price from $\hat{p}(\boldsymbol{q}^1,\bar{U}^0)$ to $\overline{p}$ results in the individual's optimal consumption bundle adjusting to the point where Line 2 and $U(\boldsymbol{q}^1)$ are tangent. The compensating variation associated with this price change corresponds to the vertical distance between points A and C. Thus when weak complementarity holds, the consumer's Hicksian consumer surplus associated with the degradation in quality (i.e., the vertical distance between points A and C) exactly equals the difference in compensating variations between $\overline{p}$ and $\hat{p}(\boldsymbol{q}^0,\bar{U}^0)$ $\overline{p}$ and $\hat{p}(\boldsymbol{q}^1,\bar{U}^0)$, respectively.

Although Mäler's duality result is elegant and potentially useful to applied researchers when preferences are quasilinear, its practical value is questionable when income effects are present and observable Marshallian and latent Hicksian demands diverge (see Bockstael and McConnell (1993), Palmquist (2004), and Smith and Banzhaf (2004) for discussions). More importantly, the development of virtually all modern empirical demand system models begins with an explicit specification of preferences represented by a direct or indirect utility function. Palmquist (2004) correctly points out that Mäler's motivation for imposing weak complementarity is less relevant in these situations because the analyst knows (or more precisely, assumes) the complete structure of preferences and welfare measurement is conceptually straightforward. Thus Mäler's rationale for imposing weak complementarity has dubious practical value for the current practice of applied demand analysis.[5]

This point does not diminish the intuitive appeal of the assumption in many applied situations, however. The main implication of weak complementarity is that all value associated with a change in a good's quality attributes arise exclusively through its use. In situations where nonuse values are thought to be absent, imposing weak complementarity *a priori* makes good sense. Moreover, in situations where the analyst believes that nonuse values are likely present but not reliably measurable, imposing weak complementarity still may be defensible. As HKP's and this paper's empirical results suggest, welfare measures derived from demand system models that are not consistent with weak complementarity can be significantly different than estimates derived from models that do. These empirical findings might suggest that nonuse values are substantial (Larson (1993)), but such inference would at best be speculative without additional stated preference data that would allow the analyst to identify more reliably total value. Moreover, the total value estimates are arbitrary in the sense that they are conditional on a specific non-weakly complementary preference ordering when in principle there are an infinite number of non-weakly complementary preference orderings that generate the same observable demand functions.

To address this limitation with welfare estimates derived from non-weakly complementary demand models, HKP have proposed decomposing total value into use and nonuse components, disregarding the unreliably measured nonuse component, and reporting only the use component. Although intuitively sensible in principle, such decomposition approaches are plagued by at least two problems in practice. Similar to total value, the use component of total value will in general depend on the non-weakly complementary preference structure

[5] Of course its relevance could increase in the future with methodological innovations in applied demand analysis.

arbitrarily chosen by the analyst. It is straightforward to show that this dependence can only be broken if the analyst restrictively assumes that preferences are quasilinear. Moreover, for policies affecting only a subset of the quality differentiated goods, Flores (2004) has argued that the decomposition will depend in general on how the analyst defines the nonuse value. To understand his argument, imagine a situation where there are two quality differentiated goods but the policy scenario considered only affects the first good's quality attributes. When defining the nonuse component of total value associated with this quality change, obviously the demand for the first good should be restricted to zero before and after the quality change, but depending on how the second good's demand is treated, the nonuse component of value will change unless the cross-price Hicksian demand elasticity between the two is zero. In other words, the nonuse (and in turn the use) component of total value will depend on whether the demand for the first good or the demands for both goods are held at zero before and after the quality change. Combined, these factors suggest that use-related welfare measures derived from decomposition approaches are sensitive to arbitrary assumptions. Of course welfare measures derived from weakly complementary models that *a priori* rule out nonuse values are also arbitrary. The point of the above discussion, however, is to suggest that the analyst might nevertheless want to assume weak complementary when nonuse values are present but not reliably measurable to avoid the difficulties inherent with decomposition approaches.

3. Empirical Strategies for Incorporating Weak Complementarity

If the analyst concludes that incorporating weak complementarity into preferences is appropriate, a relevant question is whether there are generic strategies that can be used to guide the development of weakly complementary specifications. In this section three general strategies that have been identified in the valuation literature are discussed – the repackaging, integrating back, and discontinuity approaches. In principle applied researchers can exploit each of the approaches to develop weakly complementary demand systems, but here it is argued that the repackaging approach is likely to be the most useful in practice.

3.1. The Repackaging Approach

Perhaps the oldest and most widely used strategy for developing empirical demand models consistent with weak complementarity is the repackaging approach. Preferences in this framework can be nested within the following general class of direct utility functions:

(4) $$U\left(f_{11}(x_1,\boldsymbol{q}_1),...f_{1M_1}(x_1,\boldsymbol{q}_1),f_{22}(x_2,\boldsymbol{q}_2),...f_{2M_2}(x_2,\boldsymbol{q}_2),...,f_{N1}(x_N,\boldsymbol{q}_N),...,f_{NM_N}(x_N,\boldsymbol{q}_N),\boldsymbol{x},z\right),$$

where $f_{ij}(x_i,\boldsymbol{q}_i),\forall i,j$, are alternative subfunctions that share the property that:

(5) $$\frac{\partial f_{ij}(x_i,\boldsymbol{q}_i)}{\partial \boldsymbol{q}_i}=0 \text{ if } x_i=0,\forall i,j .$$

The structure of equation (4) suggests that the $f_{ij}(x_i,\boldsymbol{q}_i)$ subfunctions aggregate or "repackage" x_i and $\boldsymbol{q}_i$ into M_i composite goods from which the consumer ultimately derives utility. It is only through these subfunctions that $\boldsymbol{q}_i$ enters consumer preferences.

There are at least three empirical repackaging specifications that have been utilized in valuation studies. The oldest and most popular is the pure repackaging framework introduced by Fisher and Shell (1968). In this framework, the primal representation of consumer preferences can be nested within the following class of direct utility functions:

(6) $$U\left(\phi_1(\boldsymbol{q}_1)x_1,\phi_2(\boldsymbol{q}_2)x_2,...,\phi_N(\boldsymbol{q}_N)x_N,z\right),$$

where $\phi_i(\boldsymbol{q}_i)>0,i=1,...,N,$ are commonly referred to as pure repackaging parameters. Muellbauer (1976) shows that the implied indirect utility and Marshallian demand functions consistent with (6) respectively take the form

(7) $$V\left(\frac{p_1}{\phi_1(\boldsymbol{q}_1)},\ldots,\frac{p_N}{\phi_N(\boldsymbol{q}_N)},y\right),$$

(8) $$x_i=\frac{1}{\phi_i(\boldsymbol{q}_i)}g_i\left(\frac{p_1}{\phi_1(\boldsymbol{q}_1)},\ldots,\frac{p_N}{\phi_N(\boldsymbol{q}_N)},y\right),i=1,\ldots,N\,.$$

The behavioral implications of the pure repackaging framework are intuitive, well known, but sometimes troubling – for example, the individual is indifferent between a doubling of x_i or $\phi_i(\boldsymbol{q}_i)$, and whether an increase in $\phi_i(\boldsymbol{q}_i)$ results in an increase in demand depends critically on whether the price elasticity of demand is less than one in absolute value. From a more practical perspective, an appealing attribute of the pure repackaging approach is that it satisfies the so-called Willig condition (Willig (1978)) which implies that a simultaneous change in p_i and $\phi_i(\boldsymbol{q}_i)$ can be transformed into a pure price change that generates the same level of satisfaction for the consumer. As von Haefen (1999) demonstrates, The Willig condition implies that even when the analyst cannot recover a closed form representation of the full structure of consumer preferences from observable Marshallian demands, she can nevertheless use Vartia's (1983) numerical algorithms to construct exact welfare measures for simultaneous price and quality changes.

A second commonly used empirical specification consistent with (4) and (5) is the cross-product repackaging approach introduced by Willig (1978). Preferences in this case can be nested within the following general structure:

(9) $$U\left(\boldsymbol{x},z+\sum_i^N\delta_i(\boldsymbol{q}_i)x_i\right).$$

where $\delta_i(\boldsymbol{q}_i), i = 1,...N,$ are commonly referred to as cross-product repackaging parameters. Assuming interior solutions for $\boldsymbol{x}$ and z, the indirect utility and Marshallian demand functions in this case can be written generally as:

(10) $V\left(p_1 - \delta_1(\boldsymbol{q}_1),..., p_N - \delta_N(\boldsymbol{q}_N), y\right)$,

(11) $x_i = g_i\left(p_1 - \delta_1(\boldsymbol{q}_1),..., p_N - \delta_N(\boldsymbol{q}_N), y\right),\ i = 1,..., N$.

As Hanemann (1984) suggests, the cross-product repackaging approach is in many ways less restrictive than the pure repackaging approach – the consumer is not necessarily indifferent between a doubling of x_i or $\phi_i(\boldsymbol{q}_i)$, and so long as the elasticity of demand is strictly negative, Marshallian demand will rise with a quality improvement. Like the pure repackaging approach, the cross-product repackaging approach implies Marshallian demand functions that are consistent with the Willig condition. As a result, exact welfare measures for simultaneous price and quality changes can be constructed using Vartia's numerical techniques regardless of the existence of closed form representations of preferences.

One feature of the cross-product repackaging framework that may limit its empirical usefulness is the possibility of negative quality adjusted prices (i.e., $(p_i - \delta_i(\boldsymbol{q}_i)) < 0$) and its implications for consumer behavior.[6] The nature of the problem can be appreciated by studying the first order conditions implied by the consumer's problem for a simple two good model where preferences can be represented by the utility function $U\left(x, z + \delta(\boldsymbol{q})x\right)$. If utility is strictly increasing in (x, z) and $(p - \delta(\boldsymbol{q})) < 0$, then

(12) $U_1 - U_2(p - \delta(\boldsymbol{q})) > 0$,

[6] A similar problem can arise when quality adjusted prices are negative with the pure repackaging approach. However, by using transformations that restrict $\phi_i(\boldsymbol{q}_i)$ to be strictly positive, these difficulties can be avoided altogether.

where U_i is the derivative of the direct utility function with respect to its *i*th argument. Equation (12) implies that a negative quality adjusted price results in the consumer spending all of her income on *x*.[7] Although possible, this outcome is extreme and unlikely to be consistent with micro data. Of course, it is only a concern to the degree that negative quality adjusted prices arise in practice which will vary from application to application. The outcome, however, is more likely to arise when prices are relatively small and individual preferences with respect to quality vary substantially. In these cases, the cross-product repackaging framework may be an undesirable approach for developing weakly complementary preferences.

A third approach for developing weakly complementary preferences was suggested indirectly by Larson (1991) and HKP and shall be referred to as the generalized translating approach. A generic direct utility function that encompasses specifications consistent with this approach is:

$$(13)\qquad U\left(\zeta_1(x_1,\boldsymbol{q}_1)-\zeta_1(x_1=0,\boldsymbol{q}_1),\zeta_2(x_2,\boldsymbol{q}_2)-\zeta_2(x_2=0,\boldsymbol{q}_2),...,\zeta_N(x_N,\boldsymbol{q}_N)-\zeta_N(x_N=0,\boldsymbol{q}_N),z\right)$$

where each $\zeta_i(x_i,\boldsymbol{q}_i)$ subfunction is strictly increasing in x_i. Loosely speaking, the $\zeta_i(x_i=0,\boldsymbol{q}_i)$ terms serve as translating parameters (Pollak and Wales (1992)) that jointly translate or shift the $\zeta_i(x_i,\boldsymbol{q}_i)$ terms and in turn the consumer's indifference curves in ways that result in weak complementarity holding. Because of this property, $\zeta_i(x_i=0,\boldsymbol{q}_i)$ is referred to here as a generalized translating parameter.

In general, little can be said about the indirect utility and Marshallian demand structures implied by (13) without placing additional structure on $\zeta_i(x_i,\boldsymbol{q}_i)$. It is uncertain, for example, whether consumption will be strictly increasing in quality or the Willig condition will be

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[7] This result carries over to the case where *x* is a vector and only one quality adjusted price is negative. The solution to the case where more than one quality adjusted price is negative is more complicated to characterize generally, but it will always be the case that *z* will not be consumed.

satisfied. This suggests that the analyst should carefully study the behavioral and welfare theoretic properties of an empirical specification to insure their plausibility in a given application.

Combined, the pure repackaging, cross-product repackaging, and generalized translating approaches represent three viable repackaging strategies for developing weakly complementary demand models for applied researchers. In addition equation (4) above suggests that several more general repackaging approaches with potentially more appealing implications for behavior and preferences are available to applied researchers. A concrete example may be instructive. Consider the following direct translog specification,

$$U = \sum_i \alpha_i \ln(x_i + b_i) + \sum_i \sum_j \beta_{ij} \ln(x_i + b_i) \ln(x_j + b_j) + \ln(z + b_z) .$$

Although it is straightforward to develop pure repackaging, cross-product repackaging, and generalized translating specifications consistent with this structure, other specifications are possible such as

$$U = \sum_i \alpha_i \ln(x_i + b_i(\boldsymbol{q}_i)) - \alpha_i \ln(b_i(\boldsymbol{q}_i)) + \sum_i \sum_j \beta_{ij} \ln(\tau_i(\boldsymbol{q}_i) x_i + b_i) \ln(\tau_i(\boldsymbol{q}_i) x_j + b_j) + \ln(z + b_z)$$

and

$$U = \sum_i \alpha_i \ln(\tau_i(\boldsymbol{q}_i) x_i + b_i) + \sum_i \sum_j \beta_{ij} (\ln(x_i + b_i(\boldsymbol{q}_i)) - \ln b_i(\boldsymbol{q}_i))(\ln(x_j + b_j(\boldsymbol{q}_j) - \ln b_j(\boldsymbol{q}_j)) + \ln(z + b_z) .$$

Finally, it is worth noting that if the analyst is working within the primal framework, developing weakly complementary empirical specifications is relatively straightforward. Beginning with any direct utility function that permits corner solutions and is nested within the general structure $U(f_{11}(x_1),...,f_{1M_1}(x_1),f_{21}(x_2),...,f_{2M_2}(x_2),...,f_{N1}(x_N),...,f_{NM_N}(x_N),z)$, the analyst should replace the $f_{ij}(x_i)$ functions with $f_{ij}(x_i,\boldsymbol{q}_i)$ that satisfy the property that

$\partial f_{ij}(x_i, \boldsymbol{q}_i) / \partial \boldsymbol{q}_i = 0$ if $x_i = 0, \forall i, j$. Within the dual framework, developing pure and cross-product repackaging approaches is straightforward (see equations (7) and (10) above), but generalized translating and other repackaging specifications are more difficult to develop in general.

3.2 The Integrating Back Approach

Larson (1991) introduced an alternative and very general strategy for developing weakly complementary empirical models that builds on an approach suggested by Hausman (1981) and LaFrance and Hanemann (1989). He assumes that the analyst begins with an integrable Marshallian demand system where goods' quality attributes enter arbitrarily. Duality theory implies that the following equalities hold:

$$(14) \qquad \frac{\partial E(\boldsymbol{p}, \boldsymbol{Q}, \bar{U}^0)}{\partial p_i} = x_i = g_i(\boldsymbol{p}, \boldsymbol{Q}, y) = g_i(\boldsymbol{p}, \boldsymbol{Q}, E(\boldsymbol{p}, \boldsymbol{Q}, \bar{U}^0)), \forall i.$$

In some cases one can use the techniques of differential calculus to solve (14) for the closed form quasi-expenditure function $\tilde{E}(\boldsymbol{p}, \boldsymbol{Q}, k(\boldsymbol{Q}, \bar{U}^0))$, or the expenditure function defined in terms of prices, quality, and a constant of integration $k(\boldsymbol{Q}, \bar{U}^0)$ that depends on quality and baseline utility. Because $\tilde{E}(\boldsymbol{p}, \boldsymbol{Q}, k(\boldsymbol{Q}, \bar{U}^0))$ is an incomplete characterization of consumer preferences with respect to quality, it can not be used to evaluate the welfare implications of policies that involve quality changes without the analyst placing additional structure on $k(\boldsymbol{Q}, \bar{U})$. Larson recognized, however, that weak complementarity places additional structure on $k(\boldsymbol{Q}, \bar{U})$ that may facilitate welfare measurement for quality changes. When $\tilde{E}(\boldsymbol{p}, \boldsymbol{Q}, k(\boldsymbol{Q}, \bar{U}^0))$ is evaluated at the Hicksian choke price for the *i*th good $\hat{p}_i(\boldsymbol{p}^{-i}, \boldsymbol{Q}, k(\boldsymbol{Q}, \bar{U}))$, or the price that drives the Hicksian demands for the *i*th quality differentiated good to zero, weak complementarity implies that, regardless of whether the other goods are consumed, the individual is not willing to pay for marginal improvements in the quality attributes of the *i*th good, i.e.,

$$(15)\qquad \frac{\partial \tilde{E}(\hat{p}_i(\boldsymbol{p}^{-i},\boldsymbol{Q},k(\boldsymbol{Q},\bar{U})),\boldsymbol{p}^{-i},\boldsymbol{Q},k(\boldsymbol{Q},\bar{U}))}{\partial \boldsymbol{q}_i} = 0, \forall i .$$

Equation (15) places restrictions on $k(\boldsymbol{Q},\bar{U})$ that can in principle be used to identify its structure up to a constant of integration that only depends on the baseline utility level, i.e., $\tilde{k}(\bar{U})$. As Hausman (1981) has argued, $\tilde{k}(\bar{U})$ can be interpreted as a monotonic transformation of utility, and thus the analyst can arbitrarily set it equal to $\bar{U}$ (i.e., $\tilde{k}(\bar{U}) = \bar{U}$) with no loss in generality. As a result, the analyst has recovered the full structure of preferences with respect to quality.

Although Larson's integrating back approach is irrefutable in its logic, two factors call into question the usefulness of the approach as a general strategy for developing weakly complementary empirical demand models. In his original paper, Larson used simple two good linear demand and linear expenditure models to illustrate the potential usefulness of the approach. A careful inspection of how quality enters each specification suggests that both can be interpreted as special cases of the repackaging approach.[8] Moreover, there have been no multi-good empirical applications that develop weakly complementary demand models via the integrating back approach since Larson suggested the approach over a decade ago. Consequently, there is little evidence that the integrating back approach offers additional insights into how applied researchers can develop weakly complementary demand models.

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[8] Larson's linear demand specification was $x = \alpha + \beta p + \delta q + \gamma y$ which can be rewritten as $x = \alpha + \beta(p + (\delta / \beta)q) + \gamma y$. Comparing this to (10) above suggests that it is consistent with the cross-product repackaging approach to introducing quality. Likewise, the weakly complementary direct utility function implied by Larson's linear expenditure system example is $U(x,\boldsymbol{q},z) = \Psi(\boldsymbol{q})\ln(x+c) - \Psi(\boldsymbol{q})\ln(c) + \ln(z+b)$ which is consistent with the generalized translating approach.

One approach to evaluating the empirical usefulness of the integrating back approach is to consider a large number of commonly used empirical demand specifications with quality entering in a variety of ways. If the integrating back approach suggests new weakly complementary demand models that could not have been derived from the repackaging approach, then its value to applied researchers is confirmed. Towards this end, 24 different single equation linear, semi-log, and log-linear specifications with quality allowed to enter in alternative ways are considered. A third of the specifications treat demand (i.e., x), another third treat expenditure ($e = px$), and the final third treat expenditure share ($s = px/y$) as the dependent variable.[9] All of these specifications or their logarithmic transformations share a simple linear in parameters structure and have been used or suggested in applied demand analysis. A linear quality index was allowed to enter through the constant, price, or income parameter separately for each model and the mechanics of the integrating back approach was used to determine if closed form solutions for weakly complementary preferences could be recovered.

For brevity the results for all 72 specifications are summarized here and reported in their entirety in a technical appendix available from the author upon request. The key finding was that the integrating back approach could be used to recover closed form weakly complementary preference specifications for 36 of the 72 models in general, although 10 of the 36 weakly complementary specifications could have also been generated by either the pure or cross-product repackaging approaches.[10,11] These findings suggest that the integrating back approach does in

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[9] All of these models assume there is a second Hicksian composite good z that is always strictly consumed.

[10] Of the 26 weakly complementary specifications that could not have been generated by the pure or cross-product repackaging approaches, it is possible that some or all could have been generated by other repackaging approaches. Determining whether this is the case would require one to derive the closed form direct utility functions. Due to time constraints, these tedious derivations were not attempted, but it is unlikely that closed form direct utility functions frequently exist.

[11] Interestingly, 4 of the 36 specifications that could not be linked back to closed form weakly complementary preferences have quality entering in ways that are consistent with either the pure or cross-product repackaging

fact have genuine value to applied researchers by expanding the menu of weakly complementary single equation models available.

An important caveat should be appended to this statement, however. In most applied situations, the researcher is concerned with developing weakly complementary models for a system of goods, and in these more general cases the marginal value of the integrating back approach is far more dubious. The key difficulty is that the restrictions on the constant of integration implied by weak complementarity will in general depend on the combination of other goods consumed in strictly positive quantities as well as their prices. To the degree that these restrictions depend on consumed goods' prices, restrictions on the constant of integration necessary for weak complementarity to hold will not exist.[12] Moreover, if these restrictions depend on the combinations of goods consumed but not their prices, the underlying preference ordering will be discontinuous in $\boldsymbol{x}$. As discussed in the following section, discontinuities in consumer preferences have behavioral implications that significantly call into question the usefulness of the integrating back approach to applied researchers.

In sum, these findings suggest that although some useful weakly complementary single equation specifications may arise from the integrating back approach, it does not represent a generic strategy that can consistently generate new and useful weakly complementary empirical demand system specifications. From a practitioner's perspective, the repackaging approach is far

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approaches. As discussed in the previous section, Vartia's numerical algorithm can thus be used with these specifications to derive exact welfare measures for price and quality changes.

[12] An example may clarify this point. Consider the demand system:

$$x_1 = \alpha_1(q_1) + \beta_{11}p_1 + \beta_{12}p_2$$
$$x_2 = \alpha_2(q_2) + \beta_{12}p_1 + \beta_{22}p_2$$

where the demand equation for the strictly positive Hicksian composite good is suppressed. Using the integrating back approach in this situation suggests that if $x_1 = 0$ but $x_2 > 0$, the constant of integration must equal $\alpha_1'(q_1)(\alpha_1(q)/\beta_{11} + (\beta_{12}/\beta_{11})p_1)$ for weak complementarity to hold, but this is internally inconsistent because the constant of integration is by assumption independent of prices.

easier to work with and holds greater promise in terms of generating useful empirical specifications.

3.3. The Discontinuity Approach

Bockstael, Hanemann and Strand (1986) suggested a third approach for developing weakly complementary demand models that exploits discontinuities in consumer preferences. Similar to traditional discrete choice models, the discontinuity approach's central building blocks are conditional indirect utility functions which are uniquely defined in terms of which of the *N* quality differentiated goods are consumed in strictly positive quantities. Since there are 2^N possible combinations of goods that are either consumed or not consumed, there are in principle 2^N conditional direct utility functions, $U_\omega(\bullet)$, where ω indexes regimes. Because each $U_\omega(\bullet)$ is by assumption only a function of the prices and quality attributes of the goods consumed in strictly positive quantities (i.e., $\partial U_\omega / \partial q_j = 0$ if $x_j = 0$), the preference ordering is consistent with weak complementary. The unconditional direct utility function takes the form:

$$(16) \qquad U(\boldsymbol{x}, \boldsymbol{Q}, z) = \max_{\omega \in \Omega} \{U_\omega(\boldsymbol{x}_\omega, \boldsymbol{Q}_\omega, z)\},$$

where Ω encompasses the full set of 2^N regimes, $\boldsymbol{x}_\omega$ is a subset of $\boldsymbol{x}$ with each element strictly positive, and $\boldsymbol{Q}_\omega$ only includes the quality attributes for the goods included in $\boldsymbol{x}_\omega$.

Although intuitive, the discontinuity approach suffers from a fundamental difficulty that casts doubt on its usefulness for applied researchers trying to develop weakly complementary demand models. In the context of a simple two good (*x*,*z*) model, Figure 2 illustrates the nature of the problem. The figure is based on the utility function

$$(17) \qquad U(x, \boldsymbol{q}, z) = \begin{cases} \Psi(\boldsymbol{q}) \ln(x+\theta) + \ln z & \text{if } x > 0 \\ \ln z & \text{if } x = 0 \end{cases},$$

where $\theta > 1$ and $\Psi(\boldsymbol{q}) > 0$. As in Figure 1, two indifference curves corresponding to the same level of utility but different levels of quality ($\boldsymbol{q}^0 > \boldsymbol{q}^1$) are drawn in (*x*,*z*) space. Notice that although both indifference curves intersect the *z* axis at point A, they do not "fan" from point A as in Figure 1. This feature arises because when the individual moves from consuming none of to a infinitesimal small quantity of *x* holding *z* and ***q*** constant, she receives a large welfare gain. As compensation for this gain, the individual is willing to forego a significant amount of *z*, which explains why point A is significantly above the points where the two indifference curves approach the *z* axis. This feature of preferences suggests that it is never rational for the individual to completely forego the consumption of *x*. The individual can always be made better off by consuming at least some infinitesimally small quantity of *x* than by completely foregoing it. Thus a strictly positive quantity of *x*, however small, becomes an essential component of a utility maximizing bundle, which is at odds with the non-essentiality condition required for weak complementarity to hold.

In principle the difficulties associated with the discontinuity approach can be avoided by placing additional structure on consumer preferences. One possibility involves imposing a minimum consumption level for *x*, say $\underline{x} > 0$, if any of the good is to be consumed at all. In Figure 2, such a minimum consumption level, in combination with a budget constraint corresponding to line 1, would imply that the individual would prefer to consume none of the good at all. Although this resolves the essentiality problem with the discontinuity approach, it implies a more complicated model of consumer choice that in practice would be more difficult to estimate. In particular, the necessary conditions for an individual to rationally choose not to consume *x* in this context are twofold: 1) $\Psi(\boldsymbol{q})/(\underline{x}+\theta) \le p/(y-p\underline{x})$ and 2) $\ln y > \Psi(\boldsymbol{q})\ln(\underline{x}+\theta)+\ln(y-px)$. If preferences were continuous (i.e.,

$U(x,\boldsymbol{q},z)=\Psi(\boldsymbol{q})\ln(x+\theta)+\ln z$), however, there would be only one necessary condition ($\Psi(\boldsymbol{q})/\theta \le p/y$). From a practitioner's perspective, the added complexity associated with deriving estimable empirical models consistent with conditions 1) and 2) are significantly greater than traditional continuous demand models. Thus the approach, while feasible in theory, is probably less useful in practice.

3.4 *Summary of Alternative Approaches*

The above discussion has several implications for applied research. Perhaps the most significant is that, among the three approaches considered, the repackaging approach offers the most helpful guidance to applied researchers wanting to develop weakly complementary demand models. The approach is also flexible and easy to implement when working within the primal framework. Moreover, the existing repackaging approaches that applied researchers have considered – i.e., the pure repackaging, cross-product repackaging, and the generalized translating approaches - by no means exhaust all of the possible structures that analysts can exploit.

4. Empirical Comparison – Alternative Specifications & Estimation Strategy

Having discussed the conceptual advantages of the alternative strategies in the previous section, a more practical question is whether they generate qualitatively different policy inference in an applied setting. As a first step toward answering this question, this section outlines the empirical specifications used to compare alternative approaches to developing weakly complementary specifications in the context of so-called "Kuhn-Tucker" models (Wales and Woodland (1983)), or continuous demand system models specified in the primal framework. Because the direct utility function is the point of departure for Kuhn-Tucker models, weakly complementary specifications derived via the integrating back approach are not considered. In addition, weakly complementary discontinuous specifications, which do not easily admit a closed-form likelihood function conditional on a vector of estimable parameters when minimum consumption thresholds are included, are also not considered. Thus all of the weakly complementary models that are compared fall within the repackaging approach. This implies that the empirical comparison is somewhat limited in scope, but the discussion in the previous section argued that the repackaging approach is by far the easiest to implement, most flexible, and most widely used of the three approaches. Moreover, the comparison encompasses examples of all approaches to developing weakly complementary demand models that have been previously used in multi-good Kuhn-Tucker demand system applications and is valuable to the degree that it informs applied researchers whether existing approaches generate qualitatively different policy inference.

All of the weakly complementary specifications included in the comparison are variations of the linear expenditure system:

(18) $$U(\boldsymbol{x},z)=\ln z+\sum_i\left[\Psi_i\ln(\phi_i x_i+\theta_i)+C_i\right],$$

where $[\Psi_i,\phi_i,\theta_i,C_i]$ are functions whose arguments vary across the alternative specifications.[13] The additively separable structure embedded in (18) restrictively implies that all goods are Hicksian substitutes and have non-negative Engel curves. For the purposes of evaluating the empirical implications of the alternative strategies for incorporating weak complementary under consideration, however, the additive separability assumption should not invalidate the comparison.

Seven separate specifications consistent with (18) are considered and summarized in Table 1. The first is the pure repackaging approach, while the second and third are variations of

[13] More general additively separable specifications used by von Haefen, Phaneuf, and Parsons (2004) were also considered and found to generate qualitatively similar results.

the generalized translating approach with the former employed by HKP. The fourth thru seventh specifications do not embed weak complementarity and are presented mainly for comparison purposes as well as to illustrate the problems arising with decomposition approaches. Although not presented here, an eighth cross-product repackaging specification was also considered. With this specification, however, quality adjusted prices were occasionally found to be negative. As discussed in section 3, such prices imply that the individual wishes to spend all of her income on quality adjusted goods. Due to the implausibility of this prediction, the specification was dropped from the comparison.

Because all seven specifications assume that each Ψ_i can be decomposed into two parts, Ψ_i^* and ε_i, where $\Psi_i = \exp(\Psi_i^* + \varepsilon_i)$, the first order conditions that implicitly define the optimal consumption bundle can be rewritten as:

(19) $$\varepsilon_i \le -\Psi_i^* + \ln(p_i/\phi_i) - \ln(y - \boldsymbol{p}^{\mathsf{T}}\boldsymbol{x}) + \ln(\phi_i x_i + \theta_i), \ \forall i .$$

Assuming that each ε_i can be treated as an iid draw from the type I extreme value distribution with common scale parameter $\mu > 0$, the likelihood of observing $\boldsymbol{x}$ conditional on a vector of estimable parameters is

(20) $$l(\boldsymbol{x}) = |\boldsymbol{J}| \prod_i \left[(\exp(-g_i/\mu)/\mu)^{1_{x_i>0}} \exp(-\exp(-g_i/\mu)) \right]$$

where g_i refers to the right hand side of (19) and $\boldsymbol{J}$ is the Jacobian of transformation. As noted by HKP, a notable feature of this likelihood is that the C_i functions do not enter. As a result, specifications 2, 4, and 6 as well as 3, 5, and 7, which differ only in terms of how the C_i functions are structured, are observationally equivalent in terms of estimation. This feature illustrates the point made earlier that weak complementarity is not a testable restriction and implies that only three separate specifications are estimated in this application.

To flexibly account for unobserved heterogeneity in preferences, all structural parameters are assumed to be normally distributed with unrestricted covariance matrix. This specification generalizes previous applications (e.g., von Haefen, Phaneuf, and Parsons, von Haefen (2004)) where for computational tractability only the parameters entering the Ψ_i functions were assumed to vary randomly across the population.[14] In all three estimated models, a total of 23 randomly distributed parameters enter the model, implying that 299 mean and covariance parameters must be estimated. Within a frequentist or classical framework, estimating such a large number of parameters using maximum simulated likelihood techniques (Gourieroux and Monfort (1996)) would represent a formidable if not prohibitively difficult econometric task.

To avoid these computational difficulties, the approach pursued in this paper is to abandon the frequentist paradigm and work within the Bayesian framework (Kim, Allenby, and Rossi (2002)). The conceptual and empirical differences between frequentist and Bayesian approaches are too numerous and subtle to summarize here, but the interested reader can consult Train (2003) for a detailed discussion. It suffices to say that while estimating all three specified models with the data described in the next section within the classical framework was confounded by computational and convergence difficulties, estimation was feasible in a single overnight run within the Bayesian framework for all three models. Moreover, as Train has pointed out, the Bernstein-von Misses theorem implies that the posterior mean Bayesian estimates, interpreted within a classical framework, are asymptotically equivalent to the maximum likelihood estimates assuming the analyst has correctly specified the data generating process. Thus, qualitative statistical inference should be similar whether one is working in a

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[14] Specifically, allowing only the Ψ_i parameters to vary randomly implies that the Jacobian of transformation is a function of only fixed parameters and thus need only be recomputed once per observation when simulating the likelihood function. With more general specifications, the analyst must calculate the Jacobian of transformation for every simulation and observation, which substantially increases the computational burden.

classical or Bayesian framework assuming one has correctly specified the data generating process and uses a sufficiently large data set.

In the Bayesian estimation framework, the analyst is assumed to have prior beliefs about the values that a set of parameters β can take. These beliefs can be formalized into a prior probability distribution, $p(\beta)$. A set of observations, $\boldsymbol{x}$, that is generated by a process that depends on β, are then observed, and the likelihood of observing $\boldsymbol{x}$ conditional on alternative values of β, $l(\boldsymbol{x} \mid \beta)$, can be constructed. Given the likelihood, the analyst updates her prior beliefs about β. By Bayes' rule, her updated beliefs can be summarized by the posterior distribution, $p(\beta \mid \boldsymbol{x})$. Because $p(\beta \mid \boldsymbol{x})$ often does not have a simple structure whose moments can be easily summarized, Bayesian econometricians have developed a number of sophisticated econometric techniques to simulate from $p(\beta \mid \boldsymbol{x})$.

In this paper, a Gibbs sampling routine in combination with an adaptive Metropolis-Hastings algorithm is used to simulate from the posterior distribution of the continuous demand system's structural parameters. The estimation strategy was first developed by Allenby and Lenk (1994) in the context of mixed logit models but is generic to any situation where the conditional likelihood function has a closed form solution as in equation (20). Diffuse priors for all parameters are assumed to limit the impact the prior distributions have on posterior inference. The basic assumptions and steps of the algorithm are sketched in the technical appendix, and the interested reader should consult Train (2003) for a more detailed discussion.

5. Data

The data set used in the empirical application comes from the 1997 Mid-Atlantic Beach Survey conducted by researchers at the University of Delaware. The survey's objective was to measure Delaware residents' beach utilization in the Mid-Atlantic region and its interaction with beach management policies. A county-stratified random sample of Delaware residents were questioned about their visits to 62 ocean beaches in New Jersey, Delaware, Maryland, and Virginia during the past year. After data cleaning, a total of 540 completed surveys remained and are the focus of the empirical application here. *PCMiler* was used to calculate round trip travel times and distances from all 540 individuals' resident zip codes to the 62 beaches, and travel cost measures were constructed assuming that travel time could be valued at the wage rate and the out-of-pocket cost of travel was $0.35 per mile.

Several beach attributes were collected and are used to represent the quality dimension of beaches in the region. Summary statistics for these characteristics as well as demographic and trip taking information are presented in Table 2. For further details on the data used in this analysis, the interested reader should consult Parsons, Massey, and Tomasi (1999) and von Haefen, Phaneuf, and Parsons (2004).

6. Estimation Results

Table 3 reports a selected set of posterior mean and variance parameter estimates for the alternative specifications estimated.[15] The estimates suggest a number of qualitative conclusions. A comparison of mean posterior conditional log-likelihood values suggest that the statistical fits of the specifications where site quality enters through the ϕ_i and θ_i functions are virtually indistinguishable and noticeably better than the fit associated with the specification where site quality enters through the Ψ_i functions. Since each model has the same number of parameters, this finding would suggest that specifications 1, 3, 5, and 7 may be more reliable for policy purposes based on purely statistical grounds. In addition, a comparison across specifications of all mean posterior variances suggests that there is considerable heterogeneity

[15] The remainder of the estimates are available from the author upon request.

across the population. In particular, the magnitudes of the posterior mean and variance point estimates for all quality attributes suggests that there is a diversity of opinions with respect to whether the alternative attributes make a site more or less attractive to visit.

Looking more closely at the quality parameters, one notices that the sign of the posterior mean estimates for the specification where quality enters through the θ_i function are generally opposite from the other specifications. Given how θ_i, ϕ_i, and Ψ_i enter preferences, however, these opposite signs are consistent with the notion that an increase in one of site *i*'s quality attributes will have the same directional impact on aggregate consumer demand. Moreover, for the weakly complementary specifications and specification 4, the directional impact on aggregate consumer utility will be the same, but the directional impact will differ for the non-weakly complementary specifications 5 and 7. This latter fact will be helpful in explaining the welfare results reported in the next section.

7. Welfare Results

7.1 Policy Scenario

The policy scenario used to evaluate the alternative specifications considers the erosion of all eleven developed (i.e., non-park) beaches in the Delaware/Maryland/Virginia area to widths of seventy-five feet or less.[16] Such an outcome might result if current state-sponsored beach nourishment programs were abandoned. For this scenario, the key parameter from the econometric models is the coefficient on the narrow beach dummy variable. Consistently across the alternative specifications, the parameter estimates suggested that *ceteris paribus* a narrow beach (i.e., a beach with width of 75 feet and less) was less frequently visited by Delaware residents on average, although some individuals found narrow beaches more attractive.

[16] In 1997 one of the beaches was less than 75 feet in width, and none were more than 200 feet in width. For further details on this scenario, see von Haefen, Phaneuf, and Parsons (2004).

7.2 Welfare Results

Summary statistics for the posterior distribution of the expected compensation surplus from the alternative specifications are presented in Table 4, and the details on how these estimates were constructed are reported in the technical appendix. One of the most striking results is that the mean estimates in panel A are similar in magnitude across specifications 1, 2, and 3, the weakly complementary models. This finding reflects the fact that all three specificationss predict similar changes in total trips for the policy scenario (-1.9541, -2.0150, and -1.9405, respectively) and rule out nonuse values. Collectively, they suggest that welfare estimates are relatively robust to the alternative weakly complementary repackaging specifications that have been previously considered in valuation studies.

Turning to specifications 4 thru 7 that do not assume weak complementarity holds but are behaviorally equivalent to specifications 2 and 3, three sets of results are reported. Beginning with the total value estimates in panel A, one finds qualitatively different welfare estimates relative to the corresponding weakly complementary specifications. For example, specification 6's mean estimate is more than triple the magnitude of specification 2's mean estimate, and specification 7's estimate is over 290 times the absolute magnitude of specification 3's and the opposite sign. As suggested in the previous section, the positive sign of specification 7's mean estimate (as well as specification 5's) arises because of the model's counterintuitive and highly questionable prediction that beach erosion causes utility to rise while demand falls. The estimates for specifications 4 and 5 are similar qualitatively to specifications 6 and 7's estimates although less extreme. Collectively, the estimates in panel A highlight that in general specifications that assume weak complementary will imply qualitatively different welfare estimates from those that do not.

Panels B and C of Table 4 report welfare estimates for specifications 4 thru 7 that exploit variations of the decomposition approach suggested by HKP. As discussed in section 2, these decomposition approaches attempt to isolate the use component of value by subtracting from the total value estimates the nonuse component. Two important judgments are required with this approach: 1) which non-weakly complementary utility function to use and 2) whether the nonuse component of value is defined when the demands for just the affected or all sites are set to zero.[17] Comparing welfare estimates between specifications 4 and 6 and 5 and 7, respectively, suggest the importance of the former assumption, and two sets of results are presented in Table 4 to suggest the importance of the latter – one where nonuse values are measured when only the affected sites' demands are restricted to zero (panel B) and another where all sites' demands equal zero before and after the quality change (panel C). Collectively, these estimates suggest that in general both sets of judgments can have important implications for policy.

Finally, some simulations and algebra not reported here show that the decomposition estimates reported in panels B and C would have generated identical welfare measures to the weakly complementary specifications in panel A if preferences were quasilinear in addition to being additively separable. Thus the reason why divergences among these estimates are found in this application are due to income effects embedded in the linear expenditure system. More generally, to the degree that preferences are not additively separable and quasilinear, one should expect at least some differences between welfare estimates from weakly complementary models and use component welfare estimates from non-weakly complementary models.

8. Conclusion

This paper has conceptually and empirically compared alternative empirical strategies for incorporating weak complementarity into continuous demand system models. Three main

[17] HKP likely appreciated these

implications for future research can be drawn from the paper's findings. First, the repackaging approach offers the most guidance to applied researchers when developing weakly complementary demand models. The approach is relatively easy to implement within the primal framework and offers the researcher considerable flexibility that remains largely untapped at present. Second, the empirical results reported in this paper suggest that among the existing menu of repackaging approaches, qualitatively similar policy implications can be expected if preferences are additively separable, but future research should study their properties in the context of non-additively separable models and more general repackaging approaches before conclusions are drawn with confidence. Third and finally, the decomposition approaches suggested by HKP for specifications that do not assume weak complementarity will in general depend on arbitrary assumptions about how to differentiate use from nonuse values. The empirical results suggested that use-related welfare estimates can be very sensitive to these judgments.

One final point is worth emphasizing in closing. Although this paper has relied upon intuitive and practical reasons to suggest why analysts should develop demand system models that are consistent with weak complementarity when only revealed preference data are available, it remains a strong and arbitrary restriction. As HKP have argued, future research should attempt to combine revealed and stated preference data in ways that allow the quality attributes of goods to enter preferences more flexibly when nonuse values are likely present (Cameron (1992)). By doing so, the analyst would in principle be able to test for weak complementarity and, when rejected, recover nonuse values. Moreover, such studies could suggest to applied researchers working with just revealed preference data when welfare estimates derived from weakly complementary models are likely to capture total value. At present, analysts must base their assessment of the appropriateness of weak complementarity entirely on intuition, but empirical evidence suggesting the conditions under which the restriction is likely to hold can only improve the credibility of valuation estimates.

Table 1
Alternative Specifications

Specification	Restrictions
1) Pure Repackaging	$\Psi_i = \exp(\tau^\top \mathbf{s} + \varepsilon_i),\ \phi_i = \exp(\delta^\top \mathbf{q}_i),\ \theta_i = \exp(\theta^*),\ C_i = 0,\ \forall i$
2) Generalized Translating #1	$\Psi_i = \exp(\tau^\top \mathbf{s} + \delta^\top \mathbf{q}_i + \varepsilon_i),\ \phi_i = 1,\ \theta_i = \exp(\theta^*),\ C_i = -\theta^* \exp(\tau^\top \mathbf{s} + \delta^\top \mathbf{q}_i + \varepsilon_i),\ \forall i$
3) Generalized Translating #2	$\Psi_i = \exp(\tau^\top \mathbf{s} + \varepsilon_i),\ \phi_i = 1,\ \theta_i = \exp(\theta^* + \delta^\top \mathbf{q}_i),\ C_i = -(\theta^* + \delta^\top \mathbf{q}_i)\exp(\tau^\top \mathbf{s} + \varepsilon_i),\ \forall i$
4) No Weak Complementarity #1	$\Psi_i = \exp(\tau^\top \mathbf{s} + \delta^\top \mathbf{q}_i + \varepsilon_i),\ \phi_i = 1,\ \theta_i = \exp(\theta^*),\ C_i = 0,\ \forall i$
5) No Weak Complementarity #2	$\Psi_i = \exp(\tau^\top \mathbf{s} + \varepsilon_i),\ \phi_i = 1,\ \theta_i = \exp(\theta^* + \delta^\top \mathbf{q}_i),\ C_i = 0,\ \forall i$
6) No Weak Complementarity #3	$\Psi_i = \exp(\tau^\top \mathbf{s} + \delta^\top \mathbf{q}_i + \varepsilon_i),\ \phi_i = 1,\ \theta_i = \exp(\theta^*),\ C_i = (\exp(\tau^\top \mathbf{s} + \delta^\top \mathbf{q}_i + \varepsilon_i)\exp(\theta^*))^2,\ \forall i$
7) No Weak Complementarity #4	$\Psi_i = \exp(\tau^\top \mathbf{s} + \varepsilon_i),\ \phi_i = 1,\ \theta_i = \exp(\theta^* + \delta^\top \mathbf{q}_i),\ C_i = (\exp(\tau^\top \mathbf{s} + \varepsilon_i)\exp(\theta^* + \delta^\top \mathbf{q}_i))^2,\ \forall i$

where:

$\mathbf{s}$ ~ vector of demographic variables

ε_i ~ unobserved heterogeneity distributed iid type I extreme value

$[\tau, \delta, \theta^*]$ ~ random parameters

For all specifications to insure $\mu > 0$, μ^* is estimated where $\mu = \exp(\mu^*)$.

Table 2
Sample Demographics and Beach Quality Characteristics

Variable	*Description*	*Mean (std. err.)*[1]
Sample demographics (540 respondents)		
Ln(age)	Natural log of respondent age	3.821 (0.334)
Kids under 10	Respondent has kids under 10 (0/1)	0.267
Kids 10 to 16	Respondent has kids between 10 and 16 (0/1)	0.206
Vacation property in DE	Respondent owns vacation home in DE (0/1)	0.037
Retired	Respondent is retired (0/1)	0.248
Student	Respondent is student (0/1)	0.0481
Income	Household annual income	49,944 (30,295)
Trips	Total visits for day trips to all sites	9.776 (14.07)
Sites visited	Number of beaches visited during 1997	2.691 (3.199)
Site quality characteristics		
Beach length	Length of beach in miles	0.624 (0.872)
Boardwalk	Boardwalk with shops and attractions (0/1)	0.403
Amusements	Amusement park near beach (0/1)	0.129
Private/limited access	Access limited (0/1)	0.258
Park	State or federal park or wildlife refuge (0/1)	0.097
Wide beach	Beach is more than 200 feet wide (0/1)	0.258
Narrow beach	Beach is less than 75 feet wide (0/1)	0.145
Atlantic City	Atlantic city indicator (0/1)	0.0161
Surfing	Recognized as good surfing location (0/1)	0.355
High rise	Highly developed beach front (0/1)	0.242
Park within	Part of the beach is a park area (0/1)	0.145
Facility	Bathrooms available (0/1)	0.484
Parking	Public parking available (0/1)	0.452
New Jersey	New Jersey beach indicator (0/1)	0.742
Travel cost	Travel Cost = (round trip travel distance) ×($0.35) + (round trip travel time)×(wage rate)	$118.42 (51.67)[2]

[1] Summary statistics for household variables are means (standard errors) over the 540 individuals. Summary statistics for site variables are means (standard errors) over the 62 sites.
[2] This statistic is the mean (standard error) of each individual's mean round trip travel cost. Each individual in the sample has a unique travel cost associated with visiting each of the 62 sites. Since prices are functions of distance, there is substantial variability in travel costs both across individuals and sites.

Table 3
Some Posterior Parameter Estimates[1]

	Specification 1		*Specifications 2 & 4*		*Specifications 3 & 5*	
Site i's quality attributes enters through	ϕ_i		Ψ_i		θ_i	
Conditional Log-Likelihood	-4,599.7 (62.153)[2]		-5,207.15 (60.793)		-4,603.2 (66.576)	
	mean	*variance*	*mean*	*variance*	*mean*	*variance*
Quality parameters						
Beach length	0.0581	0.4585	-0.0103	0.4349	0.0018	0.4802
	(0.0709)	(0.0582)	(0.0707)	(0.0682)	(0.0735)	(0.0675)
Boardwalk	-0.0229	1.1090	-0.0309	1.0738	0.2965	0.9728
	(0.1029)	(0.2088)	(0.1768)	(0.2159)	(0.1245)	(0.2560)
Amusements	1.9796	1.9766	1.9230	1.9090	-2.0466	2.0519
	(0.1432)	(0.3464)	(0.1531)	(0.4137)	(0.1397)	(0.4969)
Private/limited access	-1.1182	2.1227	-1.1681	1.7125	1.0523	1.7836
	(0.1377)	(0.4655)	(0.1601)	(0.3748)	(0.2194)	(0.4416)
Park	0.1717	2.0334	0.2291	1.9011	-.0203	2.4524
	(0.1426)	(0.4349)	(0.1907)	(0.4791)	(0.1799)	(0.5895)
Wide beach	-0.8097	1.3359	-0.8193	1.1916	0.7449	1.2932
	(0.1122)	(0.1991)	(0.1163)	(0.2493)	(0.1130)	(0.2182)
Narrow beach	-1.6687	1.8250	-1.4698	1.5513	1.6029	1.6742
	(0.2509)	(0.4184)	(0.2464)	(0.4011)	(0.2313)	(0.4624)
Atlantic City	1.3803	2.3706	1.8875	1.6759	-1.4984	2.8793
	(0.2340)	(0.7300)	(0.2219)	(0.3872)	(0.3202)	(0.6266)
Surfing	0.6470	1.3630	0.6542	1.2675	-0.6109	1.3923
	(0.1023)	(0.2420)	(0.1122)	(0.2419)	(0.1007)	(0.2643)
High rise	-0.6415	1.9916	-0.4125	1.3305	0.6171	2.1240
	(0.1399)	(0.4108)	(0.1389)	(0.2525)	(0.1651)	(0.3529)
Park within	1.6657	2.6970	1.8685	2.1328	-1.6215	3.2545
	(0.2286)	(0.5480)	(0.3904)	(0.4677)	(0.2796)	(0.7803)
Facility	-0.3740	0.7551	-0.3660	0.8591	0.4977	0.8215
	(0.1205)	(0.1236)	(0.1339)	(0.1870)	(0.1220)	(0.1430)
Parking	0.6060	1.1607	0.9368	1.1495	-0.9775	1.1368
	(0.1475)	(0.2717)	(0.2221)	(0.3022)	(0.1172)	(0.2275)
New Jersey	-2.9221	2.5998	-3.6821	2.9778	3.1803	2.9481
	(0.1688)	(0.4670)	(0.3027)	(0.8145)	(0.2964)	(0.6932)
Misc. parameters						
$\theta^* = \ln \theta$	3.0409	2.3932	0.5929	0.3789	3.0316	2.3384
	(0.1363)	(0.3712)	(0.0749)	(0.0464)	(0.1365)	(0.3811)
$\mu^* = \ln \mu$	-0.8975	0.3133	-0.4841	0.2954	-0.8764	0.3128
	(0.0563)	(0.0394)	(0.0583)	(0.0356)	(0.0706)	(0.0417)

[1] All estimates generated with 50,000 Gibbs sampling iterations. Simulations from the first 25,000 iteration were discarded as burn-in and every 10th simulation thereafter was used in constructing these estimates.
[2] Standard errors across the 2,500 simulations used to construct the point estimates are reported in parentheses. As Train (2003) notes, these can be interpreted as the asymptotic standard errors estimates within a frequentist perspective.

Table 4
Posterior Expected Hicksian Consumer Surplus Estimates for Lost Beach Width at All Delaware/Maryland/Virginia Developed Beaches

***Panel A** – Total Value Estimates*	*Mean*[1]	*Std. err.*	*95% credible set*
1) Pure Repackaging	-$94.42	18.12	[-$125.69, -$54.52]
2) Gen. Trans. #1	-$92.95	57.35	[-$159.65, $54.64]
3) Gen. Trans. #2	-$95.20	17.79	[-$132.04, -$58.47]
4) No Weak Comp. #1	-$255.77	84.63	[-$382.31, -$59.59]
5) No Weak Comp. #2	$2,773.80	353.77	[$2,167, $3,561]
6) No Weak Comp. #3	-$327.88	123.59	[-$567.87, -$68.38]
7) No Weak Comp. #4	$27,831.57	3,589.4	[$20,178, $34,780]
***Panel B** - Decomposition Approaches – nonuse value component of total value arising when demands at only affected sites are restricted to zero before and after quality change*			
4a) No Weak Comp. #1 – Use Value	-$96.32	52.66	[-$159.77, $33.58]
5a) No Weak Comp. #2 – Use Value	-$94.87	21.80	[-$134.66, -$48.45]
6a) No Weak Comp. #3 – Use Value	-$98.21	48.40	[-$159.62, $20.35]
7a) No Weak Comp. #4 – Use Value	-$237.08	88.13	[-$441.00, -$115.29]
***Panel C** - Decomposition Approaches – nonuse value component of total value arising when demands at all sites are restricted to zero before and after quality change*			
4b) No Weak Comp. #1 – Use Value	-$97.67	52.81	[-$162.00, $34.97]
5b) No Weak Comp. #2 – Use Value	-$166.44	29.79	[-$218.73, -$101.02]
6b) No Weak Comp. #3 – Use Value	-$139.07	70.08	[-$284.56, -$3.18]
7b) No Weak Comp. #4 – Use Value	-$609.23	223.24	[-$1,115, -$318]

[1] Expectations generated with 4 simulations for each of the 2,500 posterior parameter draws. Sampling weights implied by county stratified sampling designed used in all estimates.

Figure 1
Weak Complementarity Graphically (from Smith and Banzhaf (2004))

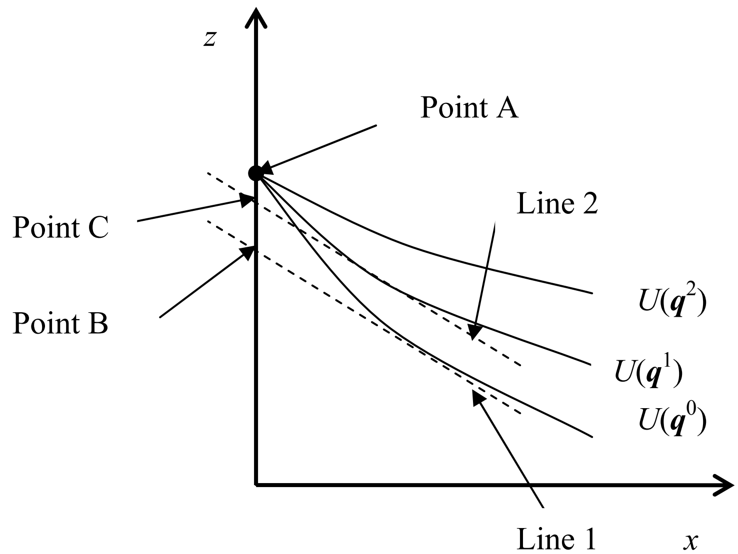
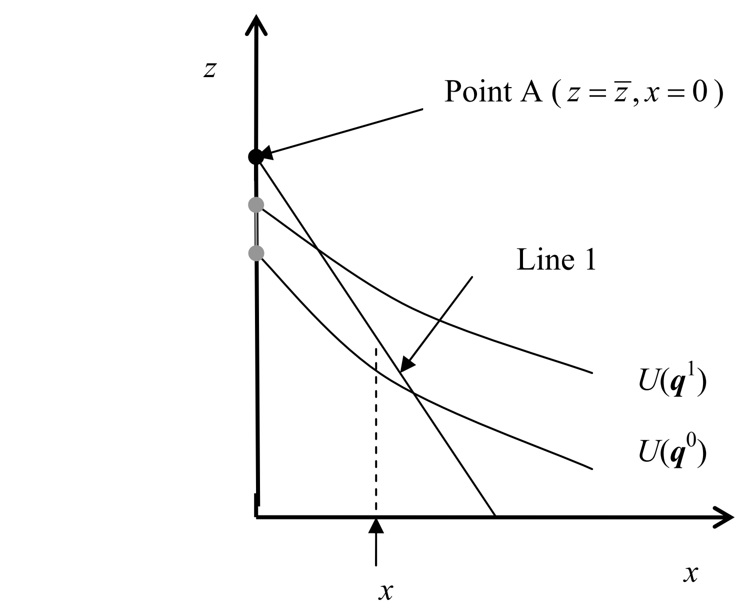


Figure 2
Discontinuity Approach to Imposing Weak Complementarity



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Technical Appendix

(Not meant for publication but available upon request)

A.1 - Integrating Back Approach

This appendix summarizes the results from an investigation into whether the integrating back approach can be used to develop closed form, weakly complementary preference specifications for 24 alternative linear, log-linear, and semi-log single equation models. The demand, expenditure, and expenditure share specifications considered are listed in Tables 1d, 1e, and 1s.

For each model, the initial tables report generic information about the structure of the quasi-expenditure and Hicksian choke price functions. In terms of price *p* and the constant of integration *k*, Tables 2d, 2e, and 2s list the corresponding structure of the closed form quasi-expenditure functions, $\tilde{E}(p,k)$, when available, and Tables 3d, 3e, and 3s report the closed form structure of the Hicksian choke prices, $\hat{p}(k)$, when they exist. Tables 4d, 4e, and 4s report the structure of the quasi-expenditure functions evaluated at the Hicksian choke price, $\tilde{E}(\hat{p}(k),k)$ when they have closed forms.

Tables 5d, 5e, and 5s report the necessary restrictions on the constant of integration, $k(q,\tilde{k})$, implied by weak complementarity when quality enters linearly through the constant term (i.e., $\alpha(q)=\alpha^{*}+\delta q$), and Tables 6d, 6e, and 6s report the corresponding expenditure function structures, $E(p,q,\bar{U})$, where $\tilde{k}=\bar{U}$. Similarly, Tables 7d, 7e, and 7s report the necessary restrictions on $k(q,\tilde{k})$ implied by weak complementarity when quality enters linearly through the price coefficient (i.e., $\beta(q)=\beta^{*}+\delta q$), and Tables 8d, 8e, and 8s report the implied structure of $E(p,q,\bar{U})$ where $\tilde{k}=\bar{U}$. Finally, Tables 9d, 9e, and 9s report the necessary restrictions on $k(q,\tilde{k})$ implied by weak complementarity when quality enters linearly through the income coefficient (i.e., $\gamma(q)=\gamma^{*}+\delta q$), and Tables 10d, 10e, and 10s report the implied structure of $E(p,q,\bar{U})$ where $\tilde{k}=\bar{U}$.

The main finding from these tables is that 36 of the 72 specifications considered can be linked to closed form weakly complementary demand specifications without placing implausible restrictions on the constant, price, and/or income coefficient. These specifications correspond to all three variations of the (x1), (x5), (x6), (x7), (x8), (e1), (e3), (e7), (e8), (s2), (s4), and (s8) models. Moreover, 10 of the 36 specifications shown to be consistent with weak complimentarity via the integrating back approach could have also been derived by either the pure or cross-product repackaging approaches (i.e., the (x1), (x5), (x6), (x7), (x8), (e3), (e7), (e8), (s4), and (s8) models where quality enters through the constant term). 4 of the 36 specifications that could not be linked back to closed form weakly complimentary preferences have quality entering in ways that are consistent with either the pure or cross-product repackaging approaches. In these cases, Vartia's (198?) numerical algorithm can be used with these specifications to derive exact welfare measures for price and quality changes.

Table 1d
Demand Specifications

Model	*Specification*[1]
(x1)	$x(p,y)=\alpha+\beta p+\gamma y$
(x2)	$x(p,y)=\alpha+\beta p+\gamma \ln y$
(x3)	$x(p,y)=\alpha+\beta \ln p+\gamma y$
(x4)	$x(p,y)=\alpha+\beta \ln p+\gamma \ln y$
(x5)	$x(p,y)=\exp(\alpha+\beta p+\gamma y)$
(x6)	$x(p,y)=\exp(\alpha+\beta p+\gamma \ln y)$
(x7)	$x(p,y)=\exp(\alpha+\beta \ln p+\gamma y)$
(x8)	$x(p,y)=\exp(\alpha+\beta \ln p+\gamma \ln y)$

[1] Although not necessary for the Slutsky matrix to be negative semi-definite, it is assumed throughout that $\beta \leq 0$ for expositional ease. No sign restrictions are placed on α and γ unless otherwise noted, however.

Table 2d
Quasi-Expenditure Functions

Model	*Conditions*	*Quasi-Expenditure Function*[1,2,3]
(x1)	$\gamma \neq 0$	$\tilde{E}(p,k) = -(\alpha + \beta p + \beta / \gamma) / \gamma + \exp(\gamma p)k$
(x1) & (x2)	$\gamma = 0$	$\tilde{E}(p,k) = \alpha p + (\beta / 2) p^2 + k$
(x2)	$\gamma \neq 0$	No closed form
(x3)	$\gamma \neq 0 \,\&\, \beta = 0$	$\tilde{E}(p,k) = -\alpha / \gamma + \exp(\gamma p)k$
(x3) & (x4)	$\gamma = 0 \,\&\, \beta \neq 0$	$\tilde{E}(p,k) = \alpha p + \beta p(\ln p - 1) + k$
(x3)	$\gamma \neq 0 \,\&\, \beta \neq 0$	No closed form
(x4)	$\gamma \neq 0$	No closed form
(x5)	$\gamma \neq 0 \,\&\, \beta \neq 0$	$-\exp(-\gamma \tilde{E}(p,k)) / \gamma = \exp(\alpha + \beta p) / \beta + k$
(x5)	$\gamma \neq 0 \,\&\, \beta = 0$	$-\exp(-\gamma \tilde{E}(p,k)) / \gamma = \exp(\alpha) p + k$
(x5)	$\gamma = 0 \,\&\, \beta \neq 0$	$\tilde{E}(p,k) = \exp(\alpha + \beta p) / \beta + k$
(x6)	$\gamma \neq 1 \,\&\, \beta \neq 0$	$\tilde{E}(p,k)^{(1-\gamma)} / (1-\gamma) = \exp(\alpha + \beta p) / \beta + k$
(x6)	$\gamma \neq 1 \,\&\, \beta = 0$	$\tilde{E}(p,k)^{(1-\gamma)} / (1-\gamma) = \exp(\alpha) p + k$
(x6)	$\gamma = 1 \,\&\, \beta \neq 0$	$\tilde{E}(p,k) = \exp(\exp(\alpha + \beta p) / \beta)k$
(x6)	$\gamma = 1 \,\&\, \beta = 0$	$\tilde{E}(p,k) = \exp(\exp(\alpha) p)k$
(x7)	$\gamma \neq 0 \,\&\, \beta \neq -1$	$-\exp(-\gamma \tilde{E}(p,k)) / \gamma = \exp(\alpha) p^{1+\beta} / (1+\beta) + k$
(x7)	$\gamma \neq 0 \,\&\, \beta = -1$	$-\exp(-\gamma \tilde{E}(p,k)) / \gamma = \exp(\alpha) \ln p + k$
(x7)	$\gamma = 0 \,\&\, \beta \neq -1$	$\tilde{E}(p,k) = \exp(\alpha) p^{1+\beta} / (1+\beta) + k$
(x7)	$\gamma = 0 \,\&\, \beta = -1$	$\tilde{E}(p,k) = \exp(\alpha) \ln p + k$
(x8)	$\gamma \neq 1 \,\&\, \beta \neq -1$	$\tilde{E}(p,k)^{(1-\gamma)} / (1-\gamma) = \exp(\alpha) p^{1+\beta} / (1+\beta) + k$
(x8)	$\gamma \neq 1 \,\&\, \beta = -1$	$\tilde{E}(p,k)^{(1-\gamma)} / (1-\gamma) = \exp(\alpha) \ln p + k$
(x8)	$\gamma = 1 \,\&\, \beta \neq -1$	$\tilde{E}(p,k) = \exp(\exp(\alpha) p^{1+\beta} / (1+\beta))k$
(x8)[4]	$\gamma = 1 \,\&\, \beta = -1$	$\tilde{E}(p,k) = p^{\exp(\alpha)} k$

[1] k is the constant of integration.
[2] Trivial cases where $\beta = \gamma = 0$ are ignored.
[3] Some of the quasi-expenditure functions are written implicitly to highlight the fact that they are not defined for all possible normalized price, normalized income and model parameter values.
[4] For economic coherence, $\alpha < 0$.

Table 3d
Hicksian Choke Prices

Model	*Conditions*	*Hicksian Choke Price*[1,2]
(x1)	$\gamma \neq 0$	$\exp(\gamma \hat{p}(k)) = \beta / (\gamma^2 k)$
(x1) & (x2)	$\gamma = 0$	$\hat{p}(k) = -\alpha / \beta$
(x3)	$\gamma < 0 \,\&\, \beta = 0$	$\hat{p}(k) = +\infty$
(x3)	$\gamma > 0 \,\&\, \beta = 0$	Does not exist
(x3) & (x4)	$\gamma = 0 \,\&\, \beta \neq 0$	$\hat{p}(k) = \exp(-\alpha / \beta)$
(x5), (x6), (x7), & (x8)	$\beta \neq 0$	$\hat{p}(k) = +\infty$

[1] Specifications where no closed form solution for the quasi-expenditure function exists are not considered.
[2] Some of the Hicksian choke prices are written implicitly to highlight the fact that they are not defined for all possible normalized price, normalized income and model parameter values.

Table 4d
Quasi-Expenditure Functions Evaluated at Hicksian Choke Prices

Model	*Conditions*	*Quasi-Expenditure Function*[1,2]
(x1)	$\gamma \neq 0$	$\tilde{E}(\hat{p}(k),k) = -\alpha/\gamma - (\beta/\gamma^2)\ln(\beta/(\gamma^2 k))$
(x1) & (x2)	$\gamma = 0$	$\tilde{E}(\hat{p}(k),k) = -\alpha^2/(2\beta) + k$
(x3)	$\gamma < 0 \,\&\, \beta = 0$	$\tilde{E}(\hat{p}(k),k) = -\alpha/\gamma$
(x3) & (x4)	$\gamma = 0, \beta \neq 0$	$\tilde{E}(\hat{p}(k),k) = -\beta\exp(-\alpha/\beta) + k$
(x5)	$\gamma \neq 0 \,\&\, \beta \neq 0$	$-\exp(-\gamma\tilde{E}(\hat{p}(k),k))/\gamma = k$
(x5)	$\gamma \neq 0 \,\&\, \beta = 0$	$-\exp(-\gamma\tilde{E}(\hat{p}(k),k))/\gamma = +\infty$
(x5)	$\gamma = 0 \,\&\, \beta \neq 0$	$\tilde{E}(\hat{p}(k),k) = k$
(x6)	$\gamma \neq 1 \,\&\, \beta \neq 0$	$\tilde{E}(\hat{p}(k),k)^{1-\gamma}/(1-\gamma) = k$
(x6)	$\gamma \neq 1 \,\&\, \beta = 0$	$\tilde{E}(\hat{p}(k),k)^{1-\gamma}/(1-\gamma) = +\infty$
(x6)	$\gamma = 1 \,\&\, \beta \neq 0$	$\tilde{E}(\hat{p}(k),k) = k$
(x6)	$\gamma = 1 \,\&\, \beta = 0$	$\tilde{E}(\hat{p}(k),k) = +\infty$
(x7)	$\gamma \neq 0 \,\&\, \beta < -1$	$-\exp(-\gamma\tilde{E}(\hat{p}(k),k))/\gamma = k$
(x7)	$\gamma \neq 0 \,\&\, \beta \geq -1$	$-\exp(-\gamma\tilde{E}(\hat{p}(k),k))/\gamma = +\infty$
(x7)	$\gamma = 0 \,\&\, \beta < -1$	$\tilde{E}(\hat{p}(k),k) = k$
(x7)	$\gamma = 0 \,\&\, \beta \geq -1$	$\tilde{E}(\hat{p}(k),k) = +\infty$
(x8)	$\gamma \neq 1 \,\&\, \beta < -1$	$\tilde{E}(\hat{p}(k),k)^{1-\gamma}/(1-\gamma) = k$
(x8)	$\gamma \neq 1 \,\&\, \beta \geq -1$	$\tilde{E}(\hat{p}(k),k)^{1-\gamma}/(1-\gamma) = +\infty$
(x8)	$\gamma = 1 \,\&\, \beta < -1$	$\tilde{E}(\hat{p}(k),k) = k$
(x8)	$\gamma = 1 \,\&\, \beta \geq -1$	$\tilde{E}(\hat{p}(k),k) = +\infty$

[1] Specifications where no closed form quasi-expenditure function or Hicksian choke price exists are not considered.
[2] For all models, Table 4d assumes that the parameter and constant of integration values are such that closed form, strictly positive solutions for the Hicksian choke prices (implicitly) defined in Table 3d exist.

Table 5d

Weak Complementarity's Implications for $k(q,\tilde{k})$ when $\alpha(q)=\alpha^{*}+\delta q$

Model	*Conditions*	*Constant of Integration*[1]
(x1)	$\gamma \neq 0$	$k(q,\tilde{k})=\exp((\gamma/\beta)(\delta q+\gamma\tilde{k}))$
(x1) & (x2)	$\gamma = 0$	$k(q,\tilde{k})=(2\alpha^{*}\delta q+(\delta q)^{2})/(2\beta)+\tilde{k}$
(x3)	$\gamma<0\ \&\ \beta=0$	Weak complementarity not satisfied
(x3) & (x4)	$\gamma=0,\beta\neq 0$	$k(q,\tilde{k})=\beta\exp(-(\alpha^{*}+\delta q)/\beta)+\tilde{k}$
(x5) & (x6)	$\beta \neq 0$	$k(q,\tilde{k})=\tilde{k}$
(x7) & (x8)	$\beta<-1$	$k(q,\tilde{k})=\tilde{k}$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered.

Table 6d

Weakly Complementary Expenditure Functions when $\alpha(q)=\alpha^{*}+\delta q$ & $\tilde{k}=\bar{U}$

Model	*Conditions*	*Quasi-Expenditure Function*[1]
(x1)	$\gamma \neq 0$	$E(p,q,\bar{U})=-(\alpha^{*}+\beta(p+\delta/\beta)q+\beta/\gamma)/\gamma+\exp(\gamma(p+(\delta/\beta)q)$ $+(\gamma^{2}/\beta)\bar{U})$
(x1) & (x2)	$\gamma = 0$	$E(p,q,\bar{U})=\alpha^{*}(p+(\delta/\beta)q)+(\beta/2)(p+(\delta/\beta)q)^{2}+\bar{U}$
(x3) & (x4)	$\gamma=0\ \&\ \beta\neq 0$	$E(p,q,\bar{U})=(\alpha^{*}+\delta q)p+\beta p(\ln p-1)+\beta\exp(-(\alpha^{*}+\delta q)/\beta)+\bar{U}$
(x5)	$\gamma\neq 0\ \&\ \beta\neq 0$	$-\exp(-\gamma E(p,q,\bar{U}))/\gamma=\exp(\alpha^{*}+\beta(p+(\delta/\beta)q)/\beta+\bar{U}$
(x5)	$\gamma=0\ \&\ \beta\neq 0$	$E(p,q,\bar{U})=\exp(\alpha^{*}+\beta(p+(\delta/\beta)q))/\beta+\bar{U}$
(x6)	$\gamma\neq 1\ \&\ \beta\neq 0$	$E(p,q,\bar{U})^{(1-\gamma)}/(1-\gamma)=\exp(\alpha^{*}+\beta(p+(\delta/\beta)q))/\beta+\bar{U}$
(x6)	$\gamma=1\ \&\ \beta\neq 0$	$E(p,q,\bar{U})=\exp(\exp(\alpha^{*}+\beta(p+(\delta/\beta)q))/\beta)\bar{U}$
(x7)	$\gamma\neq 0\ \&\ \beta<-1$	$-\exp(-\gamma E(p,q,\bar{U}))/\gamma=\exp(\alpha^{*}+\delta q)p^{1+\beta}/(1+\beta)+\bar{U}$
(x7)	$\gamma=0\ \&\ \beta<-1$	$E(p,q,\bar{U})=\exp(\alpha^{*}+\delta q)p^{1+\beta}/(1+\beta)+\bar{U}$
(x8)	$\gamma\neq 1\ \&\ \beta<-1$	$E(p,q,\bar{U})^{(1-\gamma)}/(1-\gamma)=\exp(\alpha^{*}+\delta q)p^{1+\beta}/(1+\beta)+\bar{U}$
(x8)	$\gamma=1\ \&\ \beta<-1$	$E(p,q,\bar{U})=\exp(\exp(\alpha^{*}+\delta q)p^{1+\beta}/(1+\beta))\bar{U}$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered.

Table 7d

Weak Complementarity's Implications for $k(q,\tilde{k})$ ***when*** $\beta(q)=\beta^{*}+\delta q$

Model	*Conditions*	*Constant of Integration*[1]
(x1)	$\gamma \neq 0$	$k(q,\tilde{k})=((\beta^{*}+\delta q)/\gamma^{2})\exp(\gamma^{2}\tilde{k}/(\beta^{*}+\delta q))$
(x1) & (x2)	$\gamma = 0$	$k(q,\tilde{k})=\alpha^{2}/(2\beta^{*}+2\delta q)+\tilde{k}$
(x3) & (x4)	$\gamma = 0, \beta(q)\neq 0$	$k(q,\tilde{k})=(\beta^{*}+\delta q)\exp(-\alpha/(\beta^{*}+\delta q))+\tilde{k}$
(x5) & (x6)	$\beta(q)\neq 0$	$k(q,\tilde{k})=\tilde{k}$
(x7) & (x8)	$\beta(q)<-1$	$k(q,\tilde{k})=\tilde{k}$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered.

Table 8d

Weakly Complementary Expenditure Functions when $\beta(q)=\beta^{*}+\delta q$ ***&*** $\tilde{k}=\bar{U}$

Model	*Conditions*	*Quasi-Expenditure Function*[1]
(x1)	$\gamma \neq 0$	$E(p,q,\bar{U})=-(\alpha+(\beta^{*}+\delta q)p+(\beta^{*}+\delta q)/\gamma)/\gamma$ $+((\beta^{*}+\delta q)/\gamma^{2})\exp(\gamma p+\gamma^{2}\bar{U}/(\beta^{*}+\delta q))$
(x1) & (x2)	$\gamma = 0$	$E(p,q,\bar{U})=\alpha p+((\beta^{*}+\delta q)/2)p^{2}+\alpha^{2}/(2\beta^{*}+2\delta q)+\bar{U}$
(x3) & (x4)	$\gamma = 0, \beta(q)\neq 0$	$E(p,q,\bar{U})=\alpha p+(\beta^{*}+\delta q)p(\ln p-1)$ $+(\beta^{*}+\delta q)\exp(-\alpha/(\beta^{*}+\delta q))+\bar{U}$
(x5)	$\gamma \neq 0 \,\&\, \beta(q)\neq 0$	$-\exp(-\gamma E(p,q,\bar{U}))/\gamma=\exp(\alpha+(\beta^{*}+\delta q)p)/(\beta^{*}+\delta q)+\bar{U}$
(x5)	$\gamma = 0 \,\&\, \beta(q)\neq 0$	$E(p,q,\bar{U})=\exp(\alpha+(\beta^{*}+\delta q)p)/(\beta^{*}+\delta q)+\bar{U}$
(x6)	$\gamma \neq 1 \,\&\, \beta(q)\neq 0$	$E(p,q,\bar{U})^{(1-\gamma)}/(1-\gamma)=\exp(\alpha+(\beta^{*}+\delta q)p)/(\beta^{*}+\delta q)+\bar{U}$
(x6)	$\gamma = 1 \,\&\, \beta(q)\neq 0$	$E(p,q,\bar{U})=\exp(\exp(\alpha+(\beta^{*}+\delta q)p)/(\beta^{*}+\delta q))\bar{U}$
(x7)	$\gamma \neq 0 \,\&\, \beta(q)<-1$	$-\exp(-\gamma E(p,q,\bar{U}))/\gamma=\exp(\alpha)p^{1+\beta^{*}+\delta q}/(1+\beta^{*}+\delta q)+\bar{U}$
(x7)	$\gamma = 0 \,\&\, \beta(q)<-1$	$E(p,q,\bar{U})=\exp(\alpha)p^{1+\beta^{*}+\delta q}/(1+\beta^{*}+\delta q)+\bar{U}$
(x8)	$\gamma \neq 1 \,\&\, \beta(q)<-1$	$E(p,q,\bar{U})^{(1-\gamma)}/(1-\gamma)=\exp(\alpha)p^{1+\beta^{*}+\delta q}/(1+\beta^{*}+\delta q)+\bar{U}$
(x8)	$\gamma = 1 \,\&\, \beta(q)<-1$	$E(p,q,\bar{U})=\exp(\exp(\alpha)p^{1+\beta^{*}+\delta q}/(1+\beta^{*}+\delta q))\bar{U}$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered.

Table 9d

Weak Complementarity's Implications for $k(q,\tilde{k})$ ***when*** $\gamma(q) = \gamma^* + \delta q$

Model	*Conditions*	*Constant of Integration*[1]
(x1)	$\gamma(q) \neq 0$	$k(q,\tilde{k}) = (\beta/(\gamma^* + \delta q)^2)\exp(\alpha(\gamma^* + \delta q)/\beta + \tilde{k}(\gamma^* + \delta q)^2/\beta)$
(x3)	$\gamma(q) < 0 \,\&\, \beta = 0$	Weak complementarity not satisfied
(x5)	$\gamma(q) \neq 0 \,\&\, \beta \neq 0$	$k(q,\tilde{k}) = -\exp(-(\gamma^* + \delta q)\tilde{k})/(\gamma^* + \delta q)$
(x6)	$\gamma(q) \neq 1 \,\&\, \beta \neq 0$	$k(q,\tilde{k}) = \tilde{k}^{1-\gamma^*-\delta q}/(1-\gamma^* - \delta q)$
(x7)	$\gamma(q) \neq 0 \,\&\, \beta < -1$	$k(q,\tilde{k}) = -\exp(-(\gamma^* + \delta q)\tilde{k})/(\gamma^* + \delta q)$
(x8)	$\gamma(q) \neq 1 \,\&\, \beta < -1$	$k(q,\tilde{k}) = \tilde{k}^{1-\gamma^*-\delta q}/(1-\gamma^* - \delta q)$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered. Also, the specifications where $\gamma(q) = 0$ or $\gamma(q) = 1$ are not considered.

Table 10d

Weakly Complementary Expenditure Functions when $\gamma(q) = \gamma^* + \delta q$ ***&*** $\tilde{k} = \bar{U}$

Model	*Conditions*	*Quasi-Expenditure Function*[1]
(x1)	$\gamma(q) \neq 0$	$E(p,q,\bar{U}) = (\beta/(\gamma^* + \delta q)^2)(\exp((\gamma^* + \delta q)(p + \alpha/\beta) + \bar{U}(\gamma^* + \delta q)^2/\beta) - 1) - (\alpha + \beta p)/(\gamma^* + \delta q)$
(x5)	$\gamma(q) \neq 0 \,\&\, \beta \neq 0$	$-\exp(-(\gamma^* + \delta q)E(p,q,\bar{U}))/(\gamma^* + \delta q) = \exp(\alpha + \beta p)/\beta - \exp(-(\gamma^* + \delta q)\bar{U})/(\gamma^* + \delta q)$
(x6)	$\gamma(q) \neq 1 \,\&\, \beta \neq 0$	$E(p,q,\bar{U})^{1-\gamma^*-\delta q}/(1-\gamma^* - \delta q) = \exp(\alpha + \beta p)/\beta + \bar{U}^{1-\gamma^*-\delta q}/(1-\gamma^* - \delta q)$
(x7)	$\gamma(q) \neq 0 \,\&\, \beta < -1$	$-\exp(-(\gamma^* + \delta q)E(p,q,\bar{U}))/(\gamma^* + \delta q) = \exp(\alpha)p^{1+\beta}/(1+\beta) - \exp(-(\gamma^* + \delta q)\bar{U})/(\gamma^* + \delta q)$
(x8)	$\gamma(q) \neq 1 \,\&\, \beta < -1$	$E(p,q,\bar{U})^{1-\gamma^*-\delta q}/(1-\gamma^* - \delta q) = \exp(\alpha)p^{1+\beta}/(1+\beta) + \bar{U}^{1-\gamma^*-\delta q}/(1-\gamma^* - \delta q)$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered. Also, the specifications where $\gamma(q) = 0$ or $\gamma(q) = 1$ are not considered.

Table 1e
Expenditure Specifications

Model	*Specification*[1]
(e1)	$e(p,y)=\alpha+\beta p+\gamma y$
(e2)	$e(p,y)=\alpha+\beta p+\gamma \ln y$
(e3)	$e(p,y)=\alpha+\beta \ln p+\gamma y$
(e4)	$e(p,y)=\alpha+\beta \ln p+\gamma \ln y$
(e5)	$e(p,y)=\exp(\alpha+\beta p+\gamma y)$
(e6)	$e(p,y)=\exp(\alpha+\beta p+\gamma \ln y)$
(e7)	$e(p,y)=\exp(\alpha+\beta \ln p+\gamma y)$
(e8)	$e(p,y)=\exp(\alpha+\beta \ln p+\gamma \ln y)$

[1] Although not necessary for the Slutsky matrix to be negative semi-definite, it is assumed throughout that $\beta \leq 1$ and $\beta \neq 0$ for expositional ease. No sign restrictions are placed on α and γ unless otherwise noted, however.

Table 2e
Quasi-Expenditure Functions

Model	*Conditions*	*Quasi-Expenditure Function*[1,2,3]
(e1)	$\gamma \neq 0,1$	$\tilde{E}(p,k) = \beta p/(1-\gamma) - \alpha/\gamma + p^{\gamma}k$
(e1)	$\gamma = 1$	$\tilde{E}(p,k) = \beta p \ln p - \alpha + pk$
(e1) & (e2)	$\gamma = 0 \,\&\, \beta \neq 0$	$\tilde{E}(p,k) = \alpha \ln p + \beta p + k$
(e2)	$\gamma \neq 0$	No closed form
(e3)	$\gamma \neq 0$	$\tilde{E}(p,k) = -(\alpha + \beta \ln p + \beta/\gamma)/\gamma + p^{\gamma}k$
(e3) & (e4)	$\gamma = 0$	$\tilde{E}(p,k) = \alpha \ln p + (\beta/2)(\ln p)^2 + k$
(e4)	$\gamma \neq 0$	No closed form
(e5)	$\beta \neq 0$	No closed form
(e5) & (e7)	$\beta = 0$	$-\exp(-\gamma \tilde{E}(p,k))/\gamma = \exp(\alpha)\ln p + k$
(e6)	$\beta \neq 0$	No closed form
(e6) & (e8)	$\gamma \neq 1 \,\&\, \beta = 0$	$\tilde{E}(p,k)^{1-\gamma}/(1-\gamma) = \exp(\alpha)\ln p + k$
(e6) & (e8)[4]	$\gamma = 1 \,\&\, \beta = 0$	$\tilde{E}(p,k) = p^{\exp(\alpha)}k$
(e7)	$\gamma \neq 0 \,\&\, \beta \neq 0$	$-\exp(-\gamma \tilde{E}(p,k))/\gamma = \exp(\alpha)p^{\beta}/\beta + k$
(e7)	$\gamma = 0 \,\&\, \beta \neq 0$	$\tilde{E}(p,k) = \exp(\alpha)p^{\beta}/\beta + k$
(e8)	$\gamma \neq 1 \,\&\, \beta \neq 0$	$\tilde{E}(p,k)^{1-\gamma}/(1-\gamma) = \exp(\alpha)p^{\beta}/\beta + k$
(e8)	$\gamma = 1 \,\&\, \beta \neq 0$	$\tilde{E}(p,k) = \exp(\exp(\alpha)p^{\beta}/\beta)k$

[1] k is the constant of integration.
[2] Trivial cases where $\beta = \gamma = 0$ are ignored.
[3] Some of the quasi-expenditure functions are written implicitly to highlight the fact that they are not defined for all possible normalized price, normalized income and model parameter values.
[4] For economic coherence, $\alpha < 0$.

Table 3e
Hicksian Choke Prices

Model	*Conditions*	*Hicksian Choke Price*[1,2]
(e1)	$\gamma \neq 0,1$	$\hat{p}(k)^{\gamma-1} = -\beta/((1-\gamma)\gamma k)$
(e1)	$\gamma = 1$	$\hat{p}(k) = \exp(-k/\beta - 1)$
(e1) & (e2)	$\gamma = 0 \,\&\, \beta \neq 0$	$\hat{p}(k) = -\alpha/\beta$
(e3)	$\gamma \neq 0$	$\hat{p}(k)^{\gamma} = \beta/(\gamma^2 k)$
(e3) & (e4)	$\gamma = 0$	$\hat{p}(k) = \exp(-\alpha/\beta)$
(e5) & (e6)	$\beta = 0$	$\hat{p}(k) = +\infty$
(e7) & (e8)	All	$\hat{p}(k) = +\infty$

[1] Specifications where no closed form solution for the quasi-expenditure function exists are not considered.
[2] Some of the Hicksian choke prices are written implicitly to highlight the fact that they are not defined for all possible normalized price, normalized income and model parameter values.

Table 4e
Quasi-Expenditure Functions Evaluated at Hicksian Choke Prices

Model	*Conditions*	*Quasi-Expenditure Function*[1,2]
(e1)	$\gamma \neq 0,1$	$\tilde{E}(\hat{p}(k),k) = -\alpha/\gamma + (-\beta/(1-\gamma))^{\gamma/(\gamma-1)}(1+\gamma^{-1})(\gamma k)^{-1/(1-\gamma)}$
(e1)	$\gamma = 1$	$\tilde{E}(\hat{p}(k),k) = -\beta\exp(-k/\beta - 1) - \alpha$
(e1) & (e2)	$\gamma = 0$ & $\beta \neq 0$	$\tilde{E}(\hat{p}(k),k) = \alpha(\ln(-\alpha/\beta) - 1) + k$
(e3)	$\gamma \neq 0$	$\tilde{E}(\hat{p}(k),k) = -\alpha/\gamma - (\beta/\gamma^2)\ln(\beta/(\gamma^2 k))$
(e3) & (e4)	$\gamma = 0$	$\tilde{E}(\hat{p}(k),k) = -\alpha^2/(2\beta) + k$
(e5) & (e7)	$\beta = 0$	$-\exp(-\gamma\tilde{E}(\hat{p}(k),k))/\gamma = +\infty$
(e6) & (e8)	$\gamma \neq 1$ & $\beta = 0$	$\tilde{E}(\hat{p}(k),k)^{1-\gamma}/(1-\gamma) = +\infty$
(e6) & (e8)[3]	$\gamma = 1$ & $\beta = 0$	$\tilde{E}(\hat{p}(k),k) = +\infty$
(e7)	$\gamma \neq 0$ & $\beta \neq 0$	$-\exp(-\gamma\tilde{E}(\hat{p}(k),k))/\gamma = k$
(e7)	$\gamma = 0$ & $\beta \neq 0$	$\tilde{E}(\hat{p}(k),k) = k$
(e8)	$\gamma \neq 1$ & $\beta \neq 0$	$\tilde{E}(\hat{p}(k),k)^{1-\gamma}/(1-\gamma) = k$
(e8)	$\gamma = 1$ & $\beta \neq 0$	$\tilde{E}(\hat{p}(k),k) = k$

[1] Specifications where no closed form quasi-expenditure function or Hicksian choke price exists are not considered.
[2] For all models, Table 4e assumes that the parameter and constant of integration values are such that closed form, strictly positive solutions for the Hicksian choke prices (implicitly) defined in Table 3e exist.

Table 5e

Weak Complementarity's Implications for $k(q,\tilde{k})$ when $\alpha(q)=\alpha^*+\delta q$

Model	*Conditions*	*Constant of Integration*[1]
(e1)	$\gamma \neq 0,1$	$k(q,\tilde{k}) = ((\delta/\gamma)q+\tilde{k})^{\gamma-1}(-\beta/(1-\gamma))^{\gamma}(1+\gamma^{-1})^{1-\gamma}\gamma^{-1}$
(e1)	$\gamma = 1$	$k(q,\tilde{k}) = -\beta(\ln(-(\delta q+\tilde{k})/\beta)+1)$
(e1) & (e2)	$\gamma = 0 \,\&\, \beta \neq 0$	$k(q,\tilde{k}) = -(\alpha^*+\delta q)(\ln(-(\alpha^*+\delta q)/\beta)-1)+\tilde{k}$
(e3)	$\gamma \neq 0$	$k(q,\tilde{k}) = \exp((\gamma/\beta)(\delta q+\gamma\tilde{k}))$
(e3) & (e4)	$\gamma = 0$	$k(q,\tilde{k}) = (2\alpha^*\delta q+(\delta q)^2)/(2\beta)+\tilde{k}$
(e7)	$\gamma \neq 0 \,\&\, \beta \neq 0$	$k(q,\tilde{k}) = \tilde{k}$
(e7)	$\gamma = 0 \,\&\, \beta \neq 0$	$k(q,\tilde{k}) = \tilde{k}$
(e8)	$\gamma \neq 1 \,\&\, \beta \neq 0$	$k(q,\tilde{k}) = \tilde{k}$
(e8)	$\gamma = 1 \,\&\, \beta \neq 0$	$k(q,\tilde{k}) = \tilde{k}$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered.

Table 6e

Weakly Complementary Expenditure Functions when $\alpha(q)=\alpha^*+\delta q$ & $\tilde{k}=\bar{U}$

Model	*Conditions*	*Quasi-Expenditure Function*[1]
(e1)	$\gamma \neq 0,1$	$E(p,q,\bar{U}) = \beta p/(1-\gamma)-(\alpha^*+\delta q)/\gamma + p^{\gamma}((\delta/\gamma)q+\bar{U})^{\gamma-1}$ $(-\beta/(1-\gamma))^{\gamma}(1+\gamma^{-1})^{1-\gamma}\gamma^{-1}$
(e1)	$\gamma = 1$	$E(p,q,\bar{U}) = \beta p\ln p-\alpha^*-\delta q-p\beta(\ln(-(\delta q+\bar{U})/\beta)+1)$
(e1) & (e2)	$\gamma = 0 \,\&\, \beta \neq 0$	$E(p,q,\bar{U}) = (\alpha^*+\delta q)\ln p+\beta p-(\alpha^*+\delta q)(\ln(-(\alpha^*+\delta q)/\beta)-1)+\bar{U}$
(e3)	$\gamma \neq 0$	$E(p,q,\bar{U}) = -(\alpha^*+\beta(\ln p+(\delta/\beta)q)+\beta/\gamma)/\gamma$ $+(p\exp(\delta/\beta q))^{\gamma}\exp(\gamma^2/\beta\bar{U})$
(e3) & (e4)	$\gamma = 0$	$E(p,q,\bar{U}) = \alpha^*(\ln p+(\delta/\beta)q)+(\beta/2)(\ln p+(\delta/\beta)q)^2+\bar{U}$
(e7)	$\gamma \neq 0 \,\&\, \beta \neq 0$	$-\exp(-\gamma E(p,q,\bar{U}))/\gamma = \exp(\alpha^*)(p\exp((\delta/\beta)q))^{\beta}/\beta+\bar{U}$
(e7)	$\gamma = 0 \,\&\, \beta \neq 0$	$E(p,q,\bar{U}) = \exp(\alpha^*)(p\exp((\delta/\beta)q))^{\beta}/\beta+\bar{U}$
(e8)	$\gamma \neq 1 \,\&\, \beta \neq 0$	$E(p,q,\bar{U})^{1-\gamma}/(1-\gamma) = \exp(\alpha^*)(p\exp((\delta/\beta)q))^{\beta}/\beta+\bar{U}$
(e8)	$\gamma = 1 \,\&\, \beta \neq 0$	$E(p,q,\bar{U}) = \exp(\exp(\alpha^*)(p\exp((\delta/\beta)q))^{\beta}/\beta)\bar{U}$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered.

Table 7e

Weak Complementarity's Implications for $k(q,\tilde{k})$ when $\beta(q) = \beta^* + \delta q$

Model	*Conditions*	*Constant of Integration*[1]
(e1)	$\gamma \neq 0,1$	$k(q,\tilde{k}) = \tilde{k}^{\gamma-1}(-\beta^* - \delta q)^{\gamma}$
(e1)	$\gamma = 1$	$k(q,\tilde{k}) = -(\beta^* + \delta q)(\ln(-\tilde{k}/(\beta^* + \delta q)) + 1)$
(e1) & (e2)	$\gamma = 0 \,\&\, \beta(q) \neq 0$	$k(q,\tilde{k}) = -\alpha(\ln(-\alpha/(\beta^* + \delta q)) - 1) + \tilde{k}$
(e3)	$\gamma \neq 0$	$k(q,\tilde{k}) = ((\beta^* + \delta q)/\gamma^2)\exp(\gamma^2 \tilde{k}/(\beta^* + \delta q))$
(e3) & (e4)	$\gamma = 0$	$k(q,\tilde{k}) = \alpha^2/(2(\beta^* + \delta q)) + \tilde{k}$
(e7)	$\gamma \neq 0 \,\&\, \beta(q) \neq 0$	$k(q,\tilde{k}) = \tilde{k}$
(e7)	$\gamma = 0 \,\&\, \beta(q) \neq 0$	$k(q,\tilde{k}) = \tilde{k}$
(e8)	$\gamma \neq 1 \,\&\, \beta(q) \neq 0$	$k(q,\tilde{k}) = \tilde{k}$
(e8)	$\gamma = 1 \,\&\, \beta(q) \neq 0$	$k(q,\tilde{k}) = \tilde{k}$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered.

Table 8e

Weakly Complementary Expenditure Functions when $\beta(q) = \beta^* + \delta q$ & $\tilde{k} = \bar{U}$

Model	*Conditions*	*Quasi-Expenditure Function*[1]
(e1)	$\gamma \neq 0,1$	$E(p,q,\bar{U}) = (\beta^* + \delta q)p/(1-\gamma) - \alpha/\gamma + p^{\gamma}(-\beta^* - \delta q)^{\gamma}\bar{U}^{\gamma-1}$
(e1)	$\gamma = 1$	$E(p,q,\bar{U}) = (\beta^* + \delta q)p\ln p - \alpha - p(\beta^* + \delta q)(\ln(-\bar{U}/(\beta^* + \delta q)) + 1)$
(e1) & (e2)	$\gamma = 0 \,\&\, \beta(q) \neq 0$	$E(p,q,\bar{U}) = \alpha\ln p + (\beta^* + \delta q)p - \alpha(\ln(-\alpha/(\beta^* + \delta q)) - 1) + \bar{U}$
(e3)	$\gamma \neq 0$	$E(p,q,\bar{U}) = -(\alpha + (\beta^* + \delta q)\ln p + (\beta^* + \delta q)/\gamma)/\gamma$ $+ p^{\gamma}((\beta^* + \delta q)/\gamma^2)\exp(\gamma^2\bar{U}/(\beta^* + \delta q))$
(e3) & (e4)	$\gamma = 0$	$E(p,q,\bar{U}) = \alpha\ln p + ((\beta^* + \delta q)/2)(\ln p)^2 + \alpha^2/(2(\beta^* + \delta q)) + \bar{U}$
(e7)	$\gamma \neq 0 \,\&\, \beta(q) \neq 0$	$-\exp(-\gamma E(p,q,\bar{U}))/\gamma = \exp(\alpha)p^{\beta^* + \delta q}/(\beta^* + \delta q) + \bar{U}$
(e7)	$\gamma = 0 \,\&\, \beta(q) \neq 0$	$E(p,q,\bar{U}) = \exp(\alpha)p^{(\beta^* + \delta q)}/(\beta^* + \delta q) + \bar{U}$
(e8)	$\gamma \neq 1 \,\&\, \beta(q) \neq 0$	$E(p,q,\bar{U})^{1-\gamma}/(1-\gamma) = \exp(\alpha)p^{\beta^* + \delta q}/(\beta^* + \delta q) + \bar{U}$
(e8)	$\gamma = 1 \,\&\, \beta(q) \neq 0$	$E(p,q,\bar{U}) = \exp(\exp(\alpha)p^{\beta^* + \delta q}/(\beta^* + \delta q))\bar{U}$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered.

Table 9e
Weak Complementarity's Implications for $k(q,\tilde{k})$ ***when*** $\gamma(q)=\gamma^*+\delta q$

Model	*Conditions*	*Constant of Integration*[1]
(e1)	$\gamma(q)\neq 0,1$	$k(q,\tilde{k})=(\alpha/(\gamma^*+\delta q)+\tilde{k})^{\gamma^*+\delta q-1}(-\beta/(1-\gamma^*-\delta q))^{\gamma^*+\delta q}$ $(1+(\gamma^*+\delta q)^{-1})^{1-\gamma^*-\delta q}(\gamma^*+\delta q)^{-1}$
(e3)	$\gamma(q)\neq 0$	$k(q,\tilde{k})=(\beta/(\gamma^*+\delta q)^2)\exp((\gamma^*+\delta q)^2(\tilde{k}+\alpha/(\gamma^*+\delta q))/\beta)$
(e7)	$\gamma(q)\neq 0\ \&\ \beta\neq 0$	$k(q,\tilde{k})=-\exp(-(\gamma^*+\delta q)\tilde{k})/(\gamma^*+\delta q)$
(e8)	$\gamma(q)\neq 1\ \&\ \beta\neq 0$	$k(q,\tilde{k})=\tilde{k}^{1-\gamma^*-\delta q}/(1-\gamma^*-\delta q)$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered. Also, the specifications where $\gamma(q)=0$ or $\gamma(q)=1$ are not considered.

Table 10e
Weakly Complementary Expenditure Functions when $\gamma(q)=\gamma^*+\delta q$ ***&*** $\tilde{k}=\bar{U}$

Model	*Conditions*	*Quasi-Expenditure Function*[1]
(e1)	$\gamma(q)\neq 0,1$	$E(p,q,\bar{U})=\beta p/(1-\gamma^*-\delta q)-\alpha/(\gamma^*+\delta q)$ $+p^{\gamma^*+\delta q}(\alpha/(\gamma^*+\delta q)+\bar{U})^{\gamma^*+\delta q-1}(-\beta/(1-\gamma^*-\delta q))^{-\gamma^*-\delta q}$ $(1+(\gamma^*+\delta q)^{-1})^{1-\gamma^*-\delta q}(\gamma^*+\delta q)^{-1}$
(e3)	$\gamma(q)\neq 0$	$E(p,q,\bar{U})=-(\alpha+\beta\ln p+\beta/(\gamma^*+\delta q))/(\gamma^*+\delta q)+p^{\gamma^*+\delta q}$ $(\beta/(\gamma^*+\delta q)^2)\exp((\gamma^*+\delta q)^2(\bar{U}+\alpha/(\gamma^*+\delta q))/\beta)$
(e7)	$\gamma(q)\neq 0\ \&\ \beta\neq 0$	$-\exp(-(\gamma^*+\delta q)E(p,q,\bar{U}))/(\gamma^*+\delta q)=\exp(\alpha)p^{\beta}/\beta$ $-\exp(-(\gamma^*+\delta q)\bar{U})/(\gamma^*+\delta q)$
(e8)	$\gamma(q)\neq 1\ \&\ \beta\neq 0$	$E(p,q,\bar{U})^{1-\gamma^*-\delta q}/(1-\gamma^*-\delta q)=\exp(\alpha)p^{\beta}/\beta$ $+\bar{U}^{1-\gamma^*-\delta q}/(1-\gamma^*-\delta q)$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered. Also, the specifications where $\gamma(q)=0$ or $\gamma(q)=1$ are not considered.

Table 1s
Expenditure Share Specifications

Model	*Specification*
(s1)	$s(p,y)=\alpha+\beta p+\gamma y$
(s2)	$s(p,y)=\alpha+\beta p+\gamma \ln y$
(s3)	$s(p,y)=\alpha+\beta \ln p+\gamma y$
(s4)	$s(p,y)=\alpha+\beta \ln p+\gamma \ln y$
(s5)	$s(p,y)=\exp(\alpha+\beta p+\gamma y)$
(s6)	$s(p,y)=\exp(\alpha+\beta p+\gamma \ln y)$
(s7)	$s(p,y)=\exp(\alpha+\beta \ln p+\gamma y)$
(s8)	$s(p,y)=\exp(\alpha+\beta \ln p+\gamma \ln y)$

[1] Although not necessary for the Slutsky matrix to be negative semi-definite, it is assumed throughout that $\beta \leq 1$ and $\beta \neq 0$ for expositional ease. No sign restrictions are placed on α and γ unless otherwise noted, however.

Table 2s
Quasi-Expenditure Functions

Model	*Conditions*	*Quasi-Expenditure Function*[1,2,3]
(s1)	$\gamma \neq 0 \,\&\, \beta \neq 0$	No closed form
(s1) & (s3)	$\gamma \neq 0 \,\&\, \beta = 0$	$\tilde{E}(p,k) = \alpha \exp(\alpha k) p^{\alpha} / (1 - \exp(\alpha k)\gamma p^{\alpha})$
(s1) & (s2)	$\gamma = 0 \,\&\, \beta \neq 0$	$\tilde{E}(p,k) = \exp(\alpha \ln p + \beta p) k$
(s2)	$\gamma \neq 0,1$	$\tilde{E}(p,k) = \exp(-\alpha / \gamma + \beta p / (1-\gamma) + \beta\gamma p^{\gamma} k)$
(s2)	$\gamma = 1$	$\tilde{E}(p,k) = \exp(-\alpha + \beta p(\ln p + k))$
(s3)	$\gamma \neq 0 \,\&\, \beta \neq 0$	No closed form
(s3) & (s4)	$\gamma = 0 \,\&\, \beta \neq 0$	$\tilde{E}(p,k) = \exp(\alpha \ln p + (\beta / 2)(\ln p)^2) k$
(s4)	$\gamma \neq 0$	$\tilde{E}(p,k) = \exp(-(\alpha + \beta \ln p + \beta / \gamma) / \gamma + p^{\gamma} k)$
(s5)		No closed form
(s6)	$\gamma \neq 0 \,\&\, \beta = 0$	$\tilde{E}(p,k)^{-\gamma} / \gamma = \ln p \exp(\alpha) + k$
(s6)	$\beta \neq 0$	No closed form
(s7)	$\gamma = 0 \,\&\, \beta \neq 0$	$\tilde{E}(p,k) = \exp(\exp(\alpha) p^{\beta} / \beta) k$
(s7)	$\gamma \neq 0$	No closed form
(s8)	$\gamma \neq 0 \,\&\, \beta \neq 0$	$-\tilde{E}(p,k)^{-\gamma} / \gamma = \exp(\alpha) p^{\beta} / \beta + k$
(s8)	$\gamma \neq 0 \,\&\, \beta = 0$	$-\tilde{E}(p,k)^{-\gamma} / \gamma = \exp(\alpha) \ln p + k$
(s8)	$\gamma = 0 \,\&\, \beta \neq 0$	$\tilde{E}(p,k) = \exp(\exp(\alpha) p^{\beta} / \beta) k$

[1] k is the constant of integration.
[2] Trivial cases where $\beta = \gamma = 0$ are ignored.
[3] Some of the quasi-expenditure functions are written implicitly to highlight the fact that they are not defined for all possible normalized price, normalized income and model parameter values.

Table 3s
Hicksian Choke Prices

Model	*Conditions*	*Hicksian Choke Price*[1,2]
(s1) & (s3)	$\gamma \neq 0$ & $\beta = 0$	$\hat{p}(k) = +\infty$
(s1) & (s2)	$\gamma = 0$ & $\beta \neq 0$	$\hat{p}(k) = -\alpha / \beta$
(s2)	$\gamma \neq 0,1$	$\hat{p}(k)^{1-\gamma} = -(1-\gamma)\gamma^2 k$
(s2)	$\gamma = 1$	$\hat{p}(k) = \exp(-k-1)$
(s3) & (s4)	$\gamma = 0$	$\hat{p}(k) = \exp(-\alpha / \beta)$
(s4)	$\gamma \neq 0$	$\hat{p}(k)^{\gamma} = \beta / (\gamma^2 k)$
(s6) & (s8)	$\gamma \neq 0$ & $\beta = 0$	$\hat{p}(k) = +\infty$
(s7)	$\gamma = 0$ & $\beta \neq 0$	$\hat{p}(k) = +\infty$
(s8)	$\beta \neq 0$	$\hat{p}(k) = +\infty$

[1] Specifications where no closed form quasi-expenditure function or Hicksian choke price exists are not considered.
[2] Some of the Hicksian choke prices are written implicitly to highlight the fact that they are not defined for all possible normalized price, normalized income and model parameter values.

Table 4s
Quasi-Expenditure Functions Evaluated at Hicksian Choke Prices

Model	*Conditions*	*Quasi-Expenditure Function*[1,2]
(s1) & (s3)[3]	$\gamma < 0$ & $\beta = 0$ & $\alpha > 0$	$\tilde{E}(\hat{p}(k), k) = -\alpha / \gamma$
(s1) & (s2)	$\gamma = 0$ & $\beta \neq 0$	$\tilde{E}(\hat{p}(k), k) = \exp(\alpha(\ln(-\alpha / \beta) - 1))k$
(s2)	$\gamma \neq 0,1$	$\tilde{E}(\hat{p}(k), k) = \exp(-\alpha / \gamma - \beta(1-\gamma)^{\gamma/(1-\gamma)} \gamma^{2/(1-\gamma)} (1+\gamma^{-1}) k^{1/(1-\gamma)})$
(s2)	$\gamma = 1$	$\tilde{E}(\hat{p}(k), k) = \exp(-\alpha - \beta \exp(-k-1))$
(s3) & (s4)	$\gamma = 0$ & $\beta \neq 0$	$\tilde{E}(\hat{p}(k), k) = \exp(-\alpha^2 / (2\beta))k$
(s4)	$\gamma \neq 0$	$\tilde{E}(\hat{p}(k), k) = \exp(-\alpha / \gamma - (\beta / \gamma^2) \ln(\beta / (\gamma^2 k)))$
(s6) & (s8)	$\gamma \neq 0$ & $\beta = 0$	$\tilde{E}(\hat{p}(k), k)^{-\gamma} / \gamma = +\infty$
(s7) & (s8)	$\gamma = 0$ & $\beta \neq 0$	$\tilde{E}(\hat{p}(k), k) = k$
(s8)	$\gamma \neq 0$ & $\beta \neq 0$	$-\tilde{E}(\hat{p}(k), k)^{-\gamma} / \gamma = k$

[1] Specifications where no closed form quasi-expenditure function or Hicksian choke price exists are not considered.
[2] For all models, Table 4s assumes that the parameter and constant of integration values are such that closed form, strictly positive solutions for the Hicksian choke prices (implicitly) defined in Table 3s exist.
[3] When $\alpha \leq 0$, $\tilde{E}(\hat{p}(k), k) = 0$. Since this case is uninteresting economically, it is ignored.

Table 5s

Weak Complementarity's Implications for $k(q,\tilde{k})$ when $\alpha(q)=\alpha^*+\delta q$

Model	*Conditions*	*Constant of Integration*[1]
(s1) & (s3)	$\gamma<0\ \&\ \beta=0$ & $\alpha(q)>0$	Weak complementarity not satisfied
(s1) & (s2)	$\gamma=0\ \&\ \beta\neq 0$	$k(q,\tilde{k})=\exp(-(\alpha^*+\delta q)(\ln(-(\alpha^*+\delta q)/\beta)-1))\tilde{k}$
(s2)	$\gamma\neq 0,1$	$k(q,\tilde{k})=(\delta q/\gamma+\tilde{k})^{1-\gamma}(-\beta)^{\gamma-1}(1-\gamma)^{-\gamma}\gamma^{-2}(1+\gamma^{-1})^{\gamma-1}$
(s2)	$\gamma=1$	$k(q,\tilde{k})=-(\ln(-(\delta q+\tilde{k})/\beta)+1)$
(s3) & (s4)	$\gamma=0\ \&\ \beta\neq 0$	$k(q,\tilde{k})=\exp((2\alpha^*\delta q+(\delta q)^2)/(2\beta))\tilde{k}$
(s4)	$\gamma\neq 0$	$k(q,\tilde{k})=\exp((\gamma/\beta)(\delta q+\gamma\tilde{k}))$
(s7) & (s8)	$\gamma=0\ \&\ \beta\neq 0$	$k(q,\tilde{k})=\tilde{k}$
(s8)	$\gamma\neq 0\ \&\ \beta\neq 0$	$k(q,\tilde{k})=\tilde{k}$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered.

Table 6s

Weakly Complementary Expenditure Functions when $\alpha(q)=\alpha^*+\delta q$ & $\tilde{k}=\bar{U}$

Model	*Conditions*	*Quasi-Expenditure Function*[1]
(s1) & (s2)	$\gamma=0\ \&\ \beta\neq 0$	$E(p,q,\bar{U})=\exp((\alpha^*+\delta q)\ln p+\beta p-(\alpha^*+\delta q)$ $(\ln(-(\alpha^*+\delta q)/\beta)-1))\bar{U}$
(s2)	$\gamma\neq 0,1$	$E(p,q,\bar{U})=\exp(-(\alpha^*+\delta q)/\gamma+\beta p/(1-\gamma)$ $+p^{\gamma}(\delta q/\gamma+\bar{U})^{1-\gamma}(-\beta)^{\gamma}(1-\gamma)^{-\gamma}\gamma^{-1}(1+\gamma^{-1})^{\gamma-1}$
(s2)	$\gamma=1$	$E(p,q,\bar{U})=\exp(-\alpha+\beta p(\ln p-(\ln(-(\delta q+\bar{U})/\beta)+1)))$
(s3) & (s4)	$\gamma=0\ \&\ \beta\neq 0$	$E(p,q,\bar{U})=\exp(\alpha^*(\ln p+(\delta/\beta)q)+(\beta/2)(\ln p+(\delta/\beta)q)^2)\bar{U}$
(s4)	$\gamma\neq 0$	$E(p,q,\bar{U})=\exp(-(\alpha^*+\beta(\ln p+(\delta/\beta)q)+\beta/\gamma)/\gamma$ $+(p\exp((\delta/\beta)q))^{\gamma}\exp((\gamma^2/\beta)\bar{U})$
(s7) & (s8)	$\gamma=0\ \&\ \beta\neq 0$	$E(p,q,\bar{U})=\exp(\exp(\alpha^*+\delta q)p^{\beta}/\beta)\bar{U}$
(s8)	$\gamma\neq 0\ \&\ \beta\neq 0$	$-E(p,q,\bar{U})^{-\gamma}/\gamma=\exp(\alpha^*+\delta q)p^{\beta}/\beta+\bar{U}$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered.

Table 7s

Weak Complementarity's Implications for $k(q,\tilde{k})$ when $\beta(q)=\beta^*+\delta q$

Model	*Conditions*	*Constant of Integration*[1]
(s1) & (s2)	$\gamma=0$ & $\beta(q)\neq 0$	$k(q,\tilde{k})=\exp(-\alpha\ln(-\alpha/(\beta^*+\delta q)))\tilde{k}$
(s2)	$\gamma\neq 0,1$	$k(q,\tilde{k})=-(-\beta^*-\delta q)^{\gamma-1}(1-\gamma)^{-\gamma}\gamma^{-2}(1+\gamma^{-1})^{\gamma-1}\tilde{k}^{1-\gamma}$
(s2)	$\gamma=1$	$k(q,\tilde{k})=-(\ln(-\tilde{k}/(\beta^*+\delta q))+1)$
(s3) & (s4)	$\gamma=0$ & $\beta(q)\neq 0$	$k(q,\tilde{k})=\exp(\alpha^2/(2\beta^*+2\delta q))\tilde{k}$
(s4)	$\gamma\neq 0$	$k(q,\tilde{k})=((\beta^*+\delta q)/\gamma^2)\exp(\gamma^2\tilde{k}/(\beta^*+\delta q))$
(s7) & (s8)	$\gamma=0$ & $\beta(q)\neq 0$	$k(q,\tilde{k})=\tilde{k}$
(s8)	$\gamma\neq 0$ & $\beta(q)\neq 0$	$k(q,\tilde{k})=\tilde{k}$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered. Also, the specifications where $\beta(q)=0$ are not considered.

Table 8s

Weakly Complementary Expenditure Functions when $\beta(q)=\beta^*+\delta q$ & $\tilde{k}=\bar{U}$

Model	*Conditions*	*Quasi-Expenditure Function*[1]
(s1) & (s2)	$\gamma=0$ & $\beta(q)\neq 0$	$E(p,q,\bar{U})=\exp(\alpha\ln p+(\beta^*+\delta q)p-\alpha\ln(-\alpha/(\beta^*+\delta q)))\bar{U}$
(s2)	$\gamma\neq 0,1$	$E(p,q,\bar{U})=\exp(-\alpha/\gamma+(\beta^*+\delta q)p/(1-\gamma)$ $+p^{\gamma}(\beta^*+\delta q)^{\gamma}(1-\gamma)^{-\gamma}\gamma^{-1}(1+\gamma^{-1})^{\gamma-1}\bar{U}^{1-\gamma}$
(s2)	$\gamma=1$	$E(p,q,\bar{U})=\exp(-\alpha+\beta p(\ln p-(\ln(-\bar{U}/(\beta^*+\delta q))+1)))$
(s3) & (s4)	$\gamma=0$ & $\beta(q)\neq 0$	$E(p,q,\bar{U})=\exp(\alpha\ln p+((\beta^*+\delta q)/2)(\ln p)^2)$ $\exp(\alpha^2/(2\beta^*+2\delta q))\bar{U}$
(s4)	$\gamma\neq 0$	$E(p,q,\bar{U})=\exp(-(\alpha+(\beta^*+\delta q)\ln p+(\beta^*+\delta q)/\gamma)/\gamma$ $+p^{\gamma}((\beta^*+\delta q)/\gamma^2)\exp(-\gamma^2\bar{U}/(\beta^*+\delta q)))$
(s7) & (s8)	$\gamma=0$ & $\beta(q)\neq 0$	$E(p,q,\bar{U})=\exp(\exp(\alpha)p^{\beta^*+\delta q}/(\beta^*+\delta q))\bar{U}$
(s8)	$\gamma\neq 0$ & $\beta(q)\neq 0$	$(-1/\gamma)E(p,q,\bar{U})^{-\gamma}=\exp(\alpha)p^{\beta^*+\delta q}/(\beta^*+\delta q)+\bar{U}$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered. Also, the specifications where $\beta(q)=0$ are not considered.

Table 9s

Weak Complementarity's Implications for $k(q,\tilde{k})$ ***when*** $\gamma(q)=\gamma^*+\delta q$

Model	*Conditions*	*Constant of Integration*[1]
(s1) & (s3)	$\gamma(q)<0$ & $\beta=0$ & $\alpha>0$	Weak complementarity not satisfied
(s2)	$\gamma(q)\neq 0,1$	$k(q,\tilde{k})=-(-\beta)^{\gamma^*+\delta q-1}(1-\gamma^*-\delta q)^{-\gamma^*-\delta q}(\gamma^*+\delta q)^{-2}$ $(1+(\gamma^*+\delta q)^{-1})^{\gamma^*+\delta q-1}(-\alpha/(\gamma^*+\delta q)-\tilde{k})^{1-\gamma^*-\delta q}$
(s4)	$\gamma(q)\neq 0$	$k(q,\tilde{k})=(\beta/(\gamma^*+\delta q)^2)\exp(\alpha(\gamma^*+\delta q)/\beta+\tilde{k}(\gamma^*+\delta q)^2/\beta)$
(s8)	$\gamma(q)\neq 0$ & $\beta\neq 0$	$k(q,\tilde{k})=-\tilde{k}^{-\gamma^*-\delta q}/(\gamma^*+\delta q)$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered. Also, the specifications where $\gamma(q)=0$ or $\gamma(q)=1$ are not considered.

Table 10s

Weakly Complementary Expenditure Functions when $\gamma(q)=\gamma^*+\delta q$ **&** $\tilde{k}=\bar{U}$

Model	*Conditions*	*Quasi-Expenditure Function*[1]
(s2)	$\gamma(q)\neq 0,1$	$E(p,q,\bar{U})=\exp(-\alpha/(\gamma^*+\delta q)+p/(1-\gamma^*-\delta q)-p^{\gamma^*+\delta q}(-\beta)^{\gamma^*+\delta q}$ $(1-\gamma^*-\delta q)^{-\gamma^*-\delta q}(\gamma^*+\delta q)^{-1}(1+(\gamma^*+\delta q)^{-1})^{\gamma^*+\delta q-1})$ $(-\alpha/(\gamma^*+\delta q)-\bar{U})^{1-\gamma^*-\delta q}$
(s4)	$\gamma(q)\neq 0$	$E(p,q,\bar{U})=\exp(-(\alpha+\beta\ln p+\beta/(\gamma^*+\delta q))/(\gamma^*+\delta q)+p^{\gamma^*+\delta q}$ $(\beta/(\gamma^*+\delta q)^2)\exp(-(\gamma^*+\delta q)^2(\alpha/(\gamma^*+\delta q)+\bar{U})/\beta))$
(s8)	$\gamma(q)\neq 0$ & $\beta\neq 0$	$-E(p,q,\bar{U})^{-\gamma^*-\delta q}/(\gamma^*+\delta q)=\exp(\alpha)p^{\beta}/\beta-\bar{U}^{-\gamma^*-\delta q}/(\gamma^*+\delta q)$

[1] Specifications with either 1) no Hicksian choke price or 2) no finite, closed form quasi-expenditure function evaluated at the Hicksian choke price are not considered. Also, the specifications where $\gamma(q)=0$ or $\gamma(q)=1$ are not considered.

A.2 – Additional Parameter Estimates

Additional Table
Additional Posterior Parameter Estimates[1, 2]
(not meant for publication but available upon request)

	Specification 1		*Specifications 2 & 4*		*Specifications 3 & 5*	
Site i's quality attributes enters through	ϕ_i		Ψ_i		θ_i	
	mean	*variance*	*mean*	*variance*	*mean*	*variance*
Demographic parameters entering through Ψ_i						
Constant	4.2247	2.5602	0.6815	1.8452	2.0634	2.4836
	(0.4124)	(0.7695)	(0.5326)	(0.5204)	(0.5462)	(0.6110)
Ln(age)	-0.1532	0.3657	-0.0333	0.4628	0.4060	0.3496
	(0.0992)	(0.0635)	(0.1814)	(0.0825)	(0.1415)	(0.0621)
Kids under 10	0.3409	0.9445	0.5926	1.3842	0.3957	1.0081
	(0.1568)	(0.1903)	(0.2093)	(0.3362)	(0.1487)	(0.1931)
Kids 10 to 16	0.1640	1.1893	0.1986	1.3523	0.2939	1.1321
	(0.1514)	(0.2443)	(0.2508)	(0.2779)	(0.1957)	(0.2762)
Vacation prop. in DE	0.7102	2.2747	1.7082	2.3162	1.6598	2.0904
	(0.2740)	(0.9071)	(0.4507)	(0.6667)	(0.3699)	(0.4197)
Retired	-0.6113	1.3726	-0.8241	1.7493	-0.8434	1.2624
	(0.1576)	(0.3456)	(0.2945)	(0.4282)	(0.1809)	(0.2732)
Student	0.2424	1.5048	0.5333	1.8127	0.3022	1.5893
	(0.3124)	(0.3108)	(0.4381)	(0.5560)	(0.3430)	(0.4545)

[1] All estimates generated with 50,000 Gibbs sampling iterations. Simulations from the first 25,000 iteration were discarded as burn-in and every 10th simulation thereafter was used in constructing these estimates.
[2] 95% confidence set reported in parentheses.

A.3 Bayesian Estimation Algorithm

The following prior distributional assumptions for the model parameters were used in the Gibbs Sampling estimation algorithm:

1. $\beta_t \sim N(b,\Sigma)$, where β_t is a *k* dimensional vector of random parameters (i.e., $[\tau,\delta,\theta^*,\rho_z^*,\mu^*]$) for the *t*th observation in the sample and $N(\bullet,\bullet)$ is the normal distribution
2. *f*(*b*), the prior distribution for $b \sim N(\tilde{b},\tilde{\omega}\mathbf{I})$ where $\tilde{b}$ is an arbitrarily specified vector and $1/\tilde{\omega}$ is a scalar that essentially equals 0 and **I** is a *k*-dimensional identity matrix
3. $h(\Sigma)$, the prior distribution for $\Sigma \sim IW(k,\mathbf{I})$ where **I** is a *k*-dimensional identity matrix and $IW(\bullet,\bullet)$ is the Inverse Wishert distribution.

As a result the posterior distributions for the model parameters took the form:

1. $F(\beta_t \mid b,\Sigma,\boldsymbol{x}_t) \propto l(\boldsymbol{x}_t \mid \beta_t)n(\beta_t \mid b,\Sigma), \forall t$, where $l(\boldsymbol{x}_t \mid \beta_t)$ is the conditional likelihood from equation (21) where its dependence on β_t is made explicit and $n(\beta_t \mid b,\Sigma)$ is the normal probability density function
2. $F(b \mid \beta_t,\Sigma,\boldsymbol{x}_t) \propto N(\overline{\beta},\Sigma/N)$, where *N* is the sample size & $\overline{\beta} = (1/N)\sum_t \beta_t$
3. $F(\Sigma \mid \beta_t,b,\boldsymbol{x}_t) \propto IW(k+N,(k\mathbf{I}+N\overline{S})/(k+N))$, where $\overline{S} = (1/N)\sum_t (\beta_t - b)^{\mathsf{T}}(\beta_t - b)$

To simulate from these posterior distributions, a Gibbs Sampling algorithm was employed. At each iteration *j*, *j* = 1,...,*J*, of the algorithm, the following steps were taken:

1. Simulate b^j from $N(\overline{\beta}^{j-1},\Sigma^{j-1}/N)$. To initialize the algorithm, set $\Sigma^0 = k\mathbf{I}$ and $\overline{\beta}^0 = (1/N)\sum_t \beta_t^0$ where β_t^0 is arbitrarily set to, e.g., the maximum likelihood fixed parameter estimates.
2. Simulate Σ^j from $IW(k+N,(k\mathbf{I}+N\overline{S}^{j-1})/(k+N))$.
3. Simulate each observation's β_t^j using one iteration of the following adaptive Metropolis-Hastings algorithm:
   a. For each observation, simulate a candidate vector $\tilde{\beta}_t^j$ from $N(\beta_t^{j-1},r^{j-1}\Sigma^{j-1})$, where r^{j-1} is a constant. To initialize the sequence, set $r^0 = .1$
   b. For each observation, construct the following statistic:

   $$\chi_t^j = \frac{l(\boldsymbol{x}_t \mid \beta_t^j)n(\beta_t^j \mid b^j,\Sigma^j)}{l(\boldsymbol{x}_t \mid \beta_t^{j-1})n(\beta_t^{j-1} \mid b^j,\Sigma^j)}$$

   If $\chi_t^j \geq U_t^j$ where U_t^j is a uniform random draw, accept the candidate random parameters, i.e., $\beta_t^j = \tilde{\beta}_t^j$. Otherwise, set $\beta_t^j = \beta_t^{j-1}$

c. Gelman et al. (1995) argue that the Metropolis-Hastings algorithm for the normal distribution is most efficient if the acceptance rate of candidate parameters averages between 0.23 to 0.44. Therefore, set $r^j = (1.01)r^{j-1}$ if the sample's proportion of accepted candidate parameter values is less than 0.3. Otherwise, set $r^j = (0.99)r^{j-1}$.

4. Iterate.

After a sufficiently long burn-in, this Gibbs Sampling algorithm generates random draws from the posterior distributions of (β_t, b, Σ). In the current application, 50,000 Gibbs sampling iterations were used to generate each of the three sets of estimates. Simulations from the first 25,000 iterations were discarded as burn-in and simulations from every 10th iteration thereafter were used to calculate the reported statistics. As a result, 2,500 simulations entered into the calculation of all reported estimates in Tables 3 and 4.

A.4 Solving for the Expected Hicksian Consumer Surplus

The expected Hicksian consumer surplus estimates for this policy scenario were computed as part of the Gibbs sampling routine that generated the parameter estimates in Table 4. At each iteration of the Gibbs sampling routine, individual level parameters, $(\tilde{\tau}, \tilde{\delta}, \tilde{\theta}^*, \tilde{\mu}^*)$, were also generated. For the individual level parameters corresponding to every 10th simulation after the burn-in, the expected compensating surplus for each individual, $E(CS^H \mid \tilde{\tau}, \tilde{\delta}, \tilde{\theta}^*, \tilde{\mu}^*)$, was estimated via simulation. Simulating $E(CS^H \mid \tilde{\tau}, \tilde{\delta}, \tilde{\theta}^*, \tilde{\mu}^*)$ involved the following three steps:

a) Simulate the remaining unobserved heterogeneity, $\tilde{\varepsilon}_i, \forall i$.
b) Conditional on the individual level parameters $(\tilde{\tau}, \tilde{\delta}, \tilde{\theta}^*, \mu^*)$ and $\tilde{\varepsilon}_i, \forall i$, solve for the Hicksian consumer surplus, $E(CS^H \mid \tilde{\tau}, \tilde{\delta}, \tilde{\theta}^*, \tilde{\mu}^*, \tilde{\varepsilon}_1, \tilde{\varepsilon}_2, ..., \tilde{\varepsilon}_N)$
c) Iterate S times and average.

To accomplish step a), it is important to recognize that the individual level parameters are by construction conditional on the individual's observed choice, and thus each $\tilde{\varepsilon}_i$ must also be simulated conditional on the individual's observed choice for consistency. As von Haefen, Phaneuf, and Parsons (2004) demonstrate, this can be accomplished by using the following rule:

$$\text{(1A)} \qquad \tilde{\varepsilon}_i = \begin{cases} g_i(\tilde{\tau},\tilde{\delta},\tilde{\theta}^*) \text{ if } x_i > 0 \\ -\ln(-\ln(\exp(-\exp(-g_i(\tilde{\tau},\tilde{\delta},\tilde{\theta}^*)/\tilde{\mu}))U))\tilde{\mu} \text{ otherwise} \end{cases}, \forall i,$$

where $g_i(\tilde{\tau},\tilde{\delta},\tilde{\theta}^*)$ is the right hand side of equation (19) evaluated at the individual level parameters, $\tilde{\mu} = \exp(\tilde{\mu}^*)$ is the individual level scale parameter, and U is a uniform random draw.

For step b), this paper exploits a numerical bisection algorithm that is considerably more efficient than the multi-layered numerical bisection routine developed by von Haefen, Phaneuf, and Parsons.[1] The algorithm recognizes that the Hicksian consumer surplus associated with price and quality change from $(\boldsymbol{p}^0,\boldsymbol{Q}^0)$ to $(\boldsymbol{p}^1,\boldsymbol{Q}^1)$ can be defined as:

$$\text{(2A)} \qquad CS^H = y - E(\boldsymbol{p}^1,\boldsymbol{Q}^1,U(\boldsymbol{x}^0,\boldsymbol{Q}^0,z^0)),$$

where $(\boldsymbol{x}^0,z^0)$ are the chosen levels of consumption at $(\boldsymbol{p}^0,\boldsymbol{Q}^0,y)$. Equation (2A) suggests that if the analyst knows $U(\boldsymbol{x}^0,\boldsymbol{Q}^0,z^0)$, she need only solve the single constrained minimization problem associated with $E(\boldsymbol{p}^1,\boldsymbol{Q}^1,U(\boldsymbol{x}^0,\boldsymbol{Q}^0,z^0))$ to solve for CS^H. For the problem at hand, the analyst has all the information necessary to construct $U(\boldsymbol{x}^0,\boldsymbol{Q}^0,z^0)$, and thus solving for $E(\boldsymbol{p}^1,\boldsymbol{Q}^1,U(\boldsymbol{x}^0,\boldsymbol{Q}^0,z^0))$ remains the only obstacle. To see how this can be accomplished, notice

[1] The strategy developed by von Haefen, Phaneuf and Parsons is multi-layered in the following sense. At the top layer, it uses a numerical bisection routine to solve for the income compensation that equates utility before and after the price and quality change. At the bottom layer, it uses a numerical bisection routine to solve for the individual's Marshallian demands conditional on an arbitrary income level determined by the top level numerical bisection routine.

that when preferences are additively separable (i.e., $U(\boldsymbol{x}, z) = \sum_i u_i(x_i) + u_z(z)$) the first order conditions that implicitly define the solution to the consumer's constrained minimization problem are:

$$\text{(3A)} \quad p_i \le \frac{\partial u_i(x_i^*)}{\partial x_i} \Big/ \frac{\partial u_z(z^*)}{\partial z} \ \& \ x_i^* > 0, \forall i$$

$$\text{(4A)} \quad \bar{U} = \sum_i u_i(x_i^*) + u_z(z^*)$$

The structure of equation (3A) is such that each of the *N* first order conditions depends on two endogenous arguments – x_i^* and z^*. This simple form suggests that if the analyst knew the optimal *z*, she could use (3A) to solve conditionally for $x_i^*, \forall i$. Thus, *N* + 1 dimensional constrained minimization problem collapses into a one dimensional search for z^*. The following numerical bisection routine solves for z^*:

i) At iteration *j*, set $z_a^j = (z_l^{j-1} + z_u^{j-1})/2$. To initialize the algorithm, set $z_l^0 = 0$ and $z_u^0 = u_z^{-1}(\bar{U} - \sum_i u_i(0))$.
ii) Conditional on z_a^j, solve for $x_i^j, \forall i$ using (3A).
iii) Solve for $\tilde{U}^j = U(\boldsymbol{x}^j, z_a^j)$ using (4A).
iv) If $\tilde{U}^j < \bar{U}$, set $z_l^j = z_a^j$ and $z_u^j = z_u^{j-1}$. Otherwise, set $z_l^j = z_l^{j-1}$ and $z_u^j = z_a^j$.
v) Iterate until $abs(z_l^j - z_u^j) \le c$ where *c* is arbitrarily small.

Similar to von Haefen, Phaneuf, and Parsons' numerical bisection routine for solving the consumer's constrained maximization problem, the consistency of the algorithm depends critically on the strict concavity of preferences. By totally differentiating (3A), it is straightforward to show that $dx_i / dz \ge 0$, which implies that if $z_a^j > z^*$, $U(\boldsymbol{x}^j, z_a^j) > \bar{U}$. Thus by updating the lower and upper bounds according to the rules in iv), z_a^j will eventually converge to z^*.

Finally for part c), the analyst must chose *S*, the number of simulations used to construct $E(CS^{H} \mid \tilde{\tau}, \tilde{\delta}, \tilde{\theta}^{*}, \tilde{\mu}^{*})$. Through experimentation with alternative values for *S*, it was found that as few as four simulations per set of individual level parameters was sufficient to generate mean population estimates that did not significantly change with additional simulations.